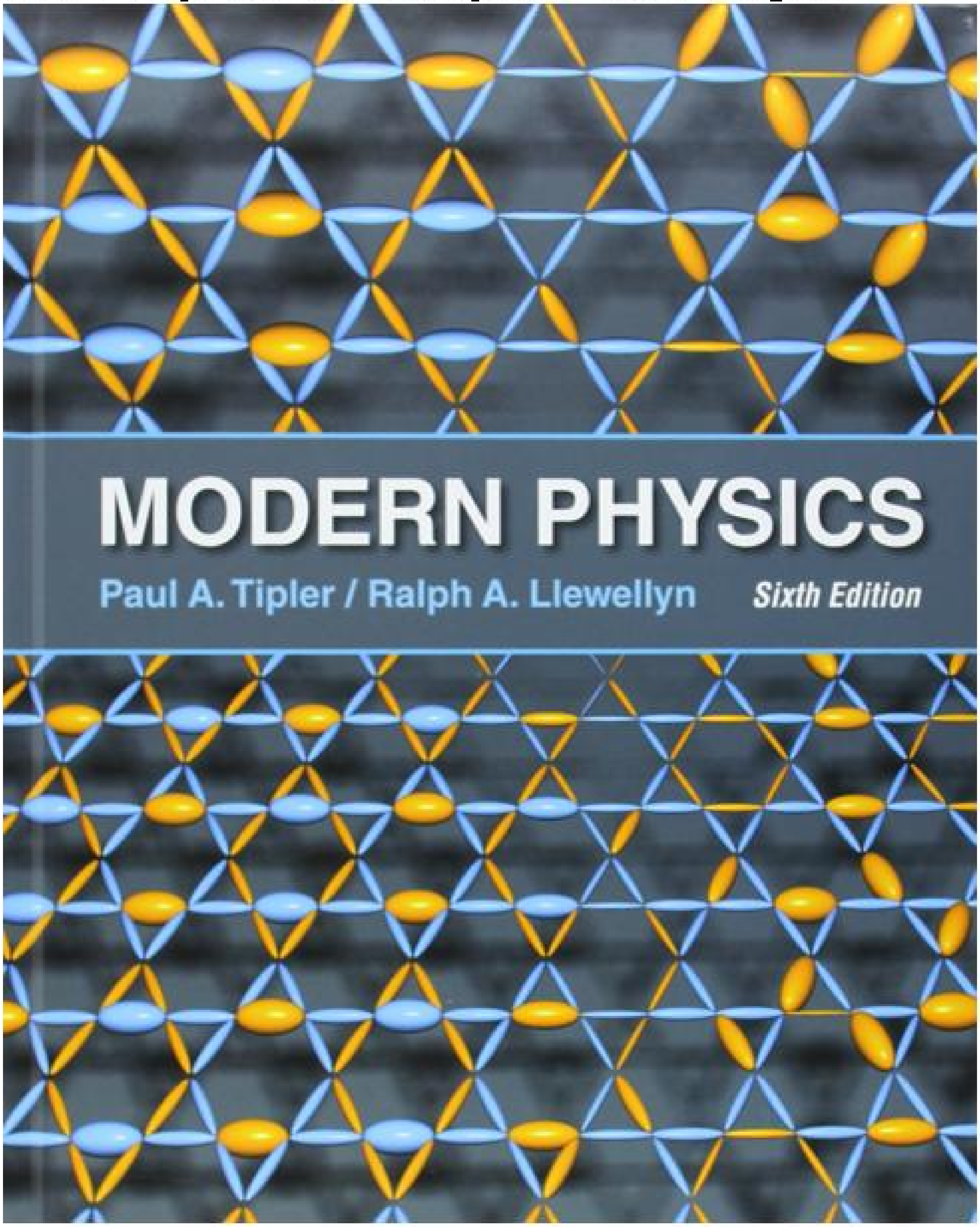
Solutions Manual

Modern Physics

6th Edition

Paul A. Tipler & Ralph Llewellyn



Instructor Solutions Manual

for

Modern Physics

Sixth Edition

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Preface

This book is an Instructor Solutions Manual for the problems which appear in *Modern Physics, Sixth Edition* by Paul A. Tipler and Ralph A. Llewellyn. This book contains solutions to every problem in the text and is not intended for class distribution to students. A separate Student Solutions Manual for *Modern Physics, Sixth Edition* is available from W. H. Freeman and Company. The Student Solutions Manual contains solutions to selected problems from each chapter, approximately one-fourth of the problems in the book.

Figure numbers, equations, and table numbers refer to those in the text. Figures in this solutions manual are not numbered and correspond only to the problem in which they appear. Notation and units parallel those in the text.

Please visit W. H. Freeman and Company's website for *Modern Physics, Sixth Edition* at www.whfreeman.com/tiplermodernphysics6e. There you will find 30 More sections that expand on high interest topics covered in the textbook, the Classical Concept Reviews that provide refreshers for many classical physics topics that are background for modern physics topics in the text, and an image gallery for Chapter 13. Some problems in the text are drawn from the More sections.

Every effort has been made to ensure that the solutions in this manual are accurate and free from errors. If you have found an error or a better solution to any of these problems, please feel free to contact me at the address below with a specific citation. I appreciate any correspondence from users of this manual who have ideas and suggestions for improving it.

Sincerely,

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Chapter 1 – Relativity I

1-1. (a) Speed of the droid relative to Hoth, according to Galilean relativity, u_{Hoth}, is

$$\begin{aligned} u_{Hoth} &= u_{spaceship} + u_{droid} \\ &= 2.3\times10^8 m/s + 2.1\times10^8 m/s \\ &= 4.4\times10^8\ m/s \end{aligned}$$

(b) No, since the droid is moving faster than light speed relative to Hoth.

1-2. (a) $t = \dfrac{2L}{c} = \dfrac{2\left(2.74\times10^4 m\right)}{3.00\times10^8 m/s} = 1.83\times10^{-4} s$

(b) From Equation 1-6 the correction $\delta t = \dfrac{2L}{c}\times\dfrac{v^2}{c^2}$

$$\delta t = \left(1.83\times10^{-4} s\right)\left(10^{-4}\right)^2 = 1.83\times10^{-12} s$$

(c) From experimental measurements $\dfrac{\delta c}{c} = \dfrac{4\,km/s}{299,796\,km/s} = 1.3\times10^{-5}$

No, the relativistic correction of order 10^{-8} is three orders of magnitude smaller than the experimental uncertainty.

1-3. $\dfrac{0.4\text{ fringe}}{\left(29.8 km/s\right)^2} = \dfrac{1.0\text{ fringe}}{\left(v\ km/s\right)^2} \to v^2 = \dfrac{1.0}{0.4}\left(29.9\ km/s\right)^2 = 2.22\times10^3 \to v = 47.1\,km/s$

1-4. (a) This is an exact analog of Example 1-1 with *L* = 12.5 *m*, *c* = 130 *mph*, and *v* = 20 *mph*. Calling the plane flying perpendicular to the wind plane #1 and the one flying parallel to the wind plane #2, plane #1 win will by Δt where

$$\Delta t = \frac{Lv^2}{c^3} = \frac{\left(12.5mi\right)\left(20mi/h\right)^2}{\left(130mi/h\right)^3} = 0.0023h = 8.2s$$

(b) Pilot #1 must use a heading $\theta = \sin^{-1}\left(20/130\right) = 8.8°$ relative to his course on both legs. Pilot #2 must use a heading of $0°$ relative to the course on both legs.

1-5. (a) In this case, the situation is analogous to Example 1-1 with $L = 3\times10^8 m$, $v = 3\times10^4 m/s$, and $c = 3\times10^8 m/s$ If the flash occurs at $t = 0$, the interior is dark until $t = 2s$ at which time a bright circle of light reflected from the circumference of the great circle plane perpendicular to the direction of motion reaches the center, the circle splits in two, one moving toward the front and the other moving toward the rear, their radii decreasing to just a point when they reach the axis 10^{-8} *s* after arrival of the first reflected light ring. Then the interior is dark again.

(b) In the frame of the seated observer, the spherical wave expands outward at *c* in all directions. The interior is dark until $t = 2s$ at which time the spherical wave (that reflected from the inner surface at $t = 1s$) returns to the center showing the entire inner surface of the sphere in reflected light, following which the interior is dark again.

1-6. Yes, you will see your image and it will look as it does now. The reason is the second postulate: All observers have the same light speed. In particular, you and the mirror are in the same frame. Light reflects from you to the mirror at speed *c* relative to you and the mirror and reflects from the mirror back to you also at speed *c*, independent of your motion.

1-7. $\Delta N = \dfrac{2Lv^2}{\lambda c^2}$ (Equation 1-10) Where $\lambda = 590\ nm$, $L = 11\ m$, and $\Delta N = 0.01\ fringe$

$$v^2 = \frac{\Delta N \lambda c^2}{2L} = (0.01\,fringe)\left(590\times10^{-9} m\right)\left(3.00\times10^8 m/s\right)^2 2(11m)$$

$$v = 4.91\times10^3 m/s \approx 5\,km/s$$

1-8. (a) No. Results depends on the relative motion of the frames.

(b) No. Results will depend on the speed of the proton relative to the frames. (This answer anticipates a discussion in Chapter 2. If by “mass”, the “rest mass” is implied, then the answer is “yes”, because that is a fundamental property of protons.)

(Problem 1-8 continued)

(c) Yes. This is guaranteed by the 2^{nd} postulate.

(d) No. The result depends on the relative motion of the frames.

(e) No. The result depends on the speeds involved.

(f) Yes. Result is independent of motion.

(g) Yes. The charge is an intrinsic property of the electron, a fundamental constant.

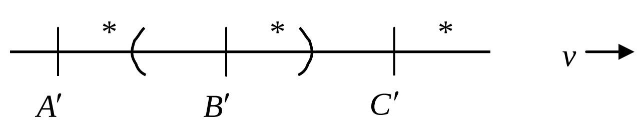
1-9. The wave from the front travels 500 *m* at speed c + (150/3.6) *m/s* and the wave from the rear travels at c – (150/3.6) *m/s*. As seen in Figure 1-14, the travel time is longer for the wave from the rear.

$$\Delta t = t_r - t_f = \frac{500m}{3.00\times10^8 m/s - (150/3.6)m/s} - \frac{500m}{3.00\times10^8 m/s + (150/3.6)m/s}$$

$$= 500\left[\frac{3\times10^8 + (150/3.6) - 3\times10^8 + (150/3.6)}{(3\times10^8) \ -2(150/3.6)(3\times10^8) - (150/3.6)^2}\right]$$

$$= 500\frac{2(150/3.6)}{(3\times10^8)^2} \approx 4.63\times10^{-13}s$$

1-10.



While the wavefront is expanding to the position shown, the original positions of A', B', and C' have moved to the * marks, according to the observer in *S*.

(a) According to an S' observer, the wavefronts arrive simultaneously at A' and B'.

(b) According to an *S* observer, the wavefronts do not arrive at A' and C' simultaneously.

(c) The wavefront arrives at A' first, according to the *S* observer, an amount Δt before arrival at C', where

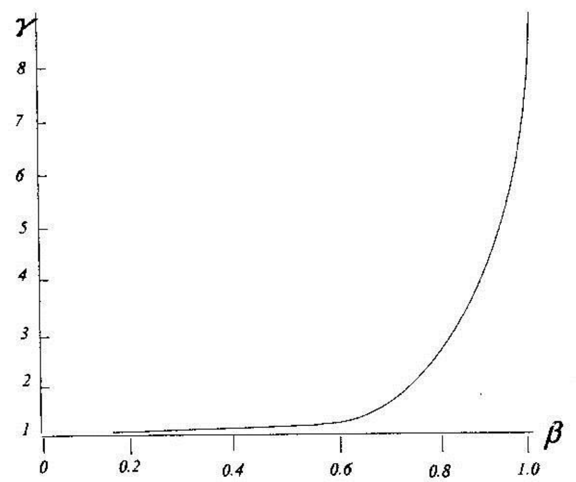
(Problem 1-10 continued)

$$\Delta t = \frac{B'C'}{c-v} - \frac{B'A'}{c+v} \quad \text{since } B'C' = B'A' = L', \text{ Thus}$$

$$\Delta t = L'\left[\frac{c+v-c+v}{c^2-v^2}\right] = L'\left[\frac{2v}{c^2-v^2}\right]$$

1-11.

β	$\gamma = 1/\left(1-\beta^2\right)^{1/2}$
0	1
0.2	1.0206
0.4	1.0911
0.6	1.2500
0.8	1.6667
0.85	1.8983
0.90	2.2942
0.925	2.6318
0.950	3.2026
0.975	4.5004
0.985	5.7953
0.990	7.0888
0.995	10.0125



1-12. $t_1 = \gamma\left(t_1' + \frac{vx_0'}{c^2}\right) \qquad t_2 = \gamma\left(t_2' + \frac{vx_0'}{c^2}\right)$ (from Equation 1-19)

(a) $t_2 - t_1 = \gamma\left(t_2' + \frac{vx_0'}{c^2} - t_1' - \frac{vx_0'}{c^2}\right) = \gamma\left(t_2' - t_1'\right)$

(b) The quantities x_1' and x_2' in Equation 1-19 are each equal to x_0', but x_1 and x_2 in Equation 1-18 are different and unknown.

1-13. (a)

$$\gamma = 1/\left(1-v^2/c^2\right)^{1/2} = 1/\left[1-(0.85c)^2/c^2\right]^{1/2} = 1.898$$

$$x' = \gamma(x-vt) = 1.898\left[75m - (0.85c)\left(2.0\times10^{-5}s\right)\right] = -9.537\times10^{3}m$$

$$y' = y = 18m$$

$$z' = z = 4.0m$$

$$t' = \gamma\left(t - vx/c^2\right) = 1.898\left[2.0\times10^{-5}s - (0.85c)(75m)/c^2\right] = 3.756\times10^{-5}s$$

(Problem 1-13 continued)

(b) $x = \gamma\left(x' + vt'\right) = 1.898\left[-9.537\times10^{3}m + \left(0.85c\right)\left(3.756\times10^{-5}s\right)\right] = 75.8m$

difference is due to rounding of γ, x', and t'.

$y = y' = 18m$

$z = z' = 4.0m$

$t = \gamma\left(t' + vx'/c^2\right) = 1.898\left[3.756\times10^{-5}s + \left(0.85c\right)\left(-9.537\times10^{3}m\right)/c^2\right] = 2.0\times10^{-5}s$

1-14. To show that $\Delta t = 0$ (refer to Figure 1-8 and Example 1-1).

$$t_1 = \frac{L}{\sqrt{c^2 - v^2}} + \frac{L}{\sqrt{c^2 - v^2}} = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2L}{c}\frac{1}{\sqrt{1 - v^2/c^2}}$$

t_2, because length parallel to motion is shortened, is given by:

$$t_2 = \frac{L\sqrt{1 - v^2/c^2}}{c + v} + \frac{L\sqrt{1 - v^2/c^2}}{c - v} = \frac{2Lc\sqrt{1 - v^2/c^2}}{c^2\left(1 - v^2/c^2\right)}$$

$$t_2 = \frac{2L}{c}\frac{\sqrt{1 - v^2/c^2}}{\left(\sqrt{1 - v^2/c^2}\right)^2} = \frac{2L}{c}\frac{1}{\sqrt{1 - v^2/c^2}} = t_1$$

Therefore, $t_2 - t_1 = 0$ and no fringe shift is expected.

1-15. (a) Let frame S be the rest frame of Earth and frame S' be the spaceship moving at speed v to the right relative to Earth. The other spaceship moving to the left relative to Earth at speed u is the "particle". Then $v = 0.9c$ and $u_x = -0.9c$.

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2} \quad \text{(Equation 1-22)}$$

$$= \frac{-0.9c - 0.9c}{1 - \left(-0.9c\right)\left(0.9c\right)/c^2} = \frac{-1.8c}{1.81} = -0.9945c$$

(b) Calculating as above with $v = 3.0\times10^{4}m/s = -u_x$

$$u'_x = \frac{-3.0\times10^{4}m/s - 3.0\times10^{4}m/s}{1 - \frac{\left(-3.0\times10^{4}m/s\right)\left(3.0\times10^{4}m/s\right)}{\left(3.0\times10^{8}m/s\right)^2}} = \frac{-6.0\times10^{4}m/s}{1 + 10^{-8}} = -6.0\times10^{4}m/s$$

1-16. $a'_x = \dfrac{du'_x}{dt'}$ where $u'_x = \dfrac{u_x - v}{1 - u_x v/c^2}$ (Equation 1-22)

And $t' = \gamma\left(t - vx/c^2\right)$ (Equation 1-18)

$$du'_x = \left(u_x - v\right)\left(v du_x / c^2\right)\left(1 - u_x v / c^2\right)^{-2} + \left(1 - u_x v / c^2\right)^{-1} du_x$$

$$= \frac{\left(\frac{v}{c^2}\right)\left(u_x - v\right)du_x + \left(1 - u_x v / c^2\right)du_x}{\left(1 - u_x v / c^2\right)^2}$$

$$dt' = \gamma\left(dt - v dx / c^2\right)$$

$$a'_x = \frac{du'_x}{dt'} = \frac{\left(\frac{v}{c^2}\right)\left(u_x - v\right)\left(du_x / dt\right) + \left(1 - u_x v / c^2\right)\left(du_x / dt\right)}{\gamma\left(1 - u_x v / c^2\right)^3}$$

$$= \frac{\left(du_x / dt\right)\left(1 - v^2 / c^2\right)}{\gamma\left(1 - u_x v / c^2\right)^3} = \frac{a_x}{\gamma^3\left(1 - u_x v / c^2\right)^3}$$

$a'_y = \dfrac{du'_y}{dt'}$ where $u'_y = \dfrac{u_y}{\gamma\left(1 - u_x v / c^2\right)}$ (Equation 1-22)

$$du'_y = \left(du_y / \gamma\right)\left(1 - u_x v / c^2\right)^{-1} + \left(u_y / \gamma\right)\left(1 - u_x v^2 / c^2\right)^{-2} du_x$$

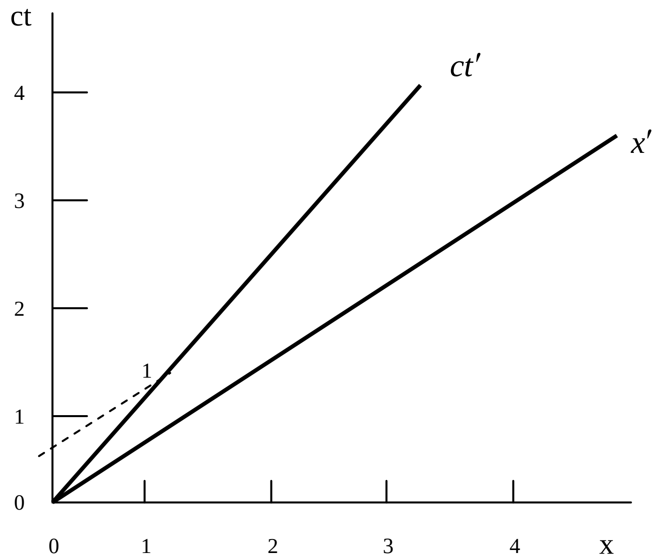
$$= \frac{\left(du_y\right)\left(1 - u_x v / c^2\right) + \left(u_y v / c^2\right)du_x}{\gamma\left(1 - u_x v / c^2\right)^2}$$

$$a'_y = \frac{du'_y}{dt'} = \frac{\left(du_y / dt\right)\left(1 - u_x v / c^2\right) + \left(u_y v / c^2\right)\left(du_x / dt\right)}{\gamma\left(1 - u_x v / c^2\right)^2 \gamma\left(1 - u_x v / c^2\right)}$$

$$= \frac{a_y\left(1 - u_x v / c^2\right) + a_x\left(u_y v / c^2\right)}{\gamma^2\left(1 - u_x v / c^2\right)^3}$$

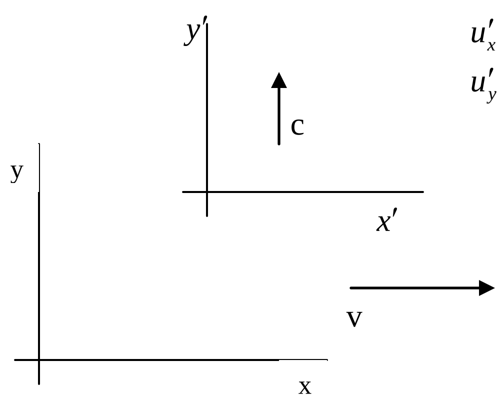
a'_z is found in the same manner and is given by: $a'_z = \dfrac{a_z\left(1 - u_x v / c^2\right) + a_x\left(u_z v / c^2\right)}{\gamma^2\left(1 - u_x v / c^2\right)^3}$

1-17. (a) As seen from the diagram, when the observer in the rocket (S') system sees 1 $c{\cdot}s$ tick by on the rocket's clock, only 0.6 $c{\cdot}s$ have ticked by on the laboratory clock.



(b) When 10 seconds have passed on the rocket's clock, only 6 seconds have passed on the laboratory clock.

1-18. (a)



$$u'_x = 0$$
$$u'_y = 0$$

$$u_x = \frac{u'_x + v}{1 + v u'_x / c^2} = \frac{0 + v}{1 + 0} = v \qquad \text{(Equation 1-23)}$$

$$u_y = \frac{u'_y}{\gamma\left(1 + v u'_x / c^2\right)} = \frac{c}{\gamma(1+0)} = \frac{c}{\gamma}$$

(b) $u = \sqrt{u_x^2 + u_y^2} = \sqrt{v^2 + c^2/\gamma^2} = \sqrt{v^2 + c^2\left(1 - v^2/c^2\right)} = c$

1-19. By analogy with Equation 1-23,

(a) $$u_x' = \frac{u_x' + v}{1 + vu_x'/c^2} = \frac{0.9c + 0.9c}{1 + (0.9c)(0.9c)/c^2} = \frac{1.8c}{1.81} = 0.9945c$$

(b) $$u_x = \frac{u_x' + v}{1 + vu_x'/c^2} = \frac{(1.8c/1.81) + 0.9c}{1 + (1.8c/1.81)(0.9c)/c^2} = \frac{1.8 + (0.9)(1.81)}{1.81 + (1.8)(0.9)}c = \frac{3.429}{3.430}c = 0.9997c$$

1-20 (a) $$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad \text{(Equation 1-16)}$$

$$= (1 - v^2/c^2)^{-1/2} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{v^2}{c^2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\frac{1}{2!}\left(-\frac{v^2}{c^2}\right)^2 + \cdots$$

$$= 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^2} + \cdots \approx 1 + \frac{1}{2}\frac{v^2}{c^2}$$

(b) $$\frac{1}{\gamma} = \sqrt{1 - v^2/c^2} = (1 - v^2/c^2)^{1/2}$$

$$= 1 + \left(\frac{1}{2}\right)\left(-\frac{v^2}{c^2}\right) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\frac{1}{2!}\left(-\frac{v^2}{c^2}\right)^2 + \cdots$$

$$= 1 - \frac{1}{2}\frac{v^2}{c^2} - \frac{1}{8}\frac{v^4}{c^2} + \cdots \approx 1 - \frac{1}{2}\frac{v^2}{c^2}$$

(c) $$\gamma - 1 = \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots \qquad 1 - \frac{1}{\gamma} = \frac{1}{2}\frac{v^2}{c^2} + \frac{1}{8}\frac{v^4}{c^4}$$

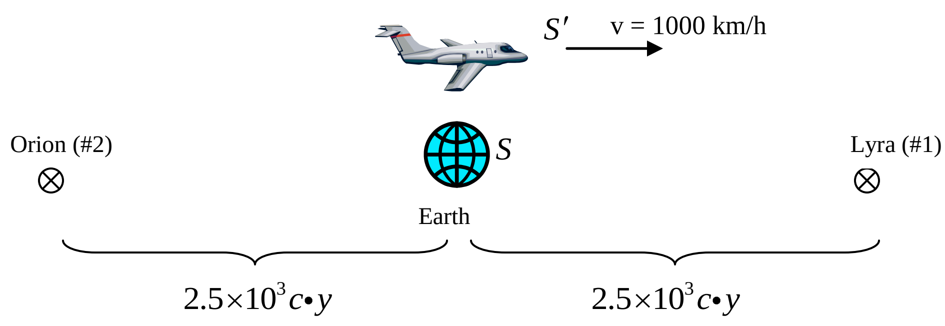
$$\therefore \quad \gamma - 1 = 1 - \frac{1}{\gamma} \approx \frac{1}{2}\frac{v^2}{c^2}$$

1-21. $$\Delta t = \gamma \Delta t' \qquad \text{(Equation 1-26)}$$

$$\frac{\Delta t - \Delta t'}{\Delta t'} = \frac{\gamma \Delta t' - \Delta t'}{\Delta t'} = \gamma - 1 \approx \frac{1}{2}\frac{v^2}{c^2} = \gamma - 1 \approx \frac{1}{2}\frac{v^2}{c^2}$$

$$v^2 = 2c^2\,\frac{\Delta t - \Delta t'}{\Delta t'} \qquad v = c\left(2 \times \frac{\Delta t - \Delta t'}{\Delta t'}\right)^{1/2} = c(2 \times 0.01)^{1/2} = 0.14c$$

1-22.



(a) Note that $\gamma = 1/\sqrt{1-v^2/c^2} \approx 1$ and $1\,c\bullet y = c\bullet\left(3.15\times10^7 s\right)$

From Equation 1-27: $\Delta t = t_2 - t_1 = 0$, since the novas are simultaneous in system S (Earth). Therefore, in S' (the aircraft)

$$\Delta t' = t_2' - t_1' = -\frac{v}{c^2}\left(x_2' - x_1'\right)$$

$$= -\frac{10^6 m/h}{\left(3600s/h\right)c^2}\left(-2.5\times10^3 - 2.5\times10^3\right)(c)\left(3.15\times10^7 s\right)$$

$$= 1.46\times10^5 s = 40.5h$$

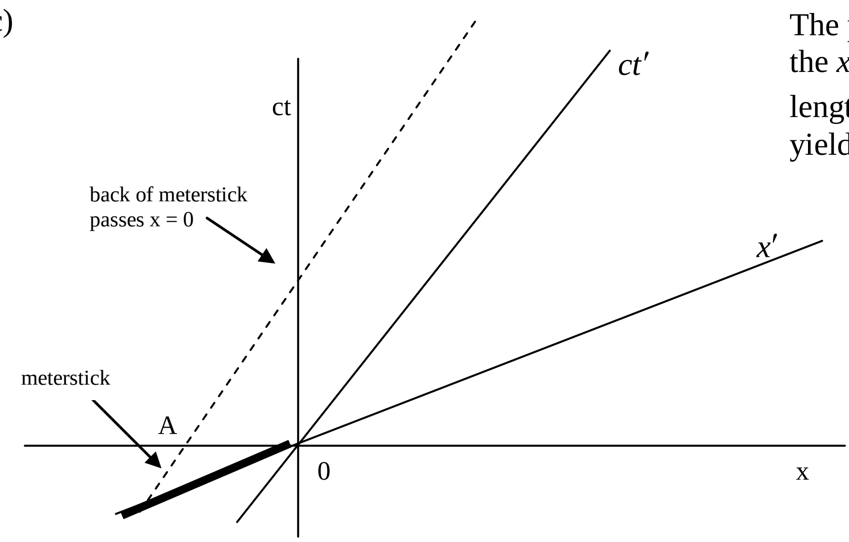
(b) Since $\Delta t'$ is positive, $t_2' > t_1'$; therefore, the nova in Lyra is detected on the aircraft before the nova in Orion.

1-23. (a) $L = L_p/\gamma$ (Equation 1-28)

$$= L_p\sqrt{1-v^2/c^2} = 1.0m\left[1-\left(0.6c\right)^2/c^2\right]^{1/2} = 0.80m$$

(b) $t = L/v = 0.80m/0.6c = 4.4\times10^{-9} s$

(c)



The projection $\overline{OA}$ on the x axis is L. The length $\overline{OB}$ on the ct axis yields t.

1-24. (a) $\Delta t = \gamma \Delta t' = \dfrac{\Delta t'}{\sqrt{1-v^2/c^2}}$ (Equation 1-26)

$$= \frac{2.6\times10^{-8}s}{\left[1-(0.9c)^2/c^2\right]^{1/2}} = \frac{2.6\times10^{-8}s}{\sqrt{0.19}} = 5.96\times10^{-8}s$$

(b) $s = v\Delta t = (0.9)(3.0\times10^8 m/s)(6.0\times10^{-8}s) = 16.1m$

(c) $s = v\Delta t = (0.9)(3.0\times10^8 m/s)(2.6\times10^{-8}s) = 7.0m$

(d) $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$ (Equation 1-31)

$$= \left[c\left(6.0\times10^{-8}\right)\right]^2 - (16.1m)^2 = 324 - 259 = 65 \quad\rightarrow\quad \Delta s = 7.8m$$

1-25. From Equation 1-28, $L = L_p/\gamma = L_p\sqrt{1-v^2/c^2}$ where $L = 85m$ and $L_p = 100m$

$\sqrt{1-v^2/c^2} = L/L_p = 85/100$

Squaring $1-v^2/c^2 = (85/100)^2$

$\therefore \quad v^2 = \left[1-(85/100)^2\right]c^2 = 0.2775c^2$ and $v = 0.527c = 1.58\times10^8 m/s$

1-26. (a) In the spaceship the length L = the proper length L_p; therefore, $t_s = \dfrac{L_p}{c} + \dfrac{L_p}{c} = \dfrac{2L_p}{c}$

(b) In the laboratory frame the length is contracted to $L = L_p/\gamma$ and the round trip time is

$$t_L = \frac{L}{c-v} + \frac{L}{c+v} = \frac{L}{c\left(1-v^2/c^2\right)}$$

$$= \frac{2L_p/\gamma}{c\left(1-v^2/c^2\right)} = \frac{2L_p\sqrt{1-v^2/c^2}}{c\left(\sqrt{1-v^2/c^2}\right)^2} = \frac{2L_p}{c\sqrt{1-v^2/c^2}}$$

(c) Yes. The time t_s measured in the spaceship is the proper time interval τ. From time dilation (Equation 1-26) the time interval in the laboratory $t_L = \gamma\tau$; therefore,

$t_L = \dfrac{1}{\sqrt{1-v^2/c^2}}\dfrac{2L_p}{c}$ which agrees with (b).

1-27. Using Equation 1-28, with L_{A_p} and L_{B_p} equal to the proper lengths of A and B and L_A = length of A measured by B and L_B = length of B measured by A.

$$L_A = L_{A_p} / \gamma = 100m\sqrt{1-(0.92c)^2} = 39.2\,m$$
$$L_{B_p} = \gamma L_B = 36 / \sqrt{1-(0.92c)^2 / c^2} = 91.9m$$

1-28. In $S' : \Delta x' = 1.0m\cos 30° = 0.866m$

$\Delta y' = 1.0m\sin 30° = 0.500m$ where $\theta' = 30°$

In S : $\Delta x = \Delta x'\sqrt{1-\beta^2} = 0.866m\sqrt{1-(0.8)^2} = 0.520m$

$\Delta y = \Delta y' = 0.500m$

where $\theta = \tan^{-1}\dfrac{0.500}{0.520} = 43.9°$

$$L = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(0.520m)^2 + (0.500)^2} = 0.721m$$

1-29. (a) In $S' : V' = a' \times b' \times c' = (2m)(2m)(4m) = 16m^3$

In S: Both a' and c' have components in the x' direction.

$a'_x = a'\sin 25° = (2m)\sin 25° = 0.84m$ and $c'_x = c'\cos 25° = (4m)\cos 25° = 3.63m$

$a_x = a'_x\sqrt{1-\beta^2} = 0.84\sqrt{1-(0.65)^2} = 0.64m$

$c_x = c'_x\sqrt{1-\beta^2} = 3.634\sqrt{1-(0.65)^2} = 2.76m$

$a_y = a'_y = a'\cos 25° = 2\cos 25° = 1.81m$ and $c_y = c'_y = c'\sin 25° = 4\sin 25° = 1.69m$

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(0.64)^2 + (1.81)^2} = 1.92m$

$c = \sqrt{c_x^2 + c_y^2} = \sqrt{(2.76)^2 + (1.69)^2} = 3.24m$

b' (in z direction) is unchanged, so $b = b' = 2m$

θ (between c and xy-plane) $= \tan^{-1}(1.69/2.76) = 31.5°$

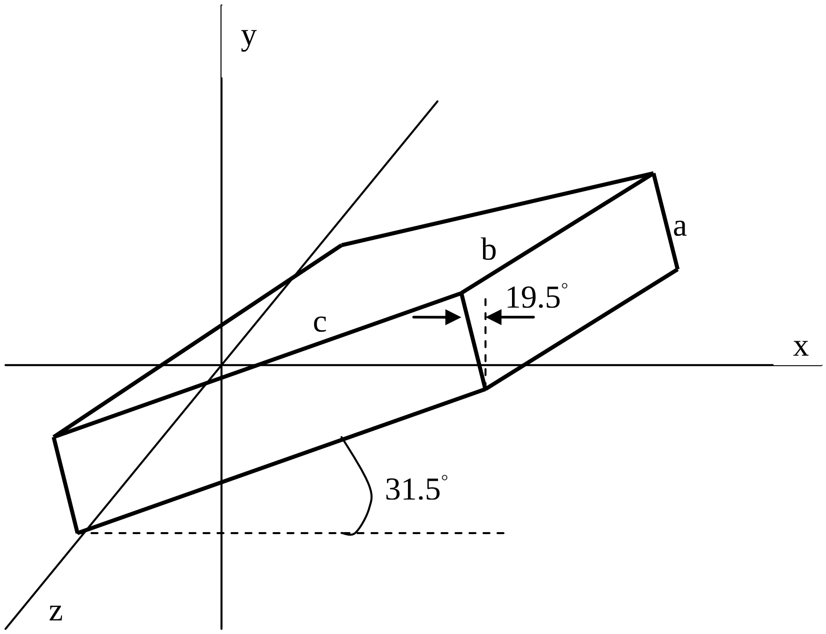
ϕ (between a and yz-plane) $= \tan^{-1}(0.64/1.81) = 19.5°$

$V =$ (area of ay face) $\bullet\, b$ (see part[b])

$V = (c \times a\sin 78°) \times b = (3.24m)(1.92m\ \sin 78°)(2m) = 12.2m^3$

(Problem 1-29 continued)

(b)



1-30. $$\lambda' = \frac{c}{f'} = \frac{c}{f_o\sqrt{\frac{1+\beta}{1-\beta}}} = \sqrt{\frac{1-v/c}{1+v/c}}\lambda_o \quad \text{(Equation 1-36)}$$

$$\left(\frac{\lambda'}{\lambda_o}\right)^2 = \frac{1-v/c}{1+v/c} \quad \rightarrow \quad \left(\frac{\lambda'}{\lambda_o}\right)^2 (1+v/c) = 1-v/c$$

Solving for v/c, $$\frac{v}{c}\left[\left(\frac{\lambda'}{\lambda_o}\right)^2 + 1\right] = 1 - \left(\frac{\lambda'}{\lambda_o}\right)^2 \quad \therefore \frac{v}{c} = \frac{1-(\lambda'/\lambda_o)^2}{1+(\lambda'/\lambda_o)^2}$$

$\lambda_o = 650nm$. For yellow $\lambda' = 590nm$. $$\frac{v}{c} = \frac{1-(590nm/650nm)^2}{1+(590nm/650nm)^2} = 0.097$$

Similarly, for green $\lambda' = 525nm \quad \rightarrow \quad \frac{v}{c} = 0.210$

and for blue $\lambda' = 460nm \quad \rightarrow \quad \frac{v}{c} = 0.333$

1-31. $$\lambda' = \frac{c}{f'} = \frac{c}{\frac{\sqrt{1-\beta}}{1+\beta} f_o} = \sqrt{\frac{1+v/c}{1-v/c}}\lambda_o \quad \text{(Equation 1-37)}$$

$$\frac{\lambda' - \lambda_o}{\lambda_o} = \frac{\lambda'}{\lambda_o} - 1 = \sqrt{\frac{1+v/c}{1-v/c}} - 1 = \left[\frac{1+\left(1.85\times10^7\, m/s\right)/\left(3.00\times10^8\, m/s\right)}{1-\left(1.85\times10\;\; m/s\right)\left(3.00\times10^8\, m/s\right)}\right]^{1/2} - 1 = 0.064$$

1-32. Because the shift is a blue shift, the star is moving toward Earth.

$$f = \sqrt{\frac{1+\beta}{1-\beta}} f_o \quad \text{where } f = 1.02 f_o$$

$$\left(1.02\right)^2 = \frac{1+\beta}{1-\beta} \quad \rightarrow \quad \beta = \frac{\left(1.02\right)^2 - 1}{\left(1.02\right)^2 + 1} = 0.0198$$

$$v = 0.0198c = 5.94\times10^6\, m/s$$

1-33. $$f = \sqrt{\frac{1-\beta}{1+\beta}} f_o \quad \rightarrow \quad \lambda = \sqrt{\frac{1+\beta}{1-\beta}}\lambda_o = \sqrt{\frac{1+\beta}{1-\beta}}\left(656.3nm\right)$$

For $\beta = 10^{-3}$: $\quad \lambda = \left(656.3nm\right)\sqrt{\frac{1+10^{-3}}{1-10^{-3}}} = 657.0nm$

For $\beta = 10^{-2}$: $\quad \lambda = \left(656.3nm\right)\sqrt{\frac{1+10^{-2}}{1-10^{-2}}} = 662.9nm$

$\beta = 10^{-1}$: $\quad \lambda = \left(656.3nm\right)\sqrt{\frac{1+10^{-1}}{1-10^{-1}}} = 725.6nm$

1-34. Let S be the rest frame of Earth, S' be Heidi's rest frame, and S'' be Hans' rest frame.

$$\gamma_{S'} = \frac{1}{\sqrt{1-\left(0.45c/c\right)^2}} = 1.1198$$

$$\gamma_{S''} = \frac{1}{\sqrt{1-\left(0.95c/c\right)^2}} = 3.206$$

When they meet, each will have traveled a distance d from Earth.

Heidi: $d = 0.45c\, t_{Heidi}$

Hans: $d = 0.95c\, t_{Hans}$

and $t_{Hans} = t_{Heidi} - 1$ in years.

Therefore, $\left(0.95c - 0.45c\right)t_{Heidi} = 0.95c$ and $t_{Heidi} = 1.90y$; $t_{Hans} = 0.90y$

(a) In her reference frame S', Heidi has aged $\Delta t'$ when she and Hans meet.

(Problem1-34 continued)

$$\Delta t' = \gamma_{S'}\left(\Delta t - \frac{v}{c^2}\Delta x\right) = 1.1198\left[1.90 - \frac{0.45c}{c^2}(0.45c \times 1.90)\right]$$
$$= 1.198(1.90y)\left[1-(0.45)^2\right] = 1.697y$$

In his reference frame S'', Hans has aged $\Delta t''$ when he and Heidi meet.

$$\Delta t'' = \gamma_{S''}\left(\Delta t - \frac{v}{c^2}\Delta x\right) = 3.3026(1.90y)\left[1-(0.95)^2\right] = 0.290y$$

The difference in their ages will be $1.697 - 0.290 = 1.407y \approx 1.4y$

(b) Heidi will be the older.

1-35. Distance to moon = $3.85 \times 10^8 m = R$

Angular velocity ω needed for $v = c$:

$\omega = v/R = C/R = (3.00 \times 10^8 m/s)/(3.85 \times 10^8 m) = 0.78 rad/s$

Information could only be transmitted by modulating the beam's frequency or intensity, but the modulation could move along the beam only at speed *c*, thus arriving at the moon only at that rate.

1-36. (a) Using Equation 1-28 and Problem 1-20(b).

$\Delta t' = \Delta t/\gamma = \Delta t(1 - v^2/2c^2) = \Delta t - \Delta t v^2/2c^2$

where $\Delta t = 3.15 \times 10^7 s/y$

$v = 2\pi R_E/T = (2\pi)(6.37 \times 10^6 m)/(108 \min)(60 s/\min)$

$v = 6.177 \times 10^3 m/s = 2.06 \times 10^{-5} c$

Time lost by satellite clock = $\Delta t v^2/c^2 = (3.15 \times 10^7 s)(2.06 \times 10^{-5})^2/2 = 0.00668s = 6.68ms$

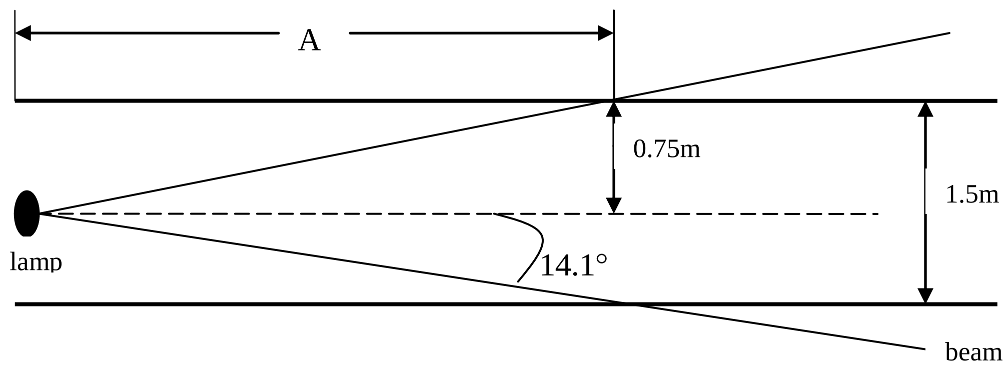
(b) $1s = \Delta t(v^2/2c^2)$

$$\Delta t = 2/(v^2/c^2) = 2/(2.06 \times 10^{-5})^2 = 4.71 \times 10^9 s = 150y$$

1-37. $\cos\theta = \dfrac{\cos\theta' + \beta}{1 + \beta\cos\theta'}$ (Equation 1-41)

where θ' = half-angle of the beam in $S' = 30°$

For $\beta = 0.65$, $\cos\theta = \dfrac{\cos 30° + 0.65}{1 + (0.65)\cos 30°} = 0.97$ or $\theta = 14.1°$



The train is A from you when the headlight disappears, where $A = \dfrac{0.75m}{\tan 14.1°} = 3.0m$

1-38. (a) $\Delta t = \gamma\Delta t_0$ For the time difference to be $1s$, $\Delta t - \Delta t_0 = 1s$

$$\Delta t - \Delta t/\gamma = 1 \quad \rightarrow \quad \Delta t\left(1 - \frac{1}{\gamma}\right) = 1$$

Substituting $\dfrac{1}{\gamma} \approx 1 - \dfrac{1}{2}\dfrac{v^2}{c^2}$ (From Problem 1-20)

$$\Delta t\left(1 - 1 + \frac{1}{2}\frac{v^2}{c^2}\right) \approx 1 \quad \rightarrow \quad \Delta t \approx 2c^2/v^2 = 2\frac{\left(3.0\times10^8\right)^2}{\left(1.5\times10^6/3.6\times10^3\right)^2}$$

$$\approx 1.04\times10^{12}\,s \approx 32{,}000\,y$$

(b) $\Delta t - \Delta t_0 = 273\times10^{-9}\,s$. Using the same substitution as in (a).

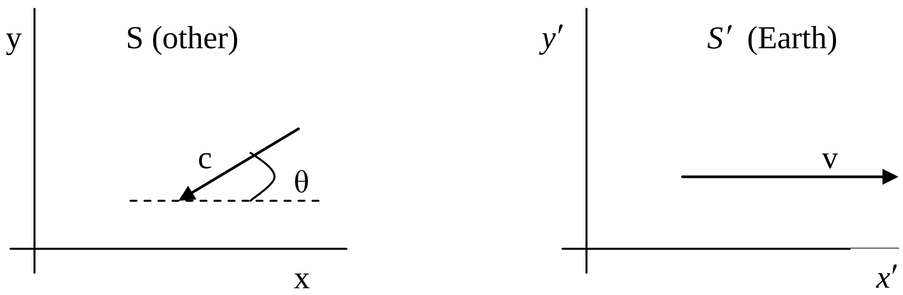
$\Delta t(1 - 1/\lambda) = 273\times10^{-9}$ and the circumference of Earth $C = 40{,}000km$, so

$4.0\times10^7 m - v\Delta t$ or $\Delta t = 4.0\times10^7/v$, and

$$4.0\times10^7/v = \left(2c^2/v^2\right)\left(273\times10^{-9}\right), \text{ or } v = \frac{2c^2\left(273\times10^{-9}\right)}{4.0\times10^7} = 1230\,m/s$$

Where v is the relative speed of the planes flying opposite directions. The speed of each plane was $(1230m/s)/2 = 615\,m/s = 2210\,km/h = 1380\,mph.$

1-39.



(a) $c_{x'} = c\cos\theta + v$

$c_{y'} = c\sin\theta$

$$\tan\theta' = \frac{c_{y'}}{c_{x'}} = \frac{c\sin\theta}{c\cos\theta + v} = \frac{\sin\theta}{\cos\theta + v/c}$$

(b) If $\theta = 90^\circ$, $\tan\theta' = \dfrac{1}{v/c} = \dfrac{1}{\beta} > 1$

$\therefore\ 90^\circ > \theta' > 45^\circ$

e.g., if $v = 0.5c$, $\theta' = 63°$

1-40. (a) Time t for information to reach front of rod is given by:

$$ct = \frac{L_p}{\gamma} + vt \quad \Rightarrow \quad t = \frac{L_p}{\gamma(c-v)}$$

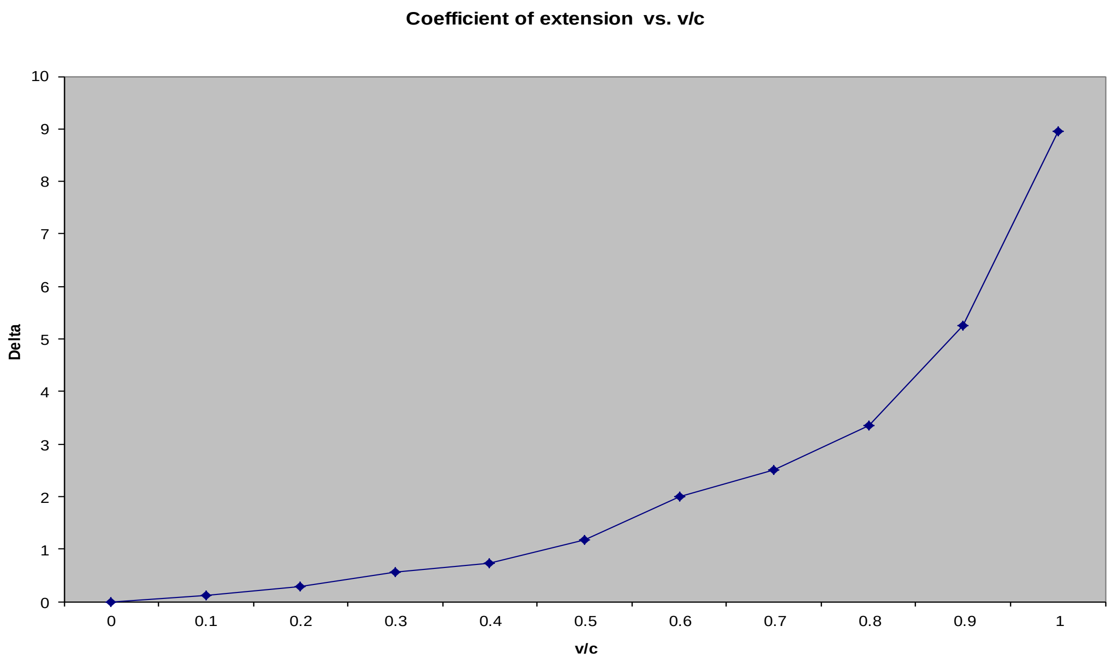
Distance information travels in time t:

$$ct = \frac{cL_p}{\gamma(c-v)} = \frac{c\sqrt{1-v^2/c^2}}{(c-v)}L_p = \frac{c\sqrt{(c-v)(c+v)/c^2}}{\sqrt{(c-v)^2}}L_p = \sqrt{\frac{c+v}{c-v}}L_p$$

Since $\sqrt{(c+v)/(c-v)} > 1$ for $v > 1$, the distance the information must travel to reach the front of the rod is $> L_p$; therefore, the rod has extended beyond its proper length.

(b) $$\Delta = \frac{1}{L_p}(ct - L_p) = \sqrt{\frac{1+v/c}{1-v/c}} - 1$$

Δ	0	0.11	0.29	0.57	0.73	1.17	2.00	2.50	3.36	5.25	8.95
v/c	0	0.10	0.25	0.40	0.50	0.65	0.80	0.85	0.90	0.95	0.98



(c) As $v \rightarrow c, \Delta \rightarrow \infty$, the maximum length of the rod $\rightarrow \infty$ also.

1-41. (a) Alpha Centauri is 4 $c{\cdot}y$ away, so the traveler went $L = \sqrt{1-\beta^2}\,(8c \cdot y)$ in $6\,y$, or

$$8c \cdot y\sqrt{1-v^2/c^2} = v(6y)$$

$$\sqrt{1-v^2/c^2} = v(6/8c) = (3/4)(v/c)$$

$$= 1-\beta^2 = (3/4)^2\,\beta^2$$

$$(3/4)^2\,\beta^2 + \beta^2 = 1$$

$$\beta^2 = 1/(1+0.5625)$$

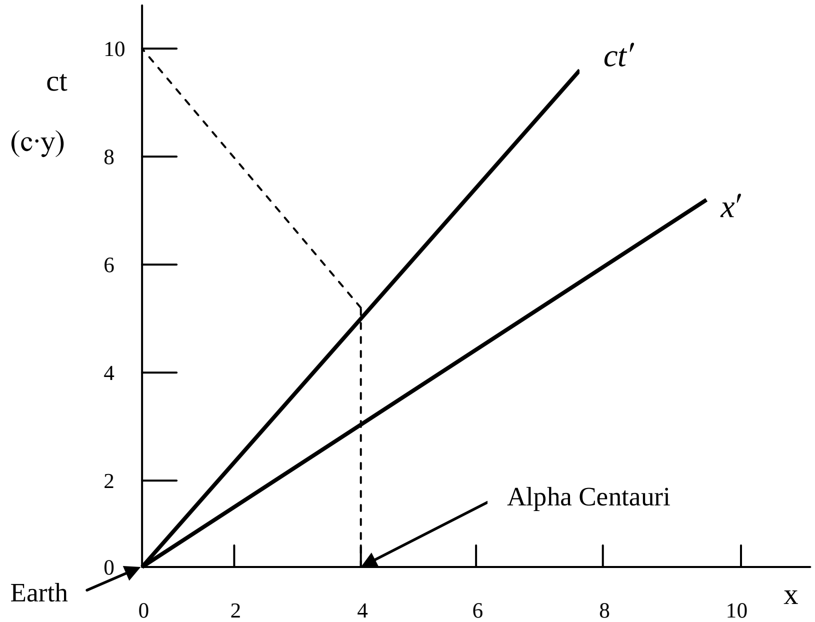
$$v = 0.8c$$

(b) $\Delta t = \gamma\Delta t_0 = \gamma(6y)$ and $\gamma = 1/\sqrt{1-\beta^2} = 1.667$

$\Delta t = 1.667(6y) = 10y$ or $4y$ older than the other traveler.

(Problem 1-41 continued)

(c)



1-42. Orbit circumference $= 4.0 \times 10^7 m$.

$$\text{Satellite speed } v = 4.0 \times 10^7 m / (90 \min \times 60 s/\min) = 7.41 \times 10^3 m/s$$

$$\Delta t - \Delta t_0 = t_{diff}$$

$$\Delta t - \Delta t/\gamma = t_{diff} = \Delta t(1 - 1/\gamma) = \Delta t\left(\frac{1}{2}\beta^2\right) \quad \text{(Problem 1-20)}$$

$$t_{diff} = (3.16 \times 10^7 s)(1/2)(7.41 \times 10^3 / 3.0 \times 10^8)^2$$

$$= 0.0096 s = 9.6 ms$$

1-43. (a) $\Delta t' = \gamma \Delta t - \frac{\gamma v}{c^2}\Delta x$ (Equation 1-20)

For events to be simultaneous in S', $\Delta t' = 0$.

$$\gamma \Delta t = \frac{\gamma v}{c^2}\Delta x \quad \rightarrow \quad 2 \times 10^{-6} s = \frac{v}{c^2}(1.5 \times 10^3 m)$$

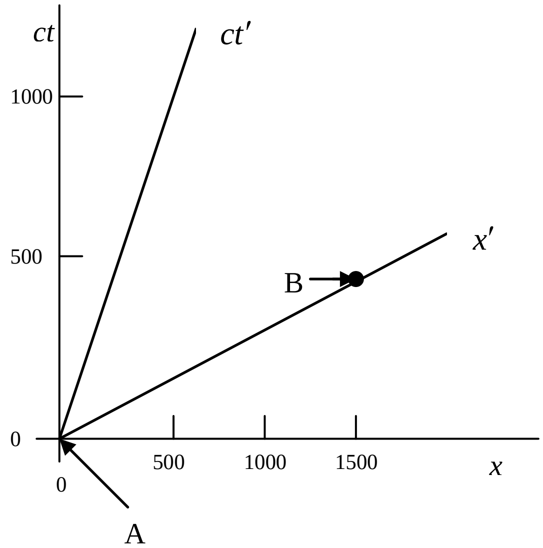
$$v = (2 \times 10^{-6} s)(3 \times 10^8 m/s) s / 1.5 \times 10^3 m$$

$$= 1.2 \times 10^8 m/s = 0.4c$$

(b) Yes.

(Problem 1-43 continued)

(c)



(d) $$(\Delta s)^2 = (\Delta x)^2 - (c\Delta t)^2 \quad \text{(Equation 1-33)}$$

$$= (1.5\times10^3)^2 - \left[(3\times10^8\, m/s)(2\times10^{-6}\, s)\right]^2$$

$$= 2.25\times10^6 - 3.6\times10^5 = 1.89\times10^6\, m^2$$

$$\Delta s = 1370m$$

$$L = \Delta s = 1370m$$

1-44. (a) $\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.92)^2} = 2.55$

(b) $\tau = 2.6\times10^{-8}s$ $\quad\Delta t(lab) = \gamma\tau = 2.55(2.6\times10^{-8}s) = 6.63\times10^{-8}s$

(c) $N(t) = N_0 e^{-t/\tau}$ (Equation 1-29) $L = \sqrt{1-\beta^2}L_0 = \sqrt{1-(0.92)^2}(50m) = 19.6m$

Where L is the distance in the pion system. At $0.92c$, the time to cover $19.6m$ is: $t = 19.6m/0.92c = 7.0\times10^{-8}s$. So, for N_0 = 50,000 pions initially, at the end of $50m$ in the lab, $N = (5.0\times10^4)e^{-7.0/2.6} = 3,390$

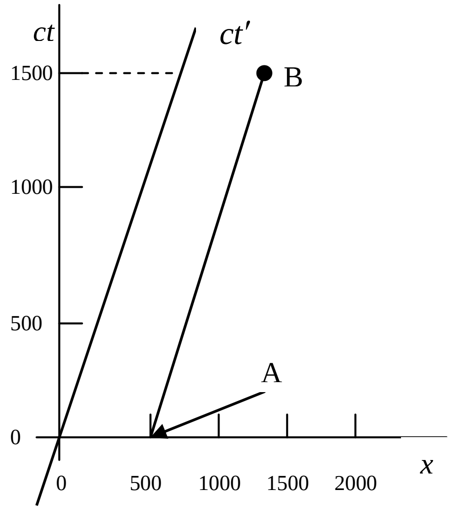
(d) 47

1-45. $\Delta L = L_p - L = L_p - L_p(1/\gamma) = L_p(1 - 1/\gamma) = L_p\left(\frac{1}{2}\frac{v^2}{c^2}\right)$ (See Problem 1-20)

For $L_p = 11m$ and $v = 3\times10^4 m/s$ $\qquad \Delta L = 11(0.5)(10^{-8}) = 5.5\times10^{-8} m$

$$\text{"Shrinkage"} = \frac{5.5\times10^{-8} m}{10^{-10} m/\text{atomic diameter}} = 550 \text{ atomic diameter}$$

1-46. (a)



(b) Slope of ct' axis $= 2.08 = 1/\beta$, so $\beta = 0.48$ and $v = 1.44\times10^8 m/s$

(c) $ct' = \gamma ct$ and $\gamma = 1/\sqrt{1-\beta^2}$ so $ct'\sqrt{1-\beta^2} = ct$

For $ct' = 1000m$ and $\beta = 0.48$ $\qquad ct = 877m \quad t' = 1.5(877)/c = 4.39\mu s$

(d) $\Delta t = \gamma\Delta t' = 1.14\Delta t' \quad \rightarrow \quad \Delta t' = 5\mu s/1.14 = 4.39\mu s$

1-47. (a) $L = L_p/\gamma = L_p\sqrt{1-u^2/c^2} = 100m\sqrt{1-(0.85)^2} = 52.7m$

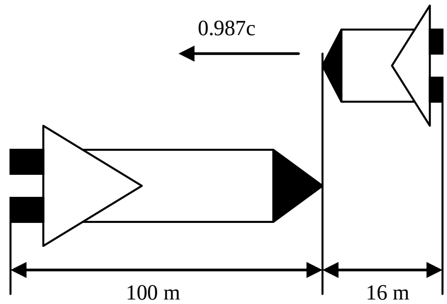
(b) $$u' = \frac{u+u}{1+uu/c^2} = \frac{0.85c + 0.85c}{1+(0.85)^2} = \frac{1.70c}{1.72} = 0.987c$$

(c) $L' = L_p/\gamma' = L_p\sqrt{1-u'^2/c^2} = 100m\sqrt{1-(1.70/1.72)^2} = 16.1m$

(d) As viewed from Earth, the ships pass in the time required for one ship to move its own contracted length. $\Delta t = \frac{L}{u} = \frac{52.7m}{0.85\times3.00\times10^8 m/s} = 2.1\times10^{-7} s$

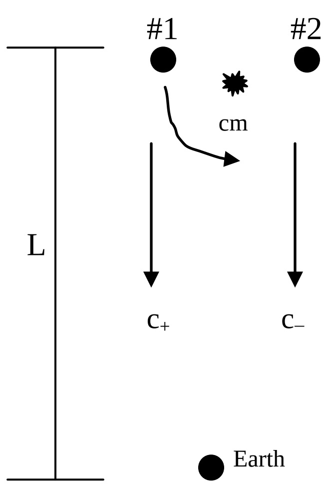
(Problem 1-47 continued)

(e)



1-48. In Doppler radar, the frequency received at the (approaching) aircraft is shifted by approximately $\Delta f / f_0 \approx v/c$. Another frequency shift in the same direction occurs at the receiver, so the total shift $\Delta f / f_0 \approx 2v/c$. $v = (c/2)\left(8\times10^{-7}\right) = 120 m/s$.

1-49.



For star #1:

$v = 32 km/s = 3.2\times10^4 m/s$

$\text{period} = 115d$

$c_+ = c + 3.2\times10^4$

$c_- = c - 3.2\times10^4$

Simultaneous images of star#1 in opposition will appear at Earth when L is at least as large as:

(Problem 1-49 continued)

$$\frac{L}{c-3.2\times10^4}=57.4d+\frac{L}{c+3.2\times10^4}$$

$$\frac{c+3.2\times10^4}{c+3.2\times10^4}\frac{L}{c-3.2\times10^4}=\left(75.5d\times24h/d\times3600s/h\right)\frac{L}{c+3.2\times10^4}\frac{c-3.2\times10^4}{c-3.2\times10^4}$$

$$\frac{\left(c+3.2\times10^4\right)L}{c^2-\left(3.2\times10^4\right)^2}=\left(57.5d\times24\times3600\right)\frac{\left(c-3.2\times10^4\right)L}{c^2-\left(3.2\times10^4\right)^2}$$

$$\left[\left(c+3.2\times10^4\right)-\left(c-3.2\times10^4\right)\right]L=\left[c^2-\left(3.2\times10^4\right)^2\right]\left(57.5\times24\times3600\right)$$

$$L=\frac{\left[c^2-\left(3.2\times10^4\right)^2\right]\left(57.5\times24\times3600\right)}{6.4\times10^4}$$

$$L=6.99\times10^{18}m=739c\bullet y$$

1-50. $t_2'-t_1'=\gamma\left(t_2-t_1\right)-\frac{\gamma v}{c^2}\left(x_b-x_a\right)$ (Equation 1-20)

(a) $t_2'-t_1'=0 \quad\rightarrow\quad \left(t_2-t_1\right)=\left(v/c^2\right)\left(x_b-x_a\right) \quad\rightarrow\quad \left(0.5-1.0\right)y=\left(v/c^2\right)\left(2.0-1.0\right)c\bullet y$

Thus, $-0.5=\left(v/c\right) \quad\rightarrow\quad v=0.5c$ in the $-x$ direction.

(b) $t'=\gamma\left(t-vx/c^2\right)$

Using the first event to calculate t' (because t' is the same for both events),

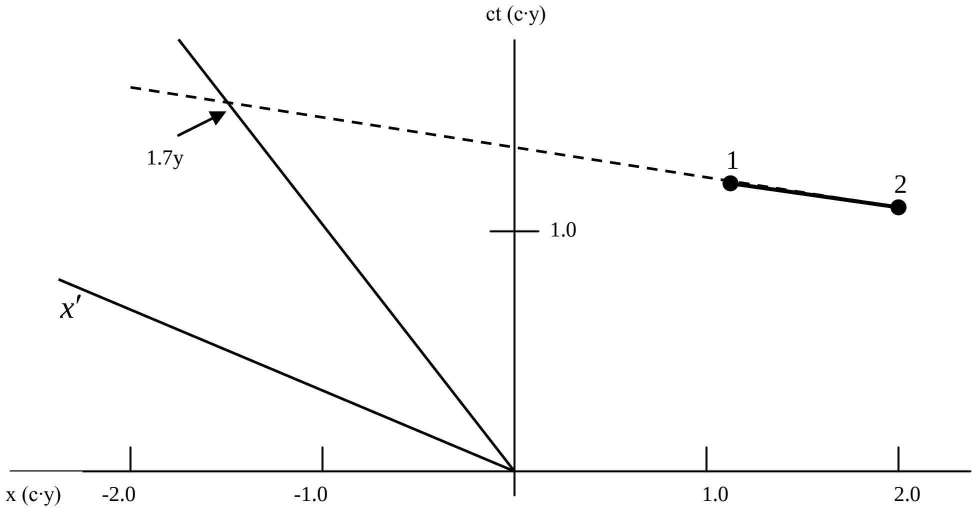
$$t'=\left(1/\sqrt{1-\left(0.5\right)^2}\right)\left[1y-\left(-0.5c\right)\left(1c\bullet y\right)/c^2\right]=1.155\left(1.5\right)=1.7y$$

(c) $\left(\Delta s\right)^2=\left(\Delta x\right)^2-\left(c\Delta t\right)^2=\left(1c\bullet y\right)^2-\left(0.5c\bullet y\right)^2=0.75\left(c\bullet y\right)^2 \quad\rightarrow\quad \Delta s=0.866c\bullet y$

(d) The interval is spacelike.

(e) $L=\Delta s=0.866c\bullet y$

1-51. (a)



Because events are simultaneous in S', line between 1 and 2 is parallel to x' axis. Its slope is $-0.5 = -\beta$. $\quad v = -0.5c$.

(b) From diagram $t' = 1.7y$.

1-52. $x_b' - x_r' = \gamma\left[\left(x_b - x_r\right) - v\left(t_b - t_r\right)\right] \qquad (1)$

$t_b' - t_r' = \gamma\left[\left(t_b - t_r\right) - v\left(x_b - x_r\right)/c^2\right] \qquad (2)$

Where $x_b - x_r = 2400m \quad t_b - t_r = 5\mu s \quad x_b' - x_r' = 2400m \quad t_b' - t_r' = -5\mu s$

Dividing (1) by (2) and inserting the values,

$$\frac{400}{-5\times10^{-6}} = \frac{2400 - v\left(5\times10^{-6}\right)}{5\times10^{6} - v\left(2400\right)/c^2} = -2400 + v\left(2400\right)^2/5\times10^{-6}c^2 = 2400 - 5\times10^{-6}v$$

$$v\left[\frac{\left(2400\right)^2}{5\times10^{-6}c^2} + 5\times10^{-6}\right] = 4800 \quad \rightarrow \quad v = 2.69\times10^{8}\,m/s \text{ in } +x \text{ direction.}$$

1-53. $u_x = 0.85c\cos 50° \qquad u_y = 0.85c\sin 50°$

$\beta = 0.72 \quad \rightarrow \quad \gamma 1/\sqrt{1-\beta^2} = 1.441 \qquad v = 0.72c$

(Problem 1-53 continued)

$$u'_x = \frac{u_x - v}{1 - vu_x/c^2} \qquad u'_y = \frac{u_y}{\gamma\left(1 - vu_x/c^2\right)}$$

$$u'_x = \frac{0.85c\cos 50° - 0.72c}{1-(0.72c)\left(0.85c\cos 50°/c^2\right)} = \frac{-0.1736c}{1-0.3934} = -0.286c$$

$$u'_y = \frac{0.85c\sin 50°}{1.441\left[1-(0.72c)\left(0.85c\cos 50°/c^2\right)\right]} = 0.745c$$

$$u' = \sqrt{u'^2_x + u'^2_y} = 0.798c$$

$\tan\theta' = u'_y/u'_x = 0.745/(-0.286)\,\theta' = 111°$ with respect to the $+x'$ axis.

1-54. This is easier to do in the xy and $x'y'$ planes. Let the center of the meterstick, which is parallel to the x-axis and moves upward with speed v_y in S, at $x = y = x' = y' = 0$ at $t = t' = 0$. The right hand end of the stick, e.g., will not be at $t' = 0$ in S' because the clocks in S' are not synchronized with those in S. In S' the components of the sticks velocity are:

$$u'_y = \frac{u_y}{\gamma\left(1 - vu_x/c^2\right)} = \frac{v_y}{\gamma} \text{ because } u_y = v_y \text{ and } u_x = 0$$

$$u'_x = \frac{u_x - v}{1 - vu_x/c^2} = -v \text{ because } u_x = 0$$

When the center of the stick is located as noted above, the right end in S' will be at:
$x' = \gamma(x - vt) = 0.5\gamma$ because $t = 0$. The S' clock there will read: $t' = \gamma\left(t - vx/c^2\right) = -0.5\gamma v/c^2$

Because $t = 0$. Therefore, when $t' = 0$ at the center, the right end is at $x'y'$ given by:

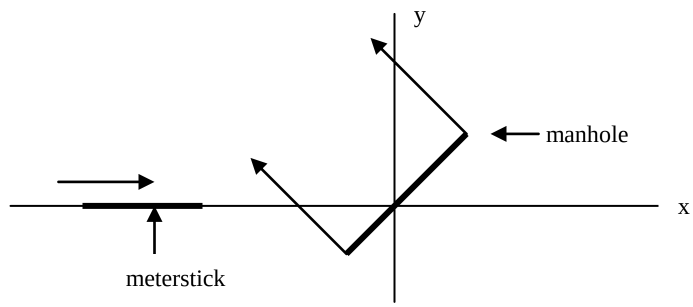
$$x' = 0.5\gamma \qquad y' = u'_y t' = \frac{v_y}{\gamma}\left(\frac{0.5\gamma v}{c^2}\right)$$

and $\theta' = \tan^{-1}\dfrac{y'}{x'} = \tan^{-1}\dfrac{v_y}{\gamma}\left(\dfrac{0.5\gamma v}{c^2}\right)/0.5\gamma = \tan^{-1}\left(v_y v/c^2\right)\sqrt{1-\beta^2}$

For $\beta = 0.65 \qquad \theta' = \left(0.494 v_y/c\right)$

1-55.
$$
\begin{aligned}
0 &= x^2 + y^2 + z^2 - (ct)^2 \\
&= \left[\gamma\left(x' + vt'\right)\right]^2 + y'^2 + z'^2 - \left[c\gamma\left(t' + vx'\right)/c^2\right]^2 \\
&= x'^2\left(\gamma^2 - c^2\gamma^2 v^2/c^4\right) + y'^2 + z'^2 + t'^2\left(\gamma^2 v^2 - c^2\gamma^2\right) + x't'\left(2\gamma^2 v - 2vc^2\gamma^2/c^2\right) \\
&= x'^2 + y'^2 + z'^2 - (ct)^2
\end{aligned}
$$

1-56. The solution to this problem is essentially the same as Problem 1-53, with the manhole taking the place of the meterstick and with the addition of the meterstick moving to the right along the *x*-axis. Following from Problem 1-53, the manhole is titled up on the right and so the meterstick passes through it; there is no collision.



1-57. (a) $t_2' = \gamma\left(t_2 - vx_2/c^2\right)$ and $t_1' = \gamma\left(t_1 - vx_1/c^2\right)$

$$t_2' - t_1' = \gamma\left[t_2 - t_1 - v\left(x_2 - x_1\right)/c^2\right] = \gamma\left[T - vD/c^2\right]$$

(b) For simultaneity in S', $t_2' = t_1'$, or $T - vD/c^2 \quad \rightarrow \quad v/c = cT/D$.

Because $v/c < 1$, cT/D is also <1 or $D > cT$.

(c) If D < cT, then $\left(T - vD/c^2\right) > \left(T - vcT/c^2\right) = T\left(1 - v/c\right)$. For T > 0 this is always positive because *v/c* < 1. Thus, $t_2' - t_1' = \gamma\left(T - vD/c^2\right)$ is always positive.

(d) Assume $T = D/c'$ with $c' > c$. Then

$$T - vD/c^2 = \left(D/c'\right) - \left(vD/c^2\right) = \left(D/c\right)\left(\frac{c}{c'} - \frac{v}{c}\right)$$

This changes sign at $v/c = c/c'$ which is still smaller than 1. For any larger v still smaller than c) $t_2' - t_1' = \gamma\left(T - vD/c^2\right) < 0$ or $t_1' > t_2'$

1-58. $v = 0.6c$ $\gamma = \frac{1}{\sqrt{1-(0.6)^2}} = 1.25$

(a) The clock in S reads $\gamma \times 60\,\text{min} = 75\,\text{min}$ when the S' clock reads 60 min and the first signal from S' is sent. At that time, the S' observer is at $v \times 75\,\text{min} = 0.6c \times 75\,\text{min} = 45c\bullet\text{min}$. The signal travels for 45 min to reach the *S* observer and arrives at 75 min + 45 min = 120 min on the *S* clock.

(b) The observer in *S* sends his first signal at 60 min and its subsequent wavefront is found at $x = c(t - 60\,\text{min})$. The S' observer is at $x = vt = 0.6ct$ and receives the wavefront when these *x* positions coincide, i.e., when

$$c(t - 60\,\text{min}) = 0.6ct$$
$$0.4ct = 60c\bullet\text{min}$$
$$t = (60c\bullet\text{min})/0.4c = 150\,\text{min}$$
$$x = 0.6c(0\,\text{min}) = 90c\bullet\text{min}$$

The confirmation signal sent by the S' observer is sent at that time and place, taking 90 min to reach the observer in *S*. It arrives at 150 min + 90 min = 240 min.

(c) Observer in *S*:

Sends first signal	60 min
Receives first signal	120 min
Receives confirmation	240 min

The S' observer makes identical observations.

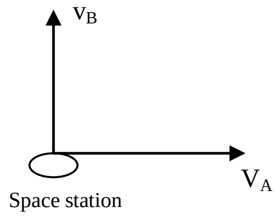
1-59. Clock at *r* moves with speed $u = r\omega$, so time dilation at that clock's location is:

$$\Delta t_0 = \gamma \Delta t \quad \rightarrow \quad \Delta t_r = \Delta t_0 \sqrt{1 - u^2/c^2} = \Delta t_0 \sqrt{1 - r^2\omega^2/c^2}$$

Or, for $r\omega \ll c$, $\Delta t_r \approx \Delta t_0 \left(1 - \frac{1}{2} r^2\omega^2/c^2\right)$

And, $\frac{\Delta t_r - \Delta t_0}{\Delta t_0} = \frac{\Delta t_0\left(1 - \frac{1}{2} r^2\omega^2/c^2\right) - \Delta t_0}{\Delta t_0} \approx -\frac{r^2\omega^2}{2C^2}$

1-60.



$$u'_x = \frac{u_x - v}{1 - u_x v / c^2} \qquad u'_y = \frac{u_y}{\gamma\left(1 - u_x v / c^2\right)}$$

(a) For v_{BA}: $v = v_B$, $v_{Ax} = 0$, $v_{Ay} = -v_A$. So, $v'_{Ax} = \dfrac{v_{Ax} - v}{1 - v_{Ax} v / c^2} = -v_B$

$$v'_{Ay} = \frac{v_{Ay}}{\gamma_B\left(1 - v_{Ax} v / c^2\right)} = \frac{-v_A}{\gamma_B} \quad \text{where } \gamma_B = \frac{1}{\sqrt{1 - v_B^2 / c^2}}$$

$$v_{BA} = \sqrt{v'^2_{Ax} + v'^2_{Ay}} = \sqrt{v_B^2 + \left(v_A / \gamma_B\right)^2}$$

$$\tan\theta'_{BA} = \frac{v'_{Ay}}{v'_{Ax}} = \frac{-v_A / \gamma_B}{-v_B} = \frac{v_A}{\gamma_B v_B}$$

(b) For v_{AB}: $v = v_A$, $v_{By} = v_B$, $v_{Bx} = 0$. So, $v'_{Bx} = \dfrac{v_{Bx} - v}{1 - v_{Bx} v / c^2} = -v_A$

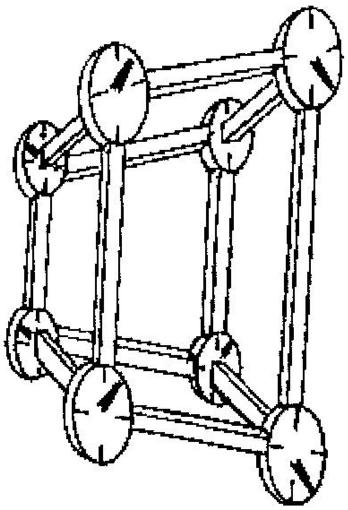
$$v'_{By} = \frac{v_{By}}{\gamma_A\left(1 - v_{Bx} v / c^2\right)} = \frac{v_B}{\gamma_A} \quad \text{where } \gamma_A = \frac{1}{\sqrt{1 - v_A^2 / c^2}}$$

$$v_{AB} = \sqrt{\left(-v_A\right)^2 + \left(v_B / \gamma_A\right)^2}$$

$$\tan\theta'_{AB} = \frac{v_B}{\gamma_A\left(-v_A\right)} = -\frac{v_B}{\gamma_A v_A}$$

(c) The situations are not symmetric. *B* viewed from *A* moves in the *+y* direction, and *A* viewed from *B* moves in the –*y* direction, so $\tan\theta'_A = -\tan\theta'_B = 45°$ only if $v_A = v_B$ and $\gamma_A = \gamma_B = 1$, which requires $v_A = v_B = 0$.

1-61.



1-62. (a) Apparent time $A \to B = T/2 - t_A + t_B$ and apparent time $B \to A = T/2 + t_A - t_B$ where t_A = light travel time from point A to Earth and t_B = light travel time from point B to Earth.

$$A \to B = \frac{T}{2} - \frac{L}{c+v} + \frac{L}{c-v} = \frac{T}{2} + \frac{2vL}{c^2 - v^2}$$

$$B \to A = \frac{T}{2} - \frac{L}{c+v} - \frac{L}{c-v} = \frac{T}{2} - \frac{2vL}{c^2 - v^2}$$

(b) Star will appear at A and B simultaneously when $t_B = T/2 + t_A$ or when the period is:

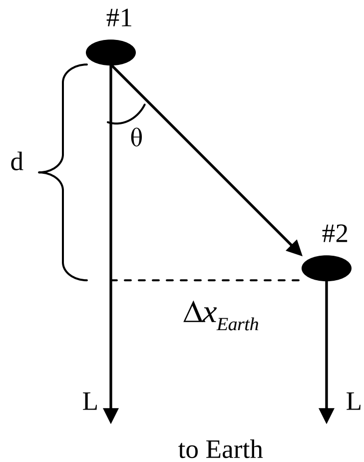
$$T = 2\left[t_B - t_A\right] = 2\left[\frac{L}{c-v} - \frac{L}{c+v}\right] = \frac{4vL}{c^2 - v^2}$$

1-63. The angle of u' with the x' axis is:

$$\tan\theta' = \frac{u'_y}{u'_x} = \frac{u_y}{\gamma\left(1 - \frac{vu_x}{c^2}\right)} \frac{\left(1 - vu_x/c^2\right)}{u_x - v}$$

$$\tan\theta' = \frac{u_y}{\gamma\left(u_x - v\right)} = \frac{u\sin\theta}{\gamma\left(u\cos\theta - v\right)} = \frac{\sin\theta}{\gamma\left(\cos\theta - v/u\right)}$$

1-64.



$$d = v\Delta t\cos\theta$$

$$\Delta x_{Earth} = v\Delta t\sin\theta$$

(a) From position #2 light reaches Earth at time $t_2 = L/c$. From position #1 light reaches Earth at time $t_1 = \frac{L}{c} + \frac{d}{c} - \Delta t$.

$$\Delta t_{Earth} = t_2 - t_1 = \frac{L}{c} - \left[\frac{L}{c} + \frac{d}{c} - \Delta t\right]$$

$$\Delta t_{Earth} = \frac{-v\Delta t\cos\theta}{c} + \Delta t$$

$$\Delta t_{Earth} = \Delta t\left(1 - \beta\cos\theta\right)$$

(b) $$B_{app} = \frac{v_{app}}{c} = \frac{\Delta x_{Earth}}{c\Delta t_{Earth}} = \frac{v\Delta t\sin\theta}{c\Delta t\left(1-\beta\cos\theta\right)} = \frac{\beta\sin\theta}{1-\beta\cos\theta}$$

(c) For $\beta = 0.75$ and $v_{app} = c \rightarrow \beta_{app} = 1$, the result in part (b) becomes

$$1 = \frac{0.75\sin\theta}{1-0.75\cos\theta} \rightarrow \sin\theta + \cos\theta = 1/0.75$$

Using the trigonometric identity

$p\sin\theta + q\cos\theta = r\sin\left(\theta + A\right)$ where $r = \sqrt{p^2+q^2}$ $\qquad \sin A = p/r \qquad \cos A = q/r$

$$\sin\theta + \cos\theta = \sqrt{2}\sin\left(\theta+45\right) = 1/0.75$$

$$\sin\left(\theta+45\right) = 1/0.75\sqrt{2}$$

$$\theta + 45 = 70.6$$

$$\theta = 25.6°$$

Chapter 2 – Relativity II

2-1. $u_{yB}^2 = u_o^2\left(1 - v^2/c^2\right) \qquad u_{xB}^2 = v^2$

$$\sqrt{1-\left(u_{xB}^2+u_B^2\right)/c^2} = \sqrt{1-v^2/c^2-\left(u_o^2/c^2\right)\left(1-v^2/c^2\right)}$$
$$= \sqrt{\left(1-v^2/c^2\right)\left(1-u_o^2/c^2\right)}$$
$$= \left(1-v^2/c^2\right)^{1/2}\left(1-u_o^2/c^2\right)^{1/2}$$

$$p_{yB} = \frac{mu_{yB}}{\sqrt{1-\left(u_{xB}^2+u_{yB}^2\right)/c^2}} = \frac{-mu_o\sqrt{1-v^2/c^2}}{\sqrt{1-v^2/c^2}\sqrt{1-u_o^2/c^2}}$$
$$= -mu_o/\sqrt{1-u_o^2/c^2} = -p_{yA}$$

2-2.
$$d\left(\gamma mu\right) = m\left(ud\gamma + \gamma du\right)$$
$$= m\left[u\left(-\frac{1}{2}\right)\left(\frac{-2u}{c^2}\right)\left(1-\frac{u^2}{c^2}\right)^{-3/2} + \left(1-\frac{u^2}{c^2}\right)^{-1/2}\right]du$$
$$= m\left[\left(\frac{u^2}{c^2}\right)\left(1-\frac{u^2}{c^2}\right)^{-3/2} + \left(1-\frac{u^2}{c^2}\right)\left(1-\frac{u^2}{c^2}\right)^{-3/2}\right]du$$
$$= m\left(1-\frac{u^2}{c^2}\right)^{-3/2} du$$

2-3. (a) $\gamma = \dfrac{1}{\sqrt{1-u^2/c^2}} = \dfrac{1}{\sqrt{1-\left(0.6c\right)^2/c^2}} = \dfrac{1}{0.8} = 1.25$

(b) $p = \gamma mu = \gamma\left(mc\right)^2\left(u/c\right)/c = 1.25\left(0.511\,MeV\right)\left(0.6\right)/c = 0.383\,MeV/c$

(c) $E = \gamma mc^2 = 1.25\left(0.511\,MeV\right) = 0.639\,MeV$

(d) $E_k = \left(\gamma - 1\right)mc^2 = 0.25\left(0.511\,MeV\right) = 0.128\,MeV$

2-4. The quantity required is the kinetic energy. $E_k = (\gamma - 1)mc^2 = \left[\left(1 - u^2/c^2\right)^{-1/2} - 1\right]mc^2$

(a) $E_k = \left[\left(1 - (0.5)^2\right)^{-1/2} - 1\right]mc^2 = 0.155mc^2$

(b) $E_k = \left[\left(1 - (0.9)^2\right)^{-1/2} - 1\right]mc^2 = 1.29mc^2$

(c) $E_k = \left[\left(1 - (0.99)^2\right)^{-1/2} - 1\right]mc^2 = 6.09mc^2$

2-5. $\Delta E = \Delta mc^2 \quad \therefore \quad \Delta m = \Delta E / c^2 = \dfrac{10J}{\left(3.08 \times 10^8 m/s\right)^2} = 1.1 \times 10^{-16} kg$

Because work is done *on* the system, the mass *increases* by this amount.

2-6. $m(u) = m / \sqrt{1 - u^2/c^2}$ (Equation 2-5)

(a) $\dfrac{m(u) - m}{m} = 0.10 \quad \rightarrow \quad \dfrac{m/\sqrt{1 - u^2/c^2} - m}{m} = 0.10$

$$\frac{1}{\sqrt{1 - u^2/c^2}} - 1 = 0.10 \quad \rightarrow \quad \frac{1}{\sqrt{1 - u^2/c^2}} = 1.10 \quad \rightarrow \quad 1 - u^2/c^2 = 1/(1.10)^2$$

Thus, $u^2/c^2 = 1 - 1/(1.10)^2 = 0.1736 \quad \rightarrow \quad u/c = 0.416$

(b) $m(u) = 5m$

$$\frac{m}{\sqrt{1 - u^2/c^2}} = 5m \quad \rightarrow \quad 1 - u^2/c^2 = 1/25$$

Thus, $u^2/c^2 = 1 - 1/25 = 0.960 \quad \rightarrow \quad u/c = 0.980$

(c) $m(u) = 20m$

$$\frac{m}{\sqrt{1 - u^2/c^2}} = 20m \quad \rightarrow \quad 1 - u^2/c^2 = 1/400$$

Thus, $u^2/c^2 = 1 - 1/400 = 0.9975 \quad \rightarrow \quad u/c = 0.99870$

2-7. (a) $v = (\text{Earth} - \text{ moon distance}) / \text{time} = 3.8 \times 10^8 m / 1.5 s = 0.84c$

(b) $E_k = \gamma mc^2 - mc^2 = mc^2(\gamma - 1)$ (Equation 2-9)

$mc^2(\text{proton}) = 938.3MeV$ $\gamma = 1/\sqrt{1-(0.84)^2} = 1.84$

$E_k = 938.3MeV(1.87 - 1) = 813MeV$

(c) $m(u) = \dfrac{m}{\sqrt{1-u^2/c^2}} = \dfrac{938.3MeV/c^2}{\sqrt{1-(0.84)^2}} = 1.730 \times 10^3 MeV/c^2 = 1.730GeV/c^2$

(d) Classically, $E_k = \frac{1}{2}mv^2 = \frac{1}{2}(938.3MeV/c^2)(0.84c)^2 = 331MeV$

$\% \text{ error} = \dfrac{813Mev - 331MeV}{813MeV} \times 100 = 59\%$

2-8. $E_k = mc^2(\gamma - 1)$ (Equation 2-9)

$E_k(u_2) - E_k(u_1) = W_{21} = mc^2(\gamma(u_2) - 1) - mc^2(\gamma(u_1) - 1)$

Or, $W_{21} = mc^2[\gamma(u_2) - \gamma(u_1)]$

(a) $W_{21} = 938.3MeV\left[(1-0.16^2)^{-1/2} - (1-0.15^2)^{-1/2}\right] = 1.51MeV$

(b) $W_{21} = 938.3MeV\left[(1-0.86^2)^{-1/2} - (1-0.85^2)^{-1/2}\right] = 57.6MeV$

(c) $W_{21} = 938.3MeV\left[(1-0.96^2)^{-1/2} - (1-0.95^2)^{-1/2}\right] = 3.35 \times 10^3 MeV = 3.35GeV$

2-9. $E = \gamma mc^2$ (Equation 2-10)

(a) $200GeV = \gamma(0.938GeV)$ where $mc^2(\text{proton}) = 0.938GeV$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{200GeV}{0.938GeV} = 213$$

$$\frac{v}{c} \approx 1 - \frac{1}{2\gamma^2} \quad \text{(Equation 2-40)}$$

$$\frac{v}{c} = 1 - \frac{1}{2(213)^2} = 1 - 0.00001102$$ Thus, $v = 0.99998898c$

(Problem 2-9 continued)

(b) $E \approx pc$ for $E \gg mc^2$ where $E = 200GeV \times 197 = 3.94\times10^4 GeV$ (Equation 2-36)

$p = E/c = 3.94\times10^4 GeV/c$

(c) Assuming one *Au* nucleus (system S') to be moving in the +*x* direction of the lab (system *S*), then *u* for the second *Au* nucleus is in the −*x* direction. The second *Au's* energy measured in the S' system is:

$$\begin{aligned} E' &= \gamma\left(E + vp_x\right) = (213)\left(3.94\times10^4 GeV + v\,3.94\times10^4 GeV/c\right) \\ &= (213)\left(3.94\times10^4 GeV\right)(1+v/c) \\ &= (213)\left(3.94\times10^4 GeV\right)(2) \\ &= 1.68\times10^7 GeV \end{aligned}$$

$$\begin{aligned} p'_x &= \gamma\left(p_x - vE/c^2\right) = (213)\left(-3.94\times10^4 GeV - v\,3.94\times10^4 GeV/c^2\right) \\ &= -(213)\left(3.94\times10^4 GeV\right)(2) \\ &= -1.68\times10^7 GeV/c \end{aligned}$$

2-10. (a) $E = mc^2 = \left(10^{-3}kg\right)c^2 = 9.0\times10^{13} J$

(b) $1kWh = 1\times10^3 J\cdot h/s\left(3600s/h\right) = 3.6\times10^6 J$

So, $9.0\times10^{13} J/3.6\times10^6 J/kWh = 2.5\times10^7 kWh$

@$\$0.10/kWh$ would sell for $\$2.5\times10^6$ or \$2.5 million.

(c) $100W = 100J/s$, so $1g$ of dirt will light the bulb for:

$$\frac{9.0\times10^{13} J}{100J/s} = 9.0\times10^{11}s = \frac{9.0\times10^{11}s}{3.16\times10^7 s/y} = 2.82\times10^4 y$$

2-11. $E = \gamma mc^2$ (Equation 2-10)

where $\gamma = 1/\sqrt{1-u^2/c^2} = 1/\sqrt{1-(0.2)^2} = 1.0206$

$E = (1.0206)(0.511MeV) = 0.5215\,MeV$

$E_k = \gamma mc^2 - mc^2 = mc^2(\gamma - 1)$ (Equation 2-9)

$= (0.511\text{MeV})(1.0206-1) = 0.01054MeV$

(Problem 2-11 continued)

$$E^2 = (pc)^2 + (mc^2)^2 \quad \text{(Equation 2-32)}$$

$$p^2 = \frac{1}{c^2}\left(E^2 - (mc^2)^2\right) = \frac{1}{c^2}\left[(0.5215MeV)^2 - (0.511MeV)^2\right]$$

$$= 0.01087\,MeV^2/c^2 \quad \rightarrow \quad p = 0.104\ MeV/c$$

2-12. $E = \gamma mc^2$ (Equation 2-10)

$$\gamma = E/mc^2 = 1400MeV/938MeV = 1.4925$$

(a) $\gamma = 1/\sqrt{1-u^2/c^2} = 1.4925 \quad \rightarrow \quad 1-u^2/c^2 = 1/(1.4925)^2$

$$u^2/c^2 = 1 - 1/(1.4925)^2 = 0.551 \quad \rightarrow \quad u = 0.74c$$

(b) $E^2 = (pc)^2 + (mc^2)^2$ (Equation 2-32)

$$p = \frac{1}{c}\left[E^2 - (mc^2)^2\right] = \frac{1}{c}\left[(1400MeV)^2 - (938MeV)^2\right] = 1040MeV/c$$

2-13. (a) $p_R = \gamma m_S v \qquad p_N = m_S v$

$$\gamma = 1/\sqrt{1-v^2/c^2} = 1/\sqrt{1-\left(2.5\times10^5/3.0\times10^8\right)^2}$$

$$\gamma = 1.00000035$$

$$\frac{p_R - p_N}{p_R} = \frac{\gamma m_S\left(2.5\times10^3\right) - m_S\left(2.5\times10^3\right)}{\gamma m_S\left(2.5\times10^3\right)} = \frac{\gamma-1}{\gamma} = \frac{3.5\times10^{-7}}{1.00000035} = 3.5\times10^{-7}$$

(b)

$$E_R = \gamma m_S c^2 - m_S c^2 = (\gamma-1)m_S c^2$$

$$E_N = \frac{1}{2}m_S v^2$$

$$\frac{E_R - E_N}{E_R} = \frac{(\gamma-1)c^2 - 0.5v^2}{(\gamma-1)c^2}$$

$$= \frac{3.5\times10^{-7}c^2 - 0.5v^2}{3.5\times10^{-7}c^2}$$

$$= 1 - \frac{0.5\left(2.5\times10^5\right)^2}{3.5\times10^{-7}c^2} = 0.0079$$

2-14. $u = 2.2\times 10^6 m/s$ and $\gamma = 1/\sqrt{1-u^2/c^2}$

(a)

$$E_k = 0.511MeV(\gamma - 1) = 0.511MeV\left(1/\sqrt{1-u^2/c^2} - 1\right) = 0.5110\left(2.689\times 10^{-5}\right)$$
$$= 1.3741\times 10^{-5} MeV$$
$$E_k(\text{classical}) = \frac{1}{2}mu^2 = \frac{1}{2}mc^2\left(u^2/c^2\right) = \left(0.5110MeV/2\right)\left(2.2\times 10^6/c^2\right)^2$$
$$= 1.374\times 10^{-5} MeV$$
$$\% \text{ difference} = \frac{1\times 10^{-9}}{1.3741\times 10^{-5}}\times 100 = 0.0073\%$$

(b)

$$p = \frac{1}{c}\sqrt{E^2 - \left(mc^2\right)^2} = \frac{1}{c}\sqrt{\left(\gamma mc^2\right)^2 - \left(mc^2\right)^2}$$
$$= mc\sqrt{\gamma^2 - 1} = \frac{mc^2}{c}\left[\left(1/\sqrt{1-\left(2.2\times 10^6/3.0\times 10^8\right)^2}\right)^2 - 1\right]^{1/2}$$
$$= 0.5110MeV/c\left(7.33\times 10^{-3}\right) = 3.74745\times 10^{-3} MeV/c$$
$$p(\text{classical}) = mu = \frac{mc^2}{c^2}\left(\frac{u}{c}\right) = \left(0.5110MeV/c\right)\left(2.2\times 10^6/3.0\times 10^8\right)$$
$$= 3.74733\times 10^{-3} MeV/c$$
$$\% \text{ difference} = \frac{1.2\times 10^{-7}}{2.74745\times 10^{-3}}\times 100 = 0.0030\%$$

2-15. (a)

$$60W = 60J/s\left(3.16\times 10^7 s/y\right) = 1.896\times 10^9 J$$
$$m = E/c^2 = 1.896\times 10^9 J/\left(3.0\times 10^8 m/s\right)^2 = 2.1\times 10^{-8} kg = 2.1\times 10^{-5} g = 21\mu g$$

(b) It would make no difference if the inner surface were a perfect reflector. The light energy would remain in the enclosure, but light has no rest mass, so the balance reading would still go down by 21 μg.

2-16. $^4He \rightarrow {}^3H + p + e$

$$Q = \left[m\left({}^3H\right) + m_p + m_e - m\left({}^4He\right)\right]c^2$$
$$= 2809.450MeV + 938.280MeV + 0.511MeV - 3728.424MeV = 19.827MeV$$

2-17. $^{3}H \rightarrow {}^{2}H + n$

Energy to remove the $n = 22.014102u\left({}^{2}H\right)+1.008665u\left(n\right)-3.016049u\left({}^{3}H\right)$

$= 0.006718u \times 931.5MeV/u = 6.26MeV$

2-18. (a) $\Delta m = \dfrac{\Delta E}{c^2} = \dfrac{\Delta Eu}{uc^2} = \dfrac{4.2eV}{931.5\times10^{6}eV}u = 4.5\times10^{-9}u$

(b) error = $\dfrac{\Delta m}{m(Na)+m(Cl)} = \dfrac{4.5\times10^{-9}u}{23u+35.5u} = 7.7\times10^{-11} = 7.7\times10^{-9}$ %

2-19. (a) $\Delta m = m\left({}^{4}He\right)-2m\left({}^{2}H\right) = \dfrac{m\left({}^{4}He\right)c^2-2m\left({}^{2}H\right)c^2}{uc^2}u$

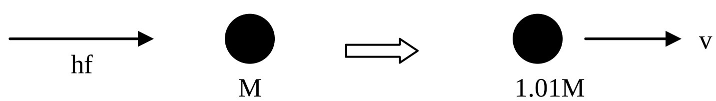
$= \left[3727.409MeV - 2\times1875.628MeV\right]u/931.5MeV$

$= -0.0256u$

(b) $\Delta E = |\Delta m|c^2 = \left(0.0256uc^2\right)\left(931.5MeV/uc^2\right) = 23.8MeV$

(c) $\dfrac{dN}{dt} = \dfrac{P}{\Delta E} = \dfrac{1W}{23.847MeV}\times\dfrac{1eV}{1.602\times10^{-19}J} = 2.62\times10^{11}/s$

2-20.



(a) before photon absorbed: after photon absorbed:

$E = hf + Mc^2$ $\qquad$ $E_f = E_{kinetic} + 1.01Mc^2$

Conservation of energy requires: $hf + Mc^2 = E_{kinetic} + 1.01Mc^2$

Rearranging, the photon energy is:

$hf = 1.01Mc^2 - Mc^2 + E_{kinetic}$

$hf = 0.01Mc^2 + E_{kinetic} > 0.01Mc^2$

(b) The photon's energy is $hf > 0.01Mc^2$ because the particle recoils from the absorption of the photon due to conservation of momentum. The recoil kinetic energy (which is greater than 0) must be supplied by the photon.

2-21. Conservation of energy requires that $E_i^2 = E_f^2$, or

$(p_i c)^2 + (2m_p c^2)^2 = (p_f c)^2 + (2m_p c^2 + m_\pi c^2)^2$ and conservation of momentum requires that $p_i = p_f$, so

$$4(m_p c^2)^2 = 4(m_p c^2)^2 + 2m_p c^2 \times 2m_\pi c^2 + (m_\pi c^2)^2$$

$$0 = 2m_p c^2 \times 2m_\pi c^2 + (m_\pi c^2)^2$$

$$0 = m_\pi c^2 \left(2 + \frac{m_\pi c^2}{2m_p c^2}\right) = m_\pi c^2 \left(2 + \frac{m_\pi}{2m_p}\right)$$

Thus, $m_\pi c^2 (2 + m_\pi / 2m_p)$ is the minimum or threshold energy E_i that a beam proton must have to produce a π^0.

$$E = m_\pi c^2 \left(2 + \frac{m_\pi c^2}{2m_p c^2}\right) = 135 MeV \left(2 + \frac{135}{2(938)}\right) = 280 MeV$$

2-22. $\Delta E = \Delta mc^2 \quad \therefore \quad \Delta m = \Delta E / c^2 = \dfrac{200 \times 10^6 eV}{(5.61 \times 10^{32} eV / g)} = 3.57 \times 10^{-25} g$

2-23. (a) $E = \frac{3}{2} kT$ per atom

$$E = \frac{3}{2}(1.38 \times 10^{-23} J / K)(1.50 \times 10^7 K)$$

$$E = 3.105 \times 10^{-16} J / atom$$

H atoms/kg $= 1kg / 1.67 \times 10^{-27} kg / atom = 6.0 \times 10^{26} atoms$

Thermal energy/kg $= (3.1 \times 10^{-16} J / atom)(6.0 \times 10^{26} atoms) = 1.86 \times 10^{11} J$

(b) $E = mc^2 \quad \Rightarrow \quad m = E / c^2$

$$m = 1.86 \times 10^{11} J / c^2 = 2.07 \times 10^{-6} kg$$

2-24. $1.0W = 1.0J/s \quad \rightarrow \quad p = E/c = (1.0J/s)/c$

(a) On being absorbed by your hand the momentum change is $\Delta p = (1.0J/s)/c$ and, from the impulse-momentum theorem,
$F\Delta t = \Delta p$ where $\Delta t = 1s \quad \rightarrow \quad F = (1.0J/s)/c\Delta t = (1.0/c)N = 3.3\times10^{-9}N$
This magnitude force would be exerted by gravity on mass *m* given by:
$m = F/g = 3.3\times10^{-9}N/(9.8m/s^2) = 3.4\times10^{-10}kg = 0.34\mu g$

(b) On being reflected from your hand the momentum change is twice the amount in part (a) by conservation of momentum. Therefore, $F = 6.6\times10^{-9}$ and $m = 0.68\mu g$.

2-25. Positronium at rest: $(2mc^2)^2 = E_i^2 + (p_i c)^2$

Because $\mathbf{p_i} = 0, \quad E_i = 2mc^2 = 2(0.511MeV) = 1.022MeV$

After photon creation; $(2mc^2)^2 = E_f^2 + (p_f c)^2$

Because $\mathbf{p_f} = 0$ and energy is conserved, $(2mc^2)^2 = E_f^2 = (1.022MeV)^2$ or

$2mc^2 = 1.022MeV$ for the photons.

2-26. $E^2 = (pc^2)^2 + (mc^2)^2$ (Equation 2-31)

$$E = \left[(pc)^2 + (mc^2)^2\right]^{1/2} = mc^2\left[1 + \left(\frac{pc}{mc^2}\right)^2\right]^{1/2}$$

$$= mc^2\left[1 + (p^2/m^2c^2)\right]^{1/2} \approx mc^2\left[1 + \frac{1}{2}(p/mc)^2 + \cdots\right] = mc^2\left[1 + \frac{p^2}{2m^2c^2}\right]$$

$$= mc^2 + p^2/2m$$

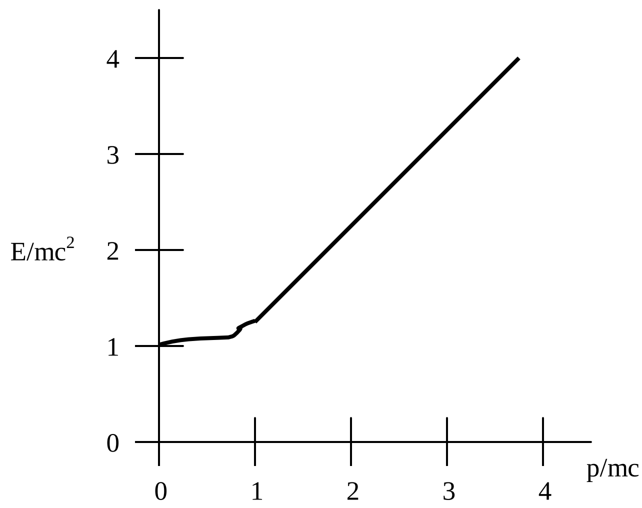
2-27. $E^2 = (pc^2)^2 + (mc^2)^2$ (Equation 2-31)

(a) $(pc)^2 = E^2 - (mc^2)^2 = (5MeV)^2 - (0.511MeV)^2 = 24.74MeV^2$
or, $p = \sqrt{24.74}\,MeV/c = 4.97MeV/c$

(b) $E = \gamma mc^2 \quad \rightarrow \quad \gamma = E/mc^2 = 1/\sqrt{1 - u^2/c^2} \quad \rightarrow \quad 1 - u^2/c^2 = (mc^2/E)^2$

$$u/c = \left[1 - (mc^2/E)^2\right]^{1/2} = \left[1 - (0.511/5.0)^2\right]^{1/2} = 0.995$$

2-28.



2-29. $E^2 = \left(pc^2\right)^2 + \left(mc^2\right)^2$ (Equation 2-31)

$$\left(1746MeV\right)^2 = \left(500MeV\right)^2 + \left(mc^2\right)^2$$

$$mc^2 = \left[\left(1746MeV\right)^2 - \left(500MeV\right)^2\right]^{1/2} = 1673MeV \quad\rightarrow\quad m = 1673MeV/c^2$$

$$E = \gamma mc^2 \quad\rightarrow\quad \gamma = 1/\sqrt{1-u^2/c^2} = E/mc^2$$

$$u/c = \left[1-\left(mc^2/E\right)^2\right]^{1/2} = \left[1-\left(1673MeV/1746MeV\right)^2\right]^{1/2} = 0.286 \quad\rightarrow\quad u = 0.286c$$

2-30. (a) $BqR = m\gamma u = p$ (Equation 2-37)

$B = \dfrac{m\gamma u}{qR}$ and $E = \gamma mc^2$ which we have written as (see Problem 2-29)

$$u/c = \left[1-\left(mc^2/E\right)^2\right]^{1/2} = \left[1-\left(0.511MeV/4.0MeV\right)^2\right]^{1/2} = 0.992$$

And $\gamma = 1/\sqrt{1-u^2/c^2} = 1/\sqrt{1-\left(0.992\right)^2} = 7.83$

Then, $B = \dfrac{\left(9.11\times10^{-31}kg\right)\left(7.83\right)\left(0.992c\right)}{\left(1.60\times10^{-19}C\right)\left(4.2\times10^{-2}m\right)} = 0.316T$

(b) γm exceeds m by a factor of $\gamma = 7.83$.

2-31. (a) $p = qBR = e\left(0.5T\right)\left(2.0\right)\times\dfrac{3.0\times10^8 m/s}{c} = 300MeV/c$

(b) $$E_k = E - mc^2 = \left[\left(pc\right)^2 + \left(mc^2\right)^2\right]^{1/2} - mc^2$$
$$= \left[\left(300MeV\right)^2 + \left(938.28MeV\right)^2\right]^{1/2} - 938.28MeV$$
$$= 46.8MeV$$

2-32. The axis of the spinning disk, system S', is the *z*-axis in cylindrical coordinates.

$$r' = r, \quad z' = z, \quad \varphi' = \varphi - \omega t \quad \Rightarrow \quad dr' = dr, \quad dz' = dz, \quad d\varphi' = d\varphi - \omega dt$$

Therefore, $d\varphi = d\varphi' + \omega dt$ and $d\varphi^2 = d\varphi'^2 + 2\omega d\varphi' dt + \omega^2 dt^2$. Substituting for $d\varphi^2$ in Equation 2-43 yields

$$ds^2 = c^2 dt^2 - \left[dr^2 + r^2(d\varphi'^2 + 2\omega d\varphi' dt + \omega^2 dt^2) + dz^2\right]$$

Simplifying, we obtain

$ds^2 = (c^2 - r^2\omega^2)dt^2 - (dr^2 + r^2 d\varphi'^2 + 2r^2\omega d\varphi' dt + dz^2)$ which is Equation 2-44.

2-33. $\alpha = 4GM/c^2R$ (Equation 2-44)

Earth radius $R = 6.37\times10^6 m$ and mass $M = 5.98\times10^{24} kg$

$$\alpha = \frac{4\left(6.67\times10^{-11} N\bullet m^2/kg^2\right)\left(5.98\times10^{24} kg\right)}{\left(3.00\times10^8 m/s\right)^2\left(6.37\times10^6 m\right)} = 2.78\times10^{-9} \text{ radians}$$

$\alpha = 2.87\times10^{-4}$ arc seconds

2-34. Because the clock furthest from Earth (where Earth's gravity is less) runs the faster, answer (c) is correct.

2-35. $\Delta\phi = \dfrac{6\pi GM}{c^2\left(1-\varepsilon^2\right)R} = \dfrac{6\pi\left(6.67\times10^{-11}N\cdot m^2/kg^2\right)\left(1.99\times10^{30}kg\right)}{\left(3.00\times10^8 m/s\right)^2\left(1-0.048^2\right)\left(7.80\times10^{11}m\right)}$ (Equation 2-51)

$= 3.64\times10^{-8}$ radians/century $= 7.55\times10^{-3}$ arc seconds/century

2-36. From Equation 2-45, $dt = dt' + \dfrac{r^2\omega\, d\varphi'}{c^2 - r^2\omega^2}$ and $dt^2 = dt'^2 + \dfrac{2r^2\omega\, d\varphi' dt'}{c^2 - r^2\omega^2} + \left(\dfrac{r^2\omega\, d\varphi'}{c^2 - r^2\omega^2}\right)^2$.

Substituting dt and dt^2 into Equation 2-44 yields

$$ds^2 = (c^2 - r^2\omega^2)\left[dt'^2 + \frac{2r^2\omega\, d\varphi' dt'}{c^2 - r^2\omega^2} + \left(\frac{r^2\omega\, d\varphi'}{c^2 - r^2\omega^2}\right)^2\right]$$
$$-\left[dr^2 + r^2\, d\varphi'^2 + 2r^2\omega\, d\varphi'\left(dt' + \frac{r^2\omega\, d\varphi'}{c^2 - r^2\omega^2}\right) + dz^2\right]$$

$$ds^2 = (c^2 - r^2\omega^2)dt'^2 + 2r^2\omega\, d\varphi'\, dt' + \frac{\left(r^2\omega\, d\varphi'\right)^2}{c^2 - r^2\omega^2}$$
$$-\left[dr^2 + r^2 d\varphi'^2 + 2r^2\omega\, d\varphi'\, dt' + \frac{2\left(r^2\omega\, d\varphi'\right)^2}{c^2 - r^2\omega^2} + dz^2\right]$$

Cancelling the $2r^2\omega\, d\varphi' dt'$ terms, one of the $-\dfrac{2\left(r^2\omega\, d\varphi'\right)^2}{c^2 - r^2\omega^2}$ terms, and noting that

$$r^2 d\varphi'^2 + \frac{\left(r^2\omega\, d\varphi'\right)^2}{c^2 - r^2\omega^2} = \frac{c^2 r^2\, d\varphi'^2}{c^2 - r^2\omega^2}$$

we have that

$$ds^2 = (c^2 - r^2\omega^2)dt'^2 - (dr^2 + \frac{c^2 r^2\, d\varphi'^2}{c^2 - r^2\omega^2} + dz^2)$$ which is Equation 2-46.

2-37. The transmission is redshifted on leaving Earth to frequency f, where $f_0 - f = f_0 gh/c^2$. Synchronous satellite orbits are at $6.623R_E$ where

$$g = \frac{GM_E}{\left(6.623R_E\right)^2} = \frac{9.9m/s^2}{\left(6.623\right)^2} = 0.223m/s^2$$

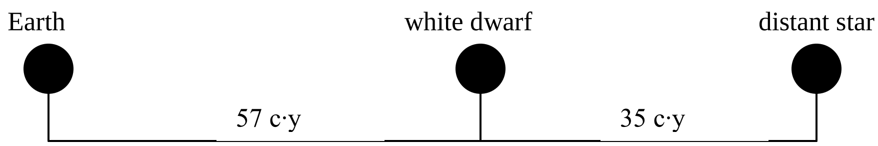
(Problem 2-37 continued)

$$h = 6.623R_E = 6.623\left(6.37\times10^6\,m\right) = 4.22\times10^7\,m$$

$$f_0 - f = \left(9.375\times10^9\,Hz\right)\left(0.223m/s^2\right)\left(4.22\times10^7\right)/\left(3.00\times10^8\right)^2 = 0.980Hz$$

$$f = f_0 - 0.980Hz = 9.374999999\times10^9\,Hz$$

2-38.



On passing "below" the white dwarf, light from the distant start is deflected through an angle:

$$\alpha = 4GM/c^2R = \frac{4\left(6.67\times10^{-11}\,N\cdot m^2/kg^2\right)(3)\left(1.99\times10^{30}\,kg\right)}{\left(3.00\times10^8\,m/s\right)^2\left(10^7\,m\right)} \quad \text{(Equation 2-44)}$$

$= 1.77\times10^{-3}$ radians $= 0.051°$ or the angle between the arcs is $2\alpha = 0.102°$

2-39. The speed v of the satellite is:

$$v = 2\pi R/T = 2\pi\left(6.37\times10^6\,m\right)/\left(90\,\text{min}\times60s/\text{min}\right) = 7.42\times10^3\,m/s$$

Special relativistic effect:

After one year the clock in orbit has recorded time $\Delta t = \Delta t/\gamma$, and the clocks differ by:

$\Delta t - \Delta t' = \Delta t - \Delta t/\gamma = \Delta t\left(1-1/\gamma\right) \approx \Delta t\left(v^2/2c^2\right)$, because $v \ll c$. Thus,

$$\Delta t - \Delta t' = \left(3.16\times10^7\,s\right)\left(7.412\times10^3\right)^2/(2)\left(3.00\times10^8\,m\right)^2 = 0.00965s = 9.65ms$$

Due to special relativity time dilation the orbiting clock is behind the Earth clock by 9.65*ms*.

(Problem 2-39 continued)

General relativistic effect:

$$\frac{\Delta f}{f_0} = \frac{gh}{c^2} = \frac{\left(9.8m/s^2\right)\left(3.0\times10^5 m\right)}{\left(3.0\times10^8 m/s\right)^2} = 3.27\times10^{-11} s/s$$

In one year the orbiting clock gains $\left(3.27\times10^{-11} s/s\right)\left(3.16\times10^7 s/y\right) = 1.03ms$.

The net difference due to both effects is a slowing of the orbiting clock by 9.65−1.03 = 8.62 ms.

2-40. The rest energy of the mass m is invariant, so observers in S' will also measure m = 4.6kg, as in Example 2-9. The total energy E' is then given by:

$$\left(mc^2\right)^2 = \left(E'\right)^2 - \left(\mathbf{p}'c\right)^2$$

Because, $\mathbf{p}' = 0$, $\quad E = mc^2 = 4.6kg\times\left(3.0\times10^8 m/s\right)^2 = 4.14\times10^{14} J$

2-41. (a)

$$E = \gamma m_e c^2 \quad \rightarrow \gamma = E/m_e c^2 = 50\times10 \;\; MeV/0.511 MeV = 9.78\times10^4$$

$$L = L_0/\gamma = 10^{-2} m$$

$$L_0 = 9.78\times10^4\left(10^{-2} m\right) = 978m \text{ (length of one bundle)}$$

The width of one bundle is the same as in the lab.

(b) An observer on the bundle "sees" the accelerator shortened to 978m from its proper length L_0, so $L_0 = \gamma(978) = 978\times10^4(978) = 9.57\times10^7 m.$ (Note that this is about 2.5 times Earth's 40,000km circumference at the equator.)

(c) The e^+ bundle is 10^{-2} m long in the lab frame, so in the e^- frame its length would be measured to be: $L = \left(10^{-2} m\right)/\gamma = 10^{-2} m/9.78\times10^4 = 1.02\times10^{-7} m$.

2-42. $E_k = \gamma mc^2 - mc^2 = mc^2(\gamma - 1)$ If $E_k = mc^2 = 938MeV$, then $\gamma = 2$.

(a) $(mc^2)^2 = E^2 - (pc)^2$ (Equation 2-32) Where $E = \gamma mc^2 = 2(938MeV)$

$$(pc)^2 = E^2 - (mc^2)^2 = (2\times 938)^2 - (938)^2 = 2.46\times 10^6$$

$$p = (2.64\times 10^6)^{1/2}/c = 1.62\times 10^3 MeV/c$$

(b) $p = \gamma mu \quad \rightarrow \quad u = p/\gamma m = (1.62\times 10^3 MeV/c)/(2)(938MeV/c^2) = 0.866c$

2-43. (a) The momentum of the ejected fuel is:

$$p = \gamma mu = mu/\sqrt{1-u^2/c^2} = 10^3 kg(c/2)/\sqrt{1-(0.5)^2} = 1.73\times 10^{11} kg\cdot m/s$$

Conservation of momentum requires that this also be the momentum p_s of the spaceship: $p_s = m_s u_s/\sqrt{1-u_s^2/c^2} = 1.73\times 10^{11} kg\cdot m/s$

Or, $m_s u_s/\sqrt{1-u_s^2/c^2} = (1.73\times 10^{11} kg\cdot m/s)^2$

$$m_s^2 c_s^2 = (1-u_s^2/c^2)(1.73\times 10^{11} kg\cdot m/s)^2 = (1.73\times 10^{11} kg\cdot m/s^2) - (3.33\times 10^5 kg^2)u_s^2$$

$$(10^6 kg)^2 u_s^2 + (3.33\times 10^5 kg^2)u_s^2 = (1.73\times 10^{11} kg\cdot m/s)^2$$

Or, $u_s = (1.73\times 10^{11} kg\cdot m/s)/10^6 kg = 1.73\times 10^5 m/s = 5.77\times 10^{-4} c$

(b) In classical mechanics, the momentum of the ejected fuel is: $mu = mc/2 = 10^3 c/2$, which must equal the magnitude of the spaceship's momentum $m_s u_s$, so

$$u_s = 10^3 (c/2)/m_s = \frac{10^3 kg(3.0\times 10^8 m/s)}{2(10^6 kg)} = 5.0\times 10^{-4} c = 1.5\times 10^5 m/s$$

(c) The initial energy E_i before the fuel was ejected is $E_i = m_s c^2$ in the ship's rest system. Following fuel ejection, the final energy E_f is:

$E_f =$ energy of fuel + energy of ship $= mc^2/\sqrt{1-u^2/c^2} + (m_s - m)c^2/\sqrt{1-u_s^2/c^2}$

Where $u = 0.5c$ and $u_s \ll c$, so $E_f = 1.155mc^2 + (m_s - m)c^2 = (1.155-1)mc^2 + m_s c^2$

The change in energy ΔE is:

$$\Delta E = E_f - E_i = \left[(0.155)(10^3 kg)c^2 + (10^6 kg)c^2\right] - \left[(10^6 kg)c^2\right]$$

$\Delta E = (155kg)c^2$ or 155 $kg = \Delta E/c^2$ of mass has been converted to energy.

2-44. The observer at the pole clock sees the light emission of the equatorial clock as transverse Doppler effect, measuring frequency *f*, where

$f_0 / f = \gamma(1-\beta\cos\theta)$ (Equation 1-35a)

$\theta = \pi/2$ for the equatorial clock, so $f / f_0 = \sqrt{1-v^2/c^2} \approx 1-\frac{1}{2}\frac{v^2}{c^2}$

$f / f_0 = 1-1.193\times10^{-12}$ (red shift)

The observer at the equatorial clock sees a gravitational blue shift for the pole clock and observes

$f / f_0 = 1-gh/c^2$ (Equation 2-45)

$f / f_0 = 1-2.897\times10^{-12}$ (blue shift)

2-45. (a) $p = 300BR(q/e)$ (Equation 2-38)

$p = 300(1.5T)(6.37\times10^6)(1) = 2.87\times10^9\,MeV$

For $E \gg mc^2$, $E = E_k$ and $E = pc$ (Equation 2-32) $\therefore$ $E_k = pc = 2.87\times10^9\,MeV$

(b) For $E = pc$, $u = c$ and

$T = 2\pi R/c = 2\pi(6.37\times10^6 m)/c = 0.133s$

2-46. $\frac{f}{f_0} = 1-GM/c^2R$ (Equation 2-47)

The fractional shift is: $\frac{f_0 - f}{f_0} = 1-\frac{f}{f_0} = GM/c^2R = 7\times10^{-4}$

The dwarf's radius is:

$$R = GM/c^2(7\times10^{-4}) = \frac{6.67\times10^{-11}N\cdot m^2/kg^2(2\times10^{30}kg)}{(3.00\times10^8 m/s)^2(7\times10^{-4})} = 2.12\times10^6 m$$

Assuming the dwarf to be spherical, the density is:

$$\rho = \frac{M}{V} = \frac{2\times10^{30}kg}{4\pi(2.12\times10^6 m)^3/3} = 5.0\times10^{10}kg/m^3$$

2-47. The minimum energy photon needed to create an $e^- - e^+$ pair is E_p = 1.022 *MeV* (see Example 2-13). At minimum energy, the pair is created at rest, i.e., with no momentum. However, the photon's momentum must be $p = E/c = 1.022MeV/c$ at minimum. Thus, momentum conservation is violated unless there is an additional mass "nearby" to absorb recoil momentum.

2-48. $$p'_y = \gamma m u'_y = \left[\frac{\gamma\left(1 - vu_x/c^2\right)}{\sqrt{1-u^2/c^2}}\right] \times m \times \left[\frac{u_y}{\gamma\left(1-vu_x/c^2\right)}\right]$$

Canceling γ and $\left(1-vu_x/c^2\right)$, gives: $p'_y = \dfrac{mu_y}{\sqrt{1-u^2/c^2}} = p_y$

In an exactly equivalent way, $p'_z = p_z$.

2-49. (a) $u' = (u-v)/\left(1-uv/c^2\right)$ where $v = -u$, so $u' = 2u/\left(1+u^2/c^2\right)$. Thus, the speed of the particle that is moving in S' is: $u' = 2u/\left(1+u^2/c^2\right)$ from which we see that:

$$1-\left(\frac{u'}{c}\right)^2 = 1-\frac{4u^2}{c^2}/\left(1+u^2/c^2\right)^2$$
$$= \left(1+2u^2/c^2+u^4/c^4-4u^2/c^2\right)/\left(1+u^2/c^2\right)$$
$$= \left(1-u^2/c^2\right)^2/\left(1+u^2/c^2\right)^2$$

And thus, $$\left[1-\left(\frac{u'}{c}\right)^2\right]^{1/2} = \frac{1-u^2/c^2}{1+u^2/c^2}$$

(b) The initial momentum p_i in S' is due to the moving particle,

$p_i = mu'/\sqrt{1-\left(u'/c\right)^2}$ where u and $\sqrt{1-\left(u'/c\right)^2}$ were given in (a).

$$p_i = m\frac{2u\left(1+u^2/c^2\right)}{\left(1+u^2/c^2\right)\left(1-u^2/c^2\right)} = 2mu/\left(1-u^2/c^2\right)$$

(c) After the collision, conservation of momentum requires that:

$$p_f = Mu/\left(1-u^2/c^2\right)^{1/2} = p_i = 2mu/\left(1-u^2/c^2\right) \quad \text{or} \quad M = 2m/\left(1-u^2/c^2\right)^{1/2}$$

(Problem 2-49 continued)

(d) In S: $E_i = 2mc^2/\sqrt{1-u^2/c^2}$ and $E_f = Mc^2$ (M is at rest.) Because we saw in (c) that $M = 2m/\left(1-u^2/c^2\right)^{1/2}$, then $E_i = E_f$ in S.

In S': $E_i = mc^2 + mc^2/\sqrt{1-\left(u'/c\right)^2}$ and substituting for the square root from (a), $E_i = 2mc^2/\left(1-u^2/c^2\right)$ and $E_f = Mc^2/\sqrt{1-u^2/c^2}$. Again substituting for M from (c), we have: $E_i = E_f$.

2-50. (a) Each proton has $E_k = m_p c^2\left(\gamma-1\right)$, and because we want $E_k = m_p c^2$, then $\gamma = 2$ and $u = 0.866c$. (See Problem 2-40.)

(b) In the lab frame S': $u'_x = \dfrac{u_x - v}{1-u_x v/c^2}$ where $u = v$ and $u_x = -u$ yields:

$$u'_x = \frac{-2u}{1+u^2/c^2} = \frac{-2\left(0.866c\right)}{1+\left(0.866c\right)} = -0.990c$$

(c) For $u = -0.990c$, $\gamma = 1/\sqrt{1-\left(0.99\right)^2} = 7.0$ and the necessary kinetic energy in the lab frame S is: $E_k = m_p c^2\left(\gamma-1\right) = m_p c^2\left(7-1\right) = 6m_p c^2$

2-51. (a) $p_i = 0 = E/c - Mv$ or $v = E/Mc$

(b) The box moves a distance $\Delta x = v\Delta t$, where $\Delta t = L/c$,

so $\Delta x = \left(E/Mc\right)\left(L/c\right) = EL/Mc^2$

(c) Let the center of the box be at $x = 0$. Radiation of mass m is emitted from the left end of the box (e.g.) and the center of mass is at:

$$x_{CM} = \frac{M\left(0\right)+m\left(L/2\right)}{M+m} = \frac{mL}{2\left(M+m\right)}$$

When the radiation is absorbed at the other end the center of mass is at:

$$x_{CM} = \frac{M\left(EL/Mc^2\right)+m\left(L/2-EL/Mc^2\right)}{M+m}$$

(Problem 2-51 continued)

Equating the two values of x_{CM} (if CM is not to move) yields:

$$m = \left(E/c^2\right)/\left(1 - E/Mc^2\right)$$

Because $E \ll Mc^2$, then $m \approx E/c^2$ and the radiation has this mass.

2-52. (a) If v mass is 0:

$$E_\mu^2 = \left(p_\mu c\right)^2 + \left(m_\mu c\right)^2 \text{ and } E_v^2 = \left(p_v c\right)^2 + 0$$

$$E_{k\mu} + E_v = 139.56755MeV - 105.65839MeV$$

$$m_\mu c^2\left(\gamma - 1\right) + E_v = 33.90916MeV$$

$$E_v = p_v c = \left(E_\mu^2 - \left(m_\mu c^2\right)^2\right)^{1/2} = 33.90916 - m_\mu c^2\left(\gamma - 1\right)$$

Squaring, we have

$$\left(m_\mu c^2\right)^2\left(\gamma - 1\right)^2 = \left(33.90916\right)^2 - 2\left(33.90916\right)\left(m_\mu c^2\right)\left(\gamma - 1\right) + \left(m_\mu c^2\right)^2\left(\gamma - 1\right)^2$$

Collecting terms, then solving for $\left(\gamma - 1\right)$,

$$\gamma - 1 = \frac{\left(33.90916\right)^2}{2\left(m_\mu c^2\right)^2 + 2\left(33.90916\right)m_\mu c^2} \quad \text{Substituting } m_\mu c^2 = 105.65839MeV$$

$$\gamma - 1 = 0.0390 \quad \rightarrow \quad \gamma = 1.0390 \text{ so,}$$

$$E_{k\mu} = 4.12MeV \text{ and } p_u = \frac{1}{c}\left[\left(109.78\right)^2 - \left(105.66\right)^2\right]^{1/2} = 29.8MeV/c$$

$$E_v = 29.8MeV \text{ and } p_v = 29.8MeV/c$$

(b) If v_μ mass = 190 keV, then $E_v^2 = \left(p_v c\right)^2 + \left(\mathrm{m}_v c^2\right)^2$ and

$$E_{k\mu} + E_{kv} = \left(139.56755MeV - 105.65839MeV\right) - 0.190MeV = 33.71916MeV$$

Solving as in (a) yields

$$E_\mu = 109.78MeV, \quad p_\mu = 29.8MeV/c, \quad E_v = 29.8MeV, \quad \text{and } p_v = 29.8MeV/c$$

2-53. $$\frac{f}{f_0} = 1 - Gm/c^2R \qquad \text{(Equation 2-47)}$$

Since $c = f\lambda$ and $c = f_0\lambda_0$,

$$\frac{c}{\lambda} \times \frac{\lambda_0}{c} = \frac{\lambda_0}{\lambda} = 1 - GM/c^2R = 1 - \frac{6.67 \times 10^{-11} N \cdot m^2/kg^2\left(1.99 \times 10^{30} kg\right)}{\left(3.00 \times 10^8 m/s\right)^2\left(6.96 \times 10^6 m\right)}$$

$$= 1 - 0.000212 = 0.999788$$

(Problem 2-53 continued)

$$\lambda = \lambda_0 / 0.999788 = 720.00nm / 0.999788 = 720.15nm$$
$$\Delta\lambda = \lambda - \lambda_0 = 0.15nm$$

2-54.
$$u_y = \left(u_y / \gamma\right)\left(1 - vu_x / c^2\right)^{-1}$$
$$a'_y = \frac{du_y}{dt} = \frac{\frac{du_y}{\gamma}\left(1 - vu_x / c^2\right)^{-1} + \frac{u_y}{\gamma}\left(\frac{vdx}{c^2}\right)\left(1 - vu_x / c^2\right)^{-2}}{\gamma\left(dt - vdx / c^2\right)}$$
$$a'_y = \frac{1}{\gamma^2}\left[\frac{\left(du_y / dt\right)\left(1 - vu_x / c^2\right)^{-1} + \left(u_y v / c^2\right)\left(du_x / dt\right)\left(1 - vu_x / c^2\right)^{-2}}{\left(1 - v\left(dx / dt\right) / c^2\right)}\right]$$
$$a'_y = \frac{a_y}{\gamma^2\left(1 - vu_x / c^2\right)^2} + \frac{a_x u_y v / c^2}{\gamma^2\left(1 - vu_x / c^2\right)^3}$$

2-55. (a)
$$F_x = \frac{dp'_x}{dt} = \frac{d\left(\gamma mv\right)}{dt} \qquad F_x = ma_x \text{ because } u_x = 0$$
$$F_x = \gamma m\left(dv / dt\right) + mv\,d\left[\left(1 - v^2 / c^2\right)^{-1/2}\right] / dt$$
$$F_x = \frac{ma_x}{\left(1 - v^2 / c^2\right)^{1/2}} + \frac{m\left(v^2 / c^2\right)a_x}{\left(1 - v^2 / c^2\right)^{3/2}}$$
$$F_x = \frac{ma_x\left(1 - v^2 / c^2\right) + m\left(v^2 / c^2\right)a_x}{\left(1 - v^2 / c^2\right)^{3/2}}$$
$$F_x = \gamma^3 ma_x$$

Because $u_x = 0$, note from Equation 2-1 (inverse form) that $a_x = a_x / \gamma^3$.

Therefore, $F_x = \gamma^3 ma_x / \gamma^3 = ma_x = F_x$

(b)
$$F_y = \frac{dp_y}{dt} = \frac{d\left(\gamma mv_y\right)}{dt} \qquad F_y = ma_y \text{ because } u_y = u_x = 0$$

$F_y = \gamma ma_y$ because S' moves in $+x$ direction and the instantaneous transverse impluse (small) changes only the direction of **v**. From the result of Problem 2-52 (inverse form) with $u_y = u_x = 0,\ a_y = a_y / \gamma^2$

Therefore, $F_y = \gamma ma_y = \gamma ma_y / \gamma^2 = ma_y / \gamma = F_y / \gamma$

2-56. (a) Energy and momentum are conserved.

Initial system: $E = Mc^2, \quad p = 0$

Invariant mass: $\left(Mc^2\right)^2 = E^2 - (pc)^2 = \left(Mc^2\right)^2 + 0$

Final system:

$$\left(2mc^2\right)^2 = \left(Mc^2\right)^2 + 0$$

For 1 particle (from symmetry)

$$\left(mc^2\right)^2 = \left(Mc^2/2\right)^2 - p^2c^2 = \left(Mc^2/2\right)^2 - (\gamma muc)^2$$

Rearranging, $1 = \left(\dfrac{Mc^2}{2mc^2}\right)^2 - (\gamma u/c)^2 \quad \to \quad \gamma^2 = \left(\dfrac{Mc^2}{2mc^2}\right)^2 = \dfrac{1}{1-u^2/c^2}$

Solving for u, $u = \left[1 - \left(\dfrac{2mc^2}{Mc^2}\right)^2\right]^{1/2} c$

(b) Energy and momentum are conserverd.

Initial system: $E = 4mc^2$

Invariant mass: $\left(Mc^2\right)^2 = \left(4mc^2\right)^2 - (pc)^2$

Final system:

Invariant mass: $\left(2mc^2\right)^2 = \left(4mc^2\right)^2 - (pc)^2$ where $(pc)^2 = \left(4mc^2\right)^2 - \left(Mc^2\right)^2$

$$\frac{u}{c} = \frac{pc}{E} = \frac{\left[\left(4mc^2\right)^2 - \left(Mc^2\right)^2\right]^{1/2}}{4mc^2}$$

$$\left(\frac{u}{c}\right)^2 = \frac{\left(4mc^2\right)^2 - \left(Mc^2\right)^2}{\left(4mc^2\right)^2} = 1 - \left(\frac{Mc^2}{4mc^2}\right)^2$$

$$u = \left[1 - \left(\frac{Mc^2}{4mc^2}\right)^2\right]^{1/2} c$$

Chapter 3 – Quantization of Charge, Light, and Energy

3-1. The radius of curvature is given by Equation 3-2.

$$R = \frac{mu}{qB} = m\left[\frac{2.5\times10^{6}\,m/s}{\left(1.60\times10^{-19}C\right)\left(0.40T\right)}\right] = m\left(3.91\times10^{25}\,m/s\cdot C\cdot T\right)$$

Substituting particle masses from Appendices A and D:

$$R(\text{protron}) = \left(1.67\times10^{-27}kg\right)\left(3.91\times10^{25}\,m/s\cdot C\cdot T\right) = 6.5\times10^{-2}m$$

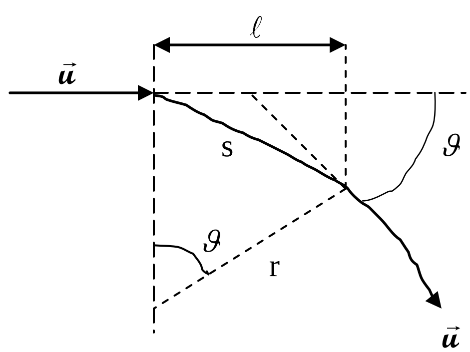
$$R(\text{electron}) = \left(9.11\times10^{-31}kg\right)\left(3.91\times10^{25}\,m/s\cdot C\cdot T\right) = 3.6\times10^{-5}m$$

$$R(\text{deuteron}) = \left(3.34\times10^{-27}kg\right)\left(3.91\times10^{25}\,m/s\cdot C\cdot T\right) = 0.13m$$

$$R(H_2) = \left(3.35\times10^{-27}kg\right)\left(3.91\times10^{25}\,m/s\cdot C\cdot T\right) = 0.13m$$

$$R(\text{helium}) = \left(6.64\times10^{-27}kg\right)\left(3.91\times10^{25}\,m/s\cdot C\cdot T\right) = 0.26m$$

3-2.



For small values of ϑ, $s \approx \ell$; therefore, $\vartheta = \frac{s}{r} \approx \frac{\ell}{r}$

Recalling that $euB = \frac{mu^2}{r} \Rightarrow r = \frac{mu}{eB} \quad \therefore \quad \vartheta \approx \frac{\ell}{mu/eB} = \frac{eB\ell}{mu}$

3-3. $B = \dfrac{\mathcal{E}}{u}$, $\quad \dfrac{u}{c} = \dfrac{pc}{E}$, and $pc = \sqrt{E^2 - \left(mc^2\right)^2}$

$$pc = \sqrt{\left(0.561MeV\right)^2 - \left(0.511MeV\right)^2} = 0.2315MeV$$

$$\frac{u}{c} = \frac{0.2315MeV}{0.561MeV} = 0.41$$

$$\therefore\ B = \frac{2.0\times10^5 V/m}{0.41c} = 1.63\times10^{-3}T = 16.3G$$

3-4. $\Delta F = quB$ and $F_G = m_p g$

$$\frac{F_B}{F_G} = \frac{quB}{m_p g} = \frac{\left(1.6\times10^{-19}C\right)\left(3.0\times10^6 m/s\right)\left(\left(3.5\times10^{-5}T\right)\right)}{\left(1.67\times10^{-27}kg\right)\left(9.80m/s^2\right)} = 1.03\times10^9$$

3-5. (a) $R = \dfrac{mu}{qB} = \dfrac{\left[\left(2E_k/e\right)\left(e/m\right)\right]^{1/2}}{\left(e/m\right)\left(B\right)}$

$$= \frac{1}{B}\sqrt{\frac{2E_k/e}{e/m}} = \frac{1}{0.325T}\left[\frac{\left(2\right)\left(4.5\times10^4 eV/e\right)}{1.76\times10^{11}kg}\right]^{1/2} = 2.2\times10^{-3}m = 2.2mm$$

(b) frequency $f = \dfrac{u}{2\pi R} = \dfrac{\sqrt{\left(2E_k/e\right)\left(e/m\right)}}{2\pi R}$

$$= \frac{\left[\left(2\right)\left(4.5\times10^4 eV/e\right)\left(1.76\times10^{11}C/kg\right)\right]^{1/2}}{2\pi\left(2.2\times10^{-3}m\right)} = 9.1\times10^9 Hz$$

period $T = 1/f = 1.1\times10^{-10}s$

3-6. (a) $1/2mu^2 = E_k$, so $u = \sqrt{\left(2E_k/e\right)\left(e/m\right)}$

$$\therefore\ u = \left[\left(2\right)\left(2000eV/e\right)\left(1.76\times10^{11}C/kg\right)\right]^{1/2} = 2.65\times10^7 m/s$$

(b) $\Delta t_1 = \dfrac{x_1}{u} = \dfrac{0.05m}{2.65\times10^7 m/s} = 1.89\times10^{-9}s = 1.89ns$

(Problem 3-6 continued)

(c) $mu_y = F\Delta t_1 = e\mathcal{E}\Delta t_1$

$$\therefore \quad u_y = (e/m)\mathcal{E}\Delta t_1 = (1.76\times10^{11}C/kg)(3.33\times10^{3}V/m)(1.89\times10^{-9}s) = 1.11\times10^{6}m/s$$

3-7. $NE_k = \Delta W = C_V\Delta T \quad \therefore$

$$N = \frac{C_V\Delta T}{E_k} = \frac{(5\times10^{-3}cal/°C)(2°C)}{2000eV}\times\frac{(4.186J/cal)}{(1.60\times10^{-19}J/eV)} = 1.31\times10^{14}$$

3-8. $Q_1 - Q_2 = (25.41-20.64)\times10^{-19}C = 4.47\times10^{-19}C = (n_1 - n_2)e$

$Q_2 - Q_3 = (20.64-17.47)\times10^{-19}C = 3.17\times10^{-19}C = (n_2 - n_3)e$

$Q_4 - Q_3 = (19.06-17.47)\times10^{-19}C = 1.59\times10^{-19}C = (n_4 - n_3)e$

$Q_4 - Q_5 = (19.06-12.70)\times10^{-19}C = 6.36\times10^{-19}C = (n_4 - n_5)e$

$Q_6 - Q_5 = (14.29-12.70)\times10^{-19}C = 1.59\times10^{-19}C = (n_6 - n_5)e$

where the n_i are integers. Assuming the smallest $\Delta n = 1$, then $\Delta n_{12} = 3.0$, $\Delta n_{23} = 2.0$, $\Delta n_{43} = 1.0$, $\Delta n_{45} = 4.0$, and $\Delta n_{65} = 1.0$. The assumption is valid and the fundamental charge implied is $1.59\times10^{-19}C$.

3-9. For the rise time to equal the field-free fall time, the net upward force must equal the weight. $q\mathcal{E} - mg = mg \quad \therefore \quad \mathcal{E} = 2mg/q.$

3-10. (See Millikan's Oil Drop Experiment on the home page at www.whfreeman.com/tiplermodernphysics6e.) The net force in the *y*-direction is $mg - bv_y = ma_y$. The net force in the *x*-direction is $q\mathcal{E} - bv_x = ma_x$. At terminal speed $a_x = a_y = 0$ and $v_x / v_t' = \sin\theta.$

$$\sin\theta = \frac{v_x}{v_t'} = \frac{(q\mathcal{E}/b)}{v_t'} = \frac{q\mathcal{E}}{bv_t'}$$

3-11. (See Millikan's Oil Drop Experiment on the home page at www.whfreeman.com/tiplermodernphysics6e.)

(a) At terminal speed $mg = bv_t$ where $m = 4/3\pi a^3 \rho_{oil}$ and $b = 6\pi\eta a$. Substituting gives

$$a^2 = \frac{18}{4}\left(\frac{\eta v_t}{\rho_{oil} g}\right) \quad \therefore \quad a = \left[\frac{(18)(1.80\times10^{-5}N\cdot s/m^2)(5.0\times10^{-3}m/20s)}{(4)(0.75)(1000kg/m^3)(9.8m/s^2)}\right]^{1/2}$$

$$= 1.66\times10^{-6}m = 1.66\times10^{-3}mm$$

$$m = 4\pi(1.66\times10^{-6}m)^3(750kg/m^3)/3 = 1.44\times10^{-14}kg$$

(b)

$$F_{\mathcal{E}} = q\mathcal{E} \text{ and } F_G = mg \quad \therefore \quad \frac{F_E}{F_G} = \frac{(2)(1.60\times10^{-19}C)(2.5\times10^5V/m)}{(1.44\times10^{-14}kg)(9.8m/s^2)} = 0.57$$

3-12. $\lambda_m T = 2.898\times10^{-3}m\cdot K$

(a) $\lambda_m = \dfrac{2.898\times10^{-3}m\cdot K}{3K} = 9.66\times10^{-4}m = 0.966mm$

(b) $\lambda_m = \dfrac{2.898\times10^{-3}m\cdot K}{300K} = 9.66\times10^{-6}m = 9.66\mu m$

(c) $\lambda_m = \dfrac{2.898\times10^{-3}m\cdot K}{3000K} = 9.66\times10^{-7}m = 966nm$

3-13. Equation 3-4: $R = \sigma T^4$. Equation 3-6: $R = \frac{1}{4}cU$.

From Example 3-4: $U = (8\pi^5 k^4 T^4)/(15h^3c^2)$

$$\sigma = \frac{R}{T^4} = \frac{(1/4)cU}{T^4} = \frac{1}{4}c(8\pi^5k^4T^4)/(15h^3c^2T^4)$$

$$= \frac{2\pi^5(1.38\times10^{-23}J/K)^4}{15(6.63\times10^{-34}J\cdot s)^3(3.00\times10^8m/s)^2} = 5.67\times10^{-8}W/m^2K^4$$

3-14. Equation 3-18: $u(\lambda) = \dfrac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$

$$u(f)\,df = u(\lambda)\,d\lambda \quad \therefore \quad u(f) = u(f)\frac{d\lambda}{df} \text{ Because } c = f\lambda, \quad \left|\frac{d\lambda}{df}\right| = c/f^2$$

$$u(f) = \frac{8\pi hc(f/c)^5}{e^{hf/kT} - 1}\left(\frac{c}{f^2}\right) = \frac{8\pi f^2}{c^3}\frac{hf}{e^{hf/kT} - 1}$$

3-15.

(a) $\lambda_m T = 2.898\times10^{-3}\,m\cdot K \quad \therefore \quad \lambda_m = \dfrac{2.898\times10^{-3}\,m\cdot K}{2.7K} = 1.07\times10^{-3}\,m = 1.07mm$

(b) $c = f\lambda \quad \therefore \quad f = \dfrac{c}{\lambda_m} = \dfrac{3.00\times10^8\,m/s}{1.07\times10^{-3}\,m} = 2.80\times10^{11}\,Hz$

(c) Equation 3-6:

$$R = \frac{1}{4}cU = \frac{c}{4}\left(8\pi^5k^4T^4/15h^3c^3\right)$$

$$= \frac{\left(3.00\times10^8\,m/s\right)\left(8\pi^5\right)\left(1.38\times10^{-23}\,J/K\right)^4\left(2.7\right)^4}{\left(4\right)\left(15\right)\left(6.63\times10^{-34}\,J\cdot s\right)^3\left(3.00\times10^8\,m/s\right)^3} = 3.01\times10^{-6}\,W/m^2$$

Area of Earth: $A = 4\pi r_E^2 = 4\pi\left(6.38\times10^6\,m\right)^2$

Total power = $RA = \left(3.01\times10^{-6}W/m^2\right)\left(4\pi\right)\left(6.38\times10^6\,m\right)^2 = 1.54\times10^9W$

3-16. $\lambda_m T = 2.898\times10^{-3}\,m\cdot K$

(a) $T = \dfrac{2.898\times10^{-3}\,m\cdot K}{700\times10^{-9}\,m} = 4140K$

(b) $T = \dfrac{2.898\times10^{-3}\,m\cdot K}{3\times10^{-2}\,m} = 9.66\times10^{-2}\,K$

(c) $T = \dfrac{2.898\times10^{-3}\,m\cdot K}{3m} = 9.66\times10^{-4}\,K$

3-17. Equation 3-4: $R_1 = \sigma T_1^4$ $\quad R_2 = \sigma T_2^4 = \sigma\left(2T_1\right)^4 = 16\sigma T_1^4 = 16R_1$

3-18. (a) Equation 3-17: $\bar{E} = \dfrac{hc/\lambda}{e^{hc/\lambda kT} - 1} = \dfrac{hc/\left(10hc/kT\right)}{e^{(hc/kT)/(10hc/kT)} - 1} = \dfrac{0.1kT}{e^{0.1} - 1} = 0.951kT$

(b) $\bar{E} = \dfrac{hc/\lambda}{e^{hc/\lambda kT} - 1} = \dfrac{hc/\left(0.1hc/kT\right)}{e^{(hc/kT)/(0.1hc/kT)} - 1} = \dfrac{10kT}{e^{10} - 1} = 4.59\times10^{-4}kT$

Equipartition theorem predicts $\bar{E} = kT$. The long wavelength value is very close to *kT*, but the short wavelength value is much smaller than the classical prediction.

3-19. (a) $\lambda_m T = 2.898\times10^{-3}m\cdot K \quad \therefore \quad T_1 = \dfrac{2.898\times10^{-3}m\cdot K}{27.0\times10^{-6}m} = 107K$

$R_1 = \sigma T_1^4$ and $R_2 = \sigma T_2^4 = 2R_1 = 2\sigma T_1^4$

$\therefore \quad T_2^4 = 2T_1^4$ or $T_2 = 2^{1/4}T_1 = \left(2^{1/4}\right)\left(107K\right) = 128K$

(b) $\lambda_m = \dfrac{2.898\times10^{-3}m\cdot K}{128K} = 23\times10^{-6}m$

3-20. (a) $\lambda_m T = 2.898\times10^{-3}m\cdot K$ (Equation 3-5)

$$\lambda_m = \frac{2.898\times10^{-3}m\cdot K}{2\times10^4 K} = 1.45\times10^{-7}m = 145nm$$

(b) λ_m is in the ultraviolet region of the electromagnetic spectrum.

3-21. Equation 3-4: $R = \sigma T^4$

$$P_{abs} = \left(1.36\times10^3 W/m^2\right)\left(\pi R_E^2 m^2\right) \text{ where } R_E = \text{radius of Earth}$$

$$P_{emit} = \left(RW/m^2\right)\left(4\pi R_E^2\right) = \left(1.36\times10^3 W/m^2\right)\left(\pi R_E^2 m^2\right)$$

$$R = \left(1.36\times10^3 W/m^2\right)\left(\frac{\pi R_E^2}{4\pi R_E^2}\right) = \frac{1.36\times10^3}{4}\frac{W}{m^2} = \sigma T^4$$

$$T^4 = \frac{1.36\times10^3 W/m^2}{4\left(5.67\times10^{-8} W/m^2\cdot K^4\right)} \quad \therefore \quad T = 278.3K = 5.3°C$$

3-22. (a) $\lambda_m T = 2.898\times10^{-3} m\cdot K \quad \therefore \quad \lambda_m = \dfrac{2.898\times10^{-3} m\cdot K}{3300K} = 8.78\times10^{-7} m = 878nm$

$$f_m = c/\lambda_m = \frac{3.00\times10^8 m/s}{8.78\times10^{-7} m} = 3.42\times10^{14} Hz$$

(b) Each photon has average energy $E = hf$ and $NE = 40J/s$.

$$N = \frac{40J/s}{hf_m} = \frac{40J/s}{\left(6.63\times10^{-34} J\cdot s\right)\left(3.42\times10^{14} Hz\right)} = 1.77\times10^{20}\ photons/s$$

(c) At 5*m* from the lamp *N* photons are distributed uniformly over an area $A = 4\pi r^2 = 100\pi m^2$. The density of photons on that sphere is $\left(N/A\right)/s\cdot m^2$.

The area of the pupil of the eye is $\pi\left(2.5\times10^{-3} m\right)^2$, so the number of photons entering the eye per second is:

$$n = \left(N/A\right)\left(\pi\right)\left(2.5\times10^{-3} m\right)^2 = \frac{\left(1.77\times10^{20}/s\right)\left(\pi\right)\left(2.5\times10^{-3} m\right)^2}{100\pi m^2}$$

$$= \left(1.77\times10^{20}/s\right)\left(\pi\right)\left(2.5\times10^{-3} m\right)^2 = 1.10\times10^{13}\ photons/s$$

3-23. Equation 3-18: $u(\lambda)=\dfrac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT}-1}$ Letting $A=\pi hc,\quad B=hc/kT$, and $U(\lambda)=\dfrac{A\lambda^{-5}}{e^{B/\lambda}-1}$

$$\frac{du}{d\lambda}=\frac{d}{d\lambda}\left[\frac{A\lambda^{-5}}{e^{B/\lambda}-1}\right]=A\left[\frac{\lambda^{-5}(-1)e^{B/\lambda}\left(-B\lambda^{-2}\right)}{\left(e^{B/\lambda}-1\right)^2}-\frac{5\lambda^{-6}}{e^{B/\lambda}-1}\right]$$
$$=\frac{A\lambda^{-6}}{\left(e^{B/\lambda}-1\right)^2}\left[\frac{B}{\lambda}e^{B/\lambda}-5\left(e^{B/\lambda}-1\right)\right]=\frac{A\lambda^{-6}e^{B/\lambda}}{\left(e^{B/\lambda}-1\right)^2}\left[\frac{B}{\lambda}-5\left(1-e^{-B/\lambda}\right)\right]=0$$

The maximum corresponds to the vanishing of the quantity in brackets. Thus, $5\lambda\left(1-e^{-B/\lambda}\right)=B$. This equation is most efficiently solved by iteration; i.e., guess at a value for *B/λ* in the expression $5\lambda\left(1-e^{-B/\lambda}\right)$, solve for a better value of *B/λ*; substitute the new value to get an even better value, and so on. Repeat the process until the calculated value no longer changes. One succession of values is: 5, 4.966310, 4.965156, 4.965116, 4.965114, 4.965114. Further iterations repeat the same value (to seven digits), so we have:

$$\frac{B}{\lambda_m}=4.965114=\frac{hc}{\lambda_m kT}\quad\therefore\quad \lambda_m T=\frac{hc}{(4.965114)k}=\frac{\left(6.63\times10^{-34}J\cdot s\right)\left(3.00\times10^{8}m/s\right)}{(4.965114)\left(1.38\times10^{-23}J/K\right)}$$
$$\lambda_m T=2.898\times10^{-3}m\cdot K\quad\text{(Equation 3-5)}$$

3-24. Photon energy $E=hf=hc/\lambda$

(a) For λ = 380*nm*: $E=(1240eV\cdot nm)/380nm=3.26eV$

For λ = 750*nm*: $E=(1240eV\cdot nm)/750nm=1.65eV$

(b) $E=hf=\left(4.14\times10^{-15}eV\cdot s\right)\left(100\times10^{6}s^{-1}\right)=4.14\times10^{-7}eV$

3-25. (a) $hf=hc/\lambda=0.47eV$.

$$\lambda_{\max}=\frac{hc}{4.87eV}=\frac{\left(4.14\times10^{-15}eV\cdot s\right)\left(3.00\times10^{8}m/s\right)}{4.87eV}=2.55\times10^{-7}m=255nm$$

(Problem 3-25 continued)

(b) It is the fraction of the total solar power with wavelengths less than 255*nm*, i.e., the area under the Planck curve (Figure 3-6) up to 255*nm* divided by the total area. The latter is: $R = \sigma T^4 = \left(5.67\times10^{-8} W/m^2\cdot K^4\right)\left(5800K\right)^4 = 6.42\times10^7 W/m^2$.

Approximating the former with $u(\lambda)\Delta\lambda$ with $\lambda = 127nm$ and $\Delta\lambda = 255nm$:

$$\left[u(127nm)\right](255nm) = \left[\frac{8\pi hc\left(127\times10^{-9}m\right)^{-5}}{e^{hc/kT\left(127\times10^{-9}\right)}-1}\right]\left(255\times10^{-9}m\right) = 1.23\times10^{-4} J/m^3$$

$$R(0-255nm) = \frac{c}{4}\left(1.23\times10^{-4} J/m^3\right) \rightarrow \frac{R(0-255nm)}{R}$$

$$= \frac{\left(3.00\times10^8 m/s\right)\left(1.23\times10^{-4} J/m^3\right)}{(4)\left(6.42\times10^7 W/m^2\right)} \quad \text{fraction} = 1.4\times10^{-4}$$

3-26. (a) $\lambda_t = \frac{hc}{\phi} = \frac{1240eV\cdot nm}{1.9ev} = 653nm, \quad f_t = \frac{\phi}{h} = \frac{1.9eV}{4.136\times10^{-15}eV\cdot s} = 4.59\times10^4 Hz$

(b) $V_0 = \frac{1}{e}\left(\frac{hc}{\lambda}-\phi\right) = \frac{1}{e}\left(\frac{1240eV\cdot nm}{300nm} - 1.9eV\right) = 2.23V$

(c) $V_0 = \frac{1}{e}\left(\frac{hc}{\lambda}-\phi\right) = \frac{1}{e}\left(\frac{1240eV\cdot nm}{400nm} - 1.9eV\right) = 1.20V$

3-27. (a) Choose λ =550*nm* for visible light. $nhf = E \rightarrow \frac{dn}{dt}hf = \frac{dE}{dt} = P$

$$\frac{dn}{dt} = \frac{P}{hf} = \frac{P\lambda}{hc} = \frac{\left(0.05\times100W\right)\left(550\times10^{-9}m\right)}{\left(6.63\times10^{-34}J\cdot s\right)\left(3.00\times10^8 m/s\right)} = 1.38\times10^{19}/s$$

(b) $flux = \frac{number\ \ radiated\ /\ unit\ time}{area\ of\ \ the\ sphere} = \frac{1.38\times10^{19}/s}{4\pi(2m)^2} = 2.75\times10^{17}/m^2\cdot s$

3-28. (a) $hf = \phi \quad \therefore \quad f_t = \dfrac{\phi}{h} = \dfrac{4.22eV}{4.14\times10^{-15}eV\cdot s} = 1.02\times10^{15}Hz$

(b) $f = c/\lambda = \dfrac{3.00\times10^{8}m/s}{560\times10^{-9}m} = 5.36\times10^{14}Hz$ No.

Available energy/photon $hf = \left(4.14\times10^{-15}eV\cdot s\right)\left(5.36\times10^{14}Hz\right) = 2.22eV$.

This is less than ϕ.

3-29. (a) $E = hf = hc/\lambda \quad \Rightarrow \quad \lambda = hc/E$

For $E = 4.26eV$: $\lambda = \left(1240eV\cdot nm\right)/\left(4.26eV\right) = 291nm$

and since $f = c/\lambda$, $f = \left(3.00\times10^{8}m/s\right)/\left(291nm\right) = 1.03\times10^{15}s^{-1}$

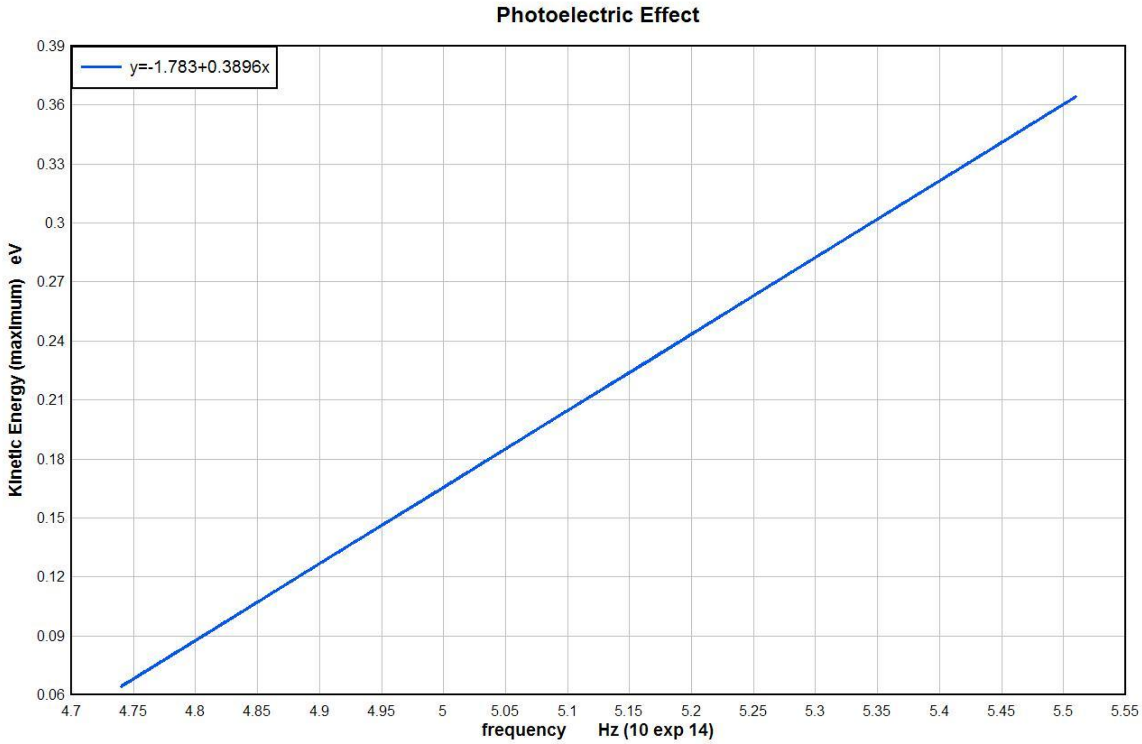
(b) This photon is in the ultraviolet region of the electromagnetic spectrum.

3-30. (a) First, add a row $f\times10^{14}$ Hz to the table in the problem, then plot a graph

$E_{k,\max}$ versus f. The slope of the graph is *h/e*; the intercept on the $-E_{k,\max}$ axis is work function. The graph below is a least squares fit to the data.

λ nm	544	594	604	612	633
$E_{k,\max}$ eV	0.360	0.199	0.156	0.117	0.062
$f\times10^{14}$ Hz	5.51	5.05	4.97	4.90	4.74

(Problem 3-30 continued)



$$\text{Slope} = h/e = 3.90\times10^{-15}\ \text{eV}\cdot\text{s}$$

(b) $\dfrac{(3.90\times10^{-15} - 4.14\times10^{-15})\,\text{eV}\cdot\text{s}}{4.14\times10^{-15}\ \text{eV}\cdot\text{s}} = -5.6\ \text{percent}$

(c) The work function is the magnitude of the intercept on the $E_{k,\max}$ axis, 1.78 eV.

(d) cesium

3-31. $$E = n\frac{hc}{\lambda} = \frac{(60)\left(6.63\times10^{-34}\,J\bullet s\right)\left(3.00\times10^{8}\,m/s\right)}{550\times10^{-9}\,m} = 2.17\times10^{-17}\,J$$

3-32. (a) $\phi = \dfrac{hc}{\lambda} = \dfrac{1240eV\bullet nm}{653nm} = 1.90eV$

(b) $E_k = \dfrac{hc}{\lambda} - \phi = \dfrac{1240eV\bullet nm}{300nm} - 1.90eV = 2.23eV$

3-33. Equation 3-25: $\lambda_2 - \lambda_1 = \Delta\lambda = \frac{h}{mc}(1-\cos\theta)$

$$\Delta\lambda = \frac{(6.63\times10^{-34}\,J\bullet s)(1-\cos135°)}{(9.11\times10^{-31}kg)(3.00\times10^{8}m/s)} = 4.14\times10^{-12}m = 4.14\times10^{-3}nm$$

$$\frac{\Delta\lambda}{\lambda_1}\times100 = \frac{4.14\times10^{-3}nm}{0.0711nm}\times100 = 5.8\%$$

3-34. Equation 3-24: $\lambda_m = \frac{1.24\times10^3}{V}nm = \frac{1.24\times10^3}{80\times10^3V} = 0.016nm$

3-35. $p = \frac{h}{\lambda} = \frac{hc}{\lambda c}$

(a) $p = \frac{1240eV\bullet nm}{c(400nm)} = 3.10eV/c = \frac{6.63\times10^{-34}\,J\bullet s}{400\times10^{-9}m} = 1.66\times10^{-27}kg\bullet m/s$

(b) $p = \frac{1240eV\bullet nm}{c(0.1nm)} = 1.24\times10^{4}eV/c = \frac{6.63\times10^{-34}\,J\bullet s}{0.1\times10^{-9}m} = 6.63\times10^{-24}kg\bullet m/s$

(c) $p = \frac{1240eV\bullet nm}{c(3\times10^{7}nm)} = 4.14\times10^{-5}eV/c = \frac{6.63\times10^{-34}\,J\bullet s}{3\times10^{-2}m} = 2.21\times10^{-32}kg\bullet m/s$

(d) $p = \frac{1240eV\bullet nm}{c(2nm)} = 620eV/c = \frac{6.63\times10^{-34}\,J\bullet s}{2\times10^{-9}m} = 3.32\times10^{-25}kg\bullet m/s$

3-36. $\lambda_2 - \lambda_1 = \frac{h}{mc}(1-\cos\theta) = \frac{(6.63\times10^{-34}\,J\bullet s)(1-\cos110°)}{(9.11\times10^{-31}kg)(3.00\times10^{8}m/s)} = 3.26\times10^{-12}m$

$$\lambda_1 = \frac{hc}{E_1} = \frac{(6.63\times10^{-34}\,J\bullet s)(3\times10^{8}m/s)}{(0.511\times10^{6}eV)(1.60\times10^{-19}J/eV)} = 2.43\times10^{-12}m$$

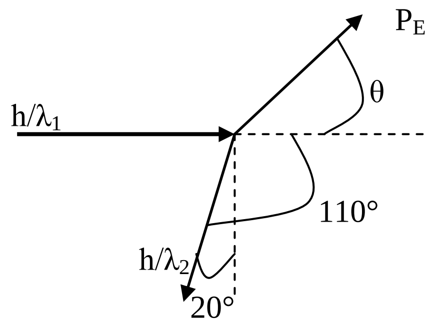
$$\lambda_2 = \lambda_1 + 3.26\times10^{-12}m = (2.43+3.26)\times10^{-12}m = 5.69\times10^{-12}m$$

(Problem 3-36 continued)

$$E_2 = \frac{hc}{\lambda_2} = \frac{1240 eV \bullet nm}{5.69\times10^{-3} nm} = 2.18\times10^5 eV = 0.218 MeV$$

Electron recoil energy $E_e = E_1 - E_2$ (Conservation of energy)

$E_e = 0.511 MeV - 0.218 MeV = 0.293 MeV$. The recoil electron momentum makes an angle θ with the direction of the initial photon.



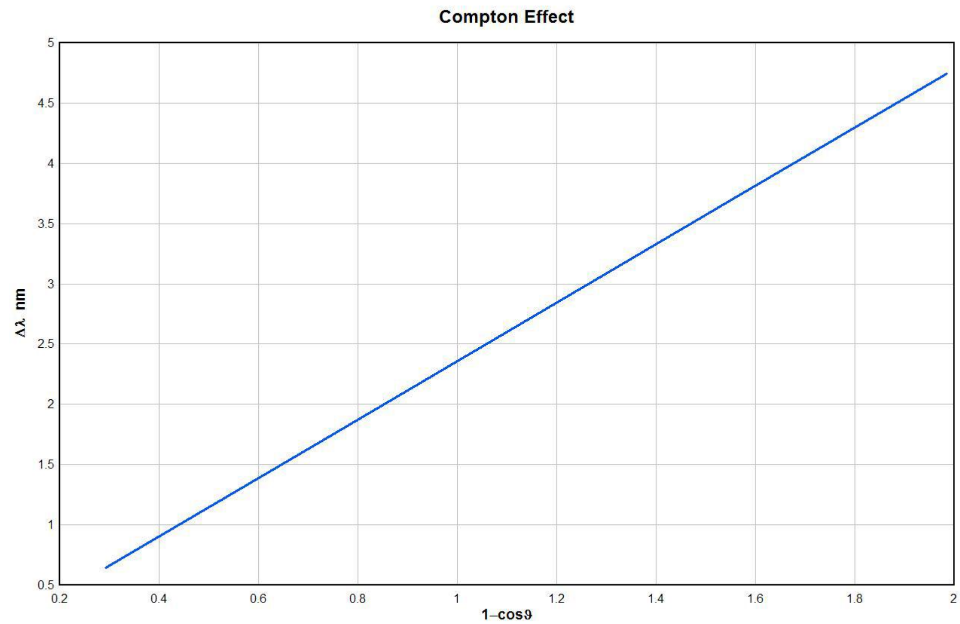
$$\frac{h}{\lambda_2}\cos 20^\circ = p_e \sin\theta = (1/c)\sqrt{E^2 - \left(mc^2\right)^2}\,\sin\theta \qquad \text{(Conservation of momentum)}$$

$$\sin\theta = \frac{\left(3.00\times10^8 m/s\right)\left(6.63\times10^{-34} J\bullet s\right)\cos 20^\circ}{\left(5.69\times10^{-12} m\right)\left[\left(0.804 MeV\right)^2 - \left(0.511 MeV\right)^2\right]^{1/2}\left(1.60\times10^{-13} J/MeV\right)}$$

$$= 0.330 \text{ or } \theta = 19.3^\circ$$

3-37. (a) First, add a row $(1-\cos\vartheta)$ to the table in the problem, then plot a graph of $\Delta\lambda$ versus $(1-\cos\vartheta)$. The slope of the graph is the Compton wavelength of the electron.

$\Delta\lambda$ pm	0.647	1.67	2.45	3.98	4.80
ϑ degrees	45	75	90	135	170
$1-\cos\vartheta$	0.293	0.741	1.000	1.707	1.985



(b) The graph above is a least squares fit to the data. The percent difference is

$$\frac{(2.43-2.426)\,\text{nm}}{2.426\,\text{nm}}\times 100=\frac{0.004\,\text{nm}}{2.426\,\text{nm}}\times 100=0.15\,\text{percent}$$

3-38. $\Delta\lambda=\lambda_2-\lambda_1=\Delta\lambda=\frac{h}{mc}(1-\cos\theta)=0.01\lambda_1$ Equation 3-25

$$\lambda_1=(100)\frac{h}{mc}(1-\cos\theta)=(100)(0.00243nm)(1-\cos 90°)=0.243nm$$

3-39. (a) $E_1=\frac{hc}{\lambda_1}=\frac{1240eV\bullet nm}{0.0711nm}=1.747\times 10^4 eV$

(b) $\lambda_2=\lambda_1+\frac{h}{mc}(1-\cos\theta)=0.0711nm+(0.00243nm)(1-\cos 180°)=0.0760nm$

(c) $E_2=\frac{hc}{\lambda_2}=\frac{1240eV\bullet nm}{0.0760nm}=1.634\times 10^4 eV$

(d) $E_e=E_1-E_2=1.128\times 10^3 eV$

3-40. (a) $\Delta\lambda = (h/mc)(1-\cos\theta)$

From protons:

$$\Delta\lambda = \left[\left(6.63\times10^{-34}\,J\bullet s\right)/\left(1.67\times10^{-27}\,kg\right)\left(3.00\times10^{8}\,m/s\right)\right]\left(1-\cos120°\right)$$

$$\Delta\lambda = 1.99\times10^{-15}\,m = 1.99\times10^{-6}\,nm$$

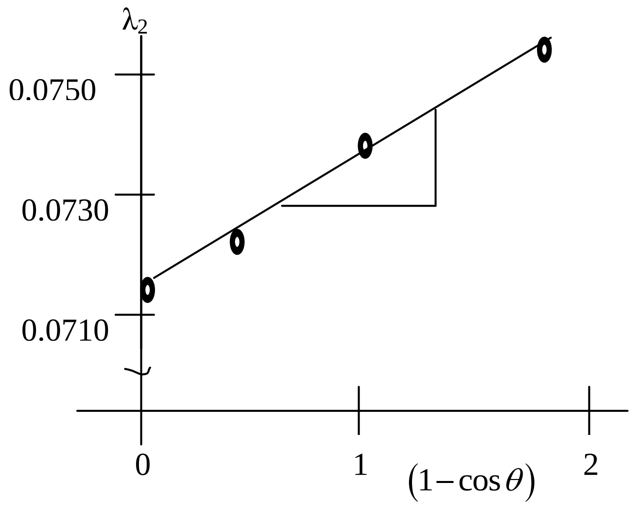
(b) Similarly, for electrons ($m = 9.11\times10^{-31}\,kg$)

$$\Delta\lambda = 2.43\times10^{-12}\,m = 2.43\times10^{-3}\,nm$$

(c) Similarly, for N_2 molecules ($m = 4.68\times10^{-26}\,kg$)

$$\Delta\lambda = 4.72\times10^{-17}\,m = 4.72\times10^{-8}\,nm$$

3-41. $\lambda_2 = \lambda_1 + \dfrac{h}{mc}(1-\cos\theta) = 0.0711 + (0.00243nm)(1-\cos\theta)$



θ	$(1-\cos\theta)$	λ_2 (nm)
0°	0	0.0711
45°	0.293	0.0718
90°	1	0.0735
135°	1.707	0.0752

Slope = $\dfrac{(0.0745-0.0720)nm}{(1.50-0.45)} = 2.381\times10^{-3}$

$$= \frac{h}{mc} \rightarrow h = \left(2.381\times10^{-3}\,nm\right)\left(9.11\times10^{-31}\,kg\right)\left(3.00\times10^{8}\,m/s\right) = 6.51\times10^{-34}\,J\bullet s$$

3-42. (a) Compton wavelength = $\dfrac{h}{mc}$

electron: $\dfrac{h}{mc} = \dfrac{6.63\times10^{-34}\,J\bullet s}{\left(9.11\times10^{-31}kg\right)\left(3.00\times10^{8}m/s\right)} = 2.43\times10^{-12}m = 0.00243nm$

proton: $\dfrac{h}{mc} = \dfrac{6.63\times10^{-34}\,J\bullet s}{\left(1.67\times10^{-27}kg\right)\left(3.00\times10^{8}m/s\right)} = 1.32\times10^{-15}m = 1.32\,fm$

(b) $E = \dfrac{hc}{\lambda}$

(i) electron: $E = \dfrac{1240eV\bullet nm}{0.00243nm} = 5.10\times10^{5}eV = 0.510MeV$

(ii) proton: $E = \dfrac{1240eV\bullet nm}{1.32\times10^{-6}nm} = 9.39\times10^{8}eV = 939MeV$

3-43. Photon energy $E = hf = hc/\lambda$ allows us to rewrite Equation 3-25 as

$$\frac{hc}{E_2} - \frac{hc}{E_1} = \frac{h}{mc}(1-\cos\vartheta)$$

Rearranging the above,

$$\frac{hc}{E_2} = \frac{h}{mc}(1-\cos\vartheta) + \frac{hc}{E_1}$$

Dividing both sides of the equation by hc yields

$$\frac{1}{E_2} = \frac{1}{mc^2}(1-\cos\vartheta) + \frac{1}{E_1} = \frac{(E_1/mc^2)(1-\cos\vartheta)+1}{E_1}$$

Or

$$E_2 = \frac{E_1}{(E_1/mc^2)(1-\cos\vartheta)+1}$$

3-44. (a) $eV_0 = hf - \phi = hc/\lambda - \phi$

$e\left(0.52V\right) = \left(hc/450nm\right) - \phi$ (i)

$e\left(1.90V\right) = \left(hc/300nm\right) - \phi$ (ii)

(Problem 3-44 continued)

Multiplying (i) by $450nm/e$ and (ii) by $300nm/e$, then subtracting (ii) from (i) and rearranging gives:

$$\frac{\phi}{e} = \frac{\left[(300nm)(1.90V) - (450nm)(0.52V)\right]}{150nm} = 2.24eV$$

(b) $\dfrac{hc}{e(300nm)} = 1.90 + 2.24 \quad \rightarrow \quad h = \dfrac{e\left(300\times10^{-9}m\right)(4.14V)}{\left(3.00\times10^{8}m/s\right)} = 6.63\times10^{-34}\,J\bullet s$

3-45. Including Earth's magnetic field in computing y_2, first show that y_2 is given by

$$y_2 = \frac{e}{m}\left[\frac{B^2 x_1 x_2}{\mathcal{E}} + \frac{1}{2}\frac{B_E x_2^2}{\mathcal{E}}\right]$$

where the second term in the brackets comes from $F_y = euB_E = ma_y$ and $y = \frac{1}{2}a_y t^2$.

Thus, $1 = \dfrac{e}{m}\left[\dfrac{B^2 x_1 x_2}{\mathcal{E}y_2} + \dfrac{1}{2}\dfrac{B_E x_2^2}{\mathcal{E}y_2}\right]$ The first term inside the brackets is the reciprocal of

$0.7\times10^{11}C$, Thomson's value for e/m. Using Thomson's data ($B = 5.5\times10^{-4}T$, $\mathcal{E} = 1.5\times10^{4}V/m$, $x_1 = 5cm$, $y_2/x_2 = 8/110$) and the modern value for e/m = $1.76\times10^{11}C/kg$ and solving for B_E.

$\dfrac{1}{2}\dfrac{B_E B x_2^2}{\mathcal{E}y_2} = -8.20\times10^{-12}$. The minus sign means that B and B_E are in opposite directions, which is why Thomson's value underestimated the actual value.

$$B_E = \frac{-\left(8.20\times10^{-12}\right)(2)\left(1.5\times10^{4}V/m\right)(8/110)^2}{\left(5.5\times10^{-4}T\right)\left(8\times10^{-2}m\right)} = -3.1\times10^{-5}T = -31\mu T$$

3-46. (a) $Q = Ne$ and $cM\Delta T = N\dfrac{mu^2}{2}$ where N = number of electrons, c = specific heat of the cup, M = mass of the cup, and u = electron's speed.

$$N = \frac{Q}{e} = \frac{2cM\Delta T}{mu^2} \quad \rightarrow \quad \frac{e}{m} = \frac{Qu^2}{2cM\Delta T}$$

(b) $\theta \approx \dfrac{eB\ell}{mu} \quad \rightarrow \quad u = \dfrac{eB\ell}{m\theta}$

Substituting u into the results of (a),

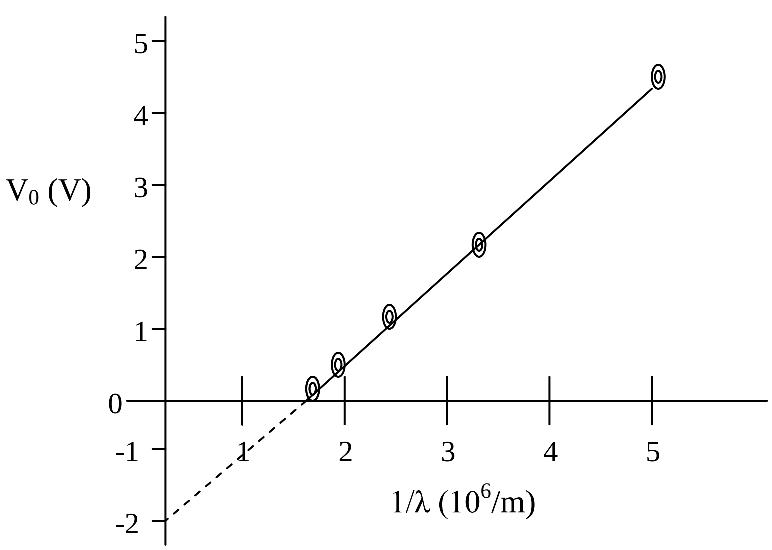
$$\frac{e}{m} = \frac{Q\left(eB\ell / m\theta\right)^2}{2cM\Delta T}$$

Solving for e/m,

$$\frac{e}{m} = \frac{2cM\theta^2\Delta T}{QB^2\ell^2}$$

3-47. Calculate $1/\lambda$ to be used in the graph.

$1/\lambda$ (10^6/m)	5.0	3.3	2.5	2.0	1.7
V_0 (V)	4.20	2.06	1.05	0.41	0.03



(a) The intercept on the vertical axis is the work function ϕ. $\phi = 2.08eV$.

(Problem 3-47 continued)

(b) The intercept on the horizontal axis corresponds to the threshold frequency.

$$\frac{1}{\lambda_t} = 1.65\times10^6 / m$$

$$f_t = \frac{c}{\lambda_t} = \left(3.00\times10^8 m/s\right)\left(1.65\times10^6 / m\right) = 4.95\times10^{14} Hz$$

(c) The slope of the graph is *h/e*. Using the vertical intercept and the largest experimental point.

$$\frac{h}{e} = \frac{1}{c}\frac{\Delta V_0}{\Delta(1/\lambda)} = \frac{4.20V-(-2.08V)}{\left(3.00\times10^8 m/s\right)\left(5.0\times10^6 /m-0\right)} = 4.19\times10^{-15} eV/Hz$$

3-48. In the center of momentum reference frame, the photon and the electron have equal and opposite momenta. $p_\gamma = E_\gamma / c = -p_e$.

The total energy is: $E_\gamma + E_e = E_\gamma + \left(p_e^2c^2 + m^2c^4\right)^{1/2} = E_\gamma + \left(E_\gamma^2 + m^2c^4\right)^{1/2}$

By conservation of momentum, the final state is an electron at rest, $p'_e = 0$. Conservation of energy requires that the final state energy E' is

$$E' = E_\gamma + E_e \quad \therefore \quad mc^2 = E_\gamma + \left(p^2c^2 + \left(mc^2\right)^2\right)^{1/2}$$

$$\therefore \quad mc^2 - E_\gamma = \left[p^2c^2 + \left(mc^2\right)^2\right]^{1/2} = \left[E_\gamma^2 + \left(mc^2\right)^2\right]^{1/2}$$

Squaring yields, $\left(mc^2\right)^2 - 2mc^2E_\gamma + E_\gamma^2 = E_\gamma^2 + \left(mc^2\right)^2 \quad \therefore \quad mc^2E_\gamma = 0.$ This can be true only if E_γ vanishes identically, i.e., if there is no photon at all.

3-49. Bragg condition: $m\lambda = 2d\sin\theta$. $\lambda = (2)(0.28nm)(\sin 20°) = 1.92\times10^{-10} m = 0.192nm$.

This is the minimum wavelength λ_m that must be produced by the X ray tube.

$$\lambda_m = \frac{1.24\times10^3}{V} nm \qquad \text{or} \qquad V = \frac{1.24\times10^3}{0.192} = 6.47\times10^3 V = 6.47kV$$

3-50. (a) $E = (100W)(10^4 s) = (100J/s)(10^4 s) = 10^6 J$

The momentum p absorbed is $p = \dfrac{E}{c} = \dfrac{10^6 J}{(3.00 \times 10^8 m/s)} = 3.33 \times 10^{-3} J \bullet s/m$

(b) $\Delta p = m(v_f - v_i) = (2 \times 10^{-3} kg)(v_f - 0) = 3.3 \times 10^{-3} J \bullet s/m$

$$\therefore v = \frac{3.33 \times 10^{-3} J \bullet s/m}{2 \times 10^{-3} kg} = 1.67 m/s$$

(c) $E = \dfrac{1}{2} m v_f^2 = \dfrac{(2 \times 10^{-3} kg)(1.67 m/s)^2}{2} = 2.78 \times 10^{-3} J$

The difference in energy has been (i) used to increase the object's temperature and (ii) radiated into space by the blackbody.

3-51. Conservation of energy: $E_1 + mc^2 = E_2 + E_k + mc^2 \quad \therefore \quad E_k = E_1 - E_2 = hf_1 - hf_2$

From Compton's equation, we have: $\lambda_2 - \lambda_1 = \dfrac{h}{mc}(1 - \cos\theta)$,

Thus, $\dfrac{1}{f_2} - \dfrac{1}{f_1} = \dfrac{h}{mc^2}(1 - \cos\theta)$

$$\frac{1}{f_2} = \frac{1}{f_1} + \frac{h}{mc^2}(1 - \cos\theta) \quad \therefore \quad f_2 = \frac{f_1 mc^2}{mc^2 + hf_1(1 - \cos\theta)}$$

Substituting this expression for f_2 into the expression for E_k (and dropping the subscript on f_1):

$$E_k = hf - \frac{hfmc^2}{mc^2 + hf(1 - \cos\theta)} = \frac{hfmc^2 + (hf)^2(1 - \cos\theta) - hfmc^2}{mc^2 + hf(1 - \cos\theta)} = \frac{hf}{1 + \dfrac{mc^2}{[hf(1 - \cos\theta)]}}$$

E_k has its maximum value when the photon energy change is maximum, i.e., when $\theta = \pi$ so $\cos\theta = -1$. Then $E_k = \dfrac{hf}{1 + \dfrac{mc^2}{2hf}}$

3-52. (a) $\lambda_m T = 2.898\times10^{-3}\,m\bullet K \quad \therefore \quad T = \dfrac{2.898\times10^{-3}\,m\bullet K}{82.8\times10^{-9}\,m} = 3.50\times10^{4}\,K$

(b) Equation 3-18: $\dfrac{u(70nm)}{u(82.8nm)} = \dfrac{(70nm)^{-5}/\left(e^{hc/(70nm)kT}-1\right)}{(82.8nm)^{-5}/\left(e^{hc/(82.8nm)kT}-1\right)}$

where $\dfrac{hc}{(70nm)kT} = \dfrac{(6.63\times10^{-34}\,J\bullet s)(3.00\times10^{8}\,m/s)}{(70\times10^{-9}\,m)(1.38\times10^{-23}\,J/K)(3.5\times10^{4}\,K)} = 5.88$ and

$\dfrac{hc}{(82.8nm)kT} = 4.97 \qquad \dfrac{u(70nm)}{u(82.8nm)} = \dfrac{(70nm)^{-5}/\left(e^{5.88}-1\right)}{(82.8nm)^{-5}/\left(e^{4.97}-1\right)} = 0.929$

Similarly, $\dfrac{u(100nm)}{u(82.8nm)} = \dfrac{(100nm)^{-5}/\left(e^{4.12}-1\right)}{(82.8nm)^{-5}/\left(e^{4.97}-1\right)} = 0.924$

3-53. Fraction of radiated solar energy in the visible region of the spectrum is the area under the Planck curve (Figure 3-6) between 350*nm* and 700*nm* divided by the total area. The latter is $6.42\times10^{7}\,W/m^2$ (see solution to Problem 3-25). Evaluating $u(\lambda)\Delta\lambda$ with $\lambda = 525nm$ (midpoint of visible) and $\Delta\lambda = 700nm - 350nm = 350nm.$

$$u(\lambda)\Delta\lambda = \frac{8\pi(6.63\times10^{-34}\,J\bullet s)(3.00\times10^{8}\,m/s)(525nm)^{-5}(350nm)}{\exp\left[\dfrac{(6.63\times10^{-34}\,J\bullet s)(3.00\times10^{8}\,m/s)}{(1.38\times10^{-23}\,J/k)(5800K)(525nm)}\right]-1} = 0.389\,J/m^3$$

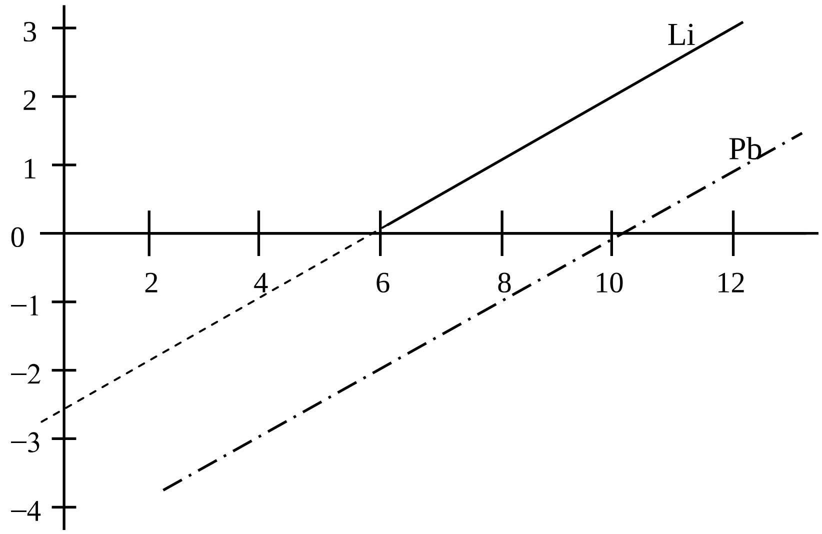
$$R = (350-700) = \frac{c}{4}u = (3.00\times10^{8}\,m/s)(0.389\,J/m^3)/4 = 2.92\times10^{7}\,W/m^2$$

Fraction in visible = $R(350-700)/R = (2.92\times10^{7}\,W/m^2)/(6.42\times10^{7}\,W/m^2) = 0.455$

3-54. (a) Make a table of $f = c/\lambda$ vs. V_0.

$f\,(\times10^{14}\,Hz)$	11.83	9.6	8.22	7.41	6.91
V_0 (V)	2.57	1.67	1.09	0.73	0.55

(Problem 3-54 continued)



The work function for Li (intercept on the vertical axis) is $\phi = 2.40eV$.

(b) The slope of the graph is *h/e*. Using the largest V_0 and the intercept on the vertical axis, $\frac{h}{e} = \frac{2.57V - (-2.40V)}{11.53\times 10^{14}Hz - 0}$ or, $h = \frac{(4.97V)(1.60\times 10^{-19}C)}{11.53\times 10^{14}Hz} = 6.89\times 10^{-34}J\bullet s$

(c) The slope is the same for all metals. Draw a line parallel to the Li graph with the work function (vertical intercept) of Pb, $\phi = 4.14eV$. Reading from the graph, the threshold frequency for Pb is $9.8\times 10^{14}Hz$; therefore, no photon wavelengths larger than $\lambda = c/f_t = (3.00\times 10^8 m/s)(9.8\times 10^{14}Hz) = 306nm$ will cause emission of photoelectrons from Pb.

3-55. (a) Equation 3-18: $u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$ Letting $C = 8\pi hc$ and $a = hc/kT$ gives $u(\lambda) = \frac{C\lambda^{-5}}{e^{a/\lambda} - 1}$

(b)

$$\frac{du}{d\lambda} = \frac{d}{d\lambda}\left[\frac{C\lambda^{-5}}{e^{a/\lambda} - 1}\right] = C\left[\frac{\lambda^{-5}(-1)e^{a/\lambda}(-a\lambda^{-2})}{(e^{a/\lambda} - 1)^2} - \frac{5\lambda^{-6}}{e^{a/\lambda} - 1}\right]$$

$$= \frac{C\lambda^{-6}}{(e^{a/\lambda} - 1)^2}\left[\frac{a}{\lambda}e^{a/\lambda} - 5(e^{a/\lambda} - 1)\right] = \frac{C\lambda^{-6}e^{a/\lambda}}{(e^{a/\lambda} - 1)^2}\left[\frac{a}{\lambda} - 5(1 - e^{a/\lambda})\right] = 0$$

(Problem 3-55 continued)

The maximum corresponds to the vanishing of the quantity in brackets.
Thus, $5\lambda\left(1-e^{-a/\lambda}\right)=a$

(c) This equation is most efficiently solved by trial and error; i.e., guess at a value for a/λ in the expression $5\lambda\left(1-e^{-a/\lambda}\right)=a$, solve for a better value of a/λ; substitute the new value to get an even better value, and so on. Repeat the process until the calculated value no longer changes. One succession of values is 5, 4.966310, 4.965156, 4.965116, 4.965114, 4.965114. Further iterations repeat the same value (to seven digits), so we have $\dfrac{a}{\lambda_m}=4.965114=\dfrac{hc}{\lambda_m kT}$

(d) $$\lambda_m T=\frac{hc}{(4.965114)k}=\frac{\left(6.63\times10^{-34}J\bullet s\right)\left(3.00\times10^{8}m/s\right)}{(4.965114)\left(1.38\times10^{-23}J/K\right)}$$

Therefore, $\lambda_m T=2.898\times10^{-3}m\bullet K$ Equation 3-5

3-56. (a) $$I=\frac{P}{4\pi R^2}=\frac{1W}{4\pi(1m)^2}\left(\frac{1}{1.602\times10^{-19}J/eV}\right)=4.97\times10^{17}eV/m^2\bullet s$$

(b) Let the atom occupy an area of $(0.1nm)^2$.

$$\frac{dW}{dt}=IA=\left(4.97\times10^{17}eV/m^2\bullet s\right)(0.1nm)^2\left(10^{-9}m/nm\right)^2=4.97\times10^{-3}eV/s$$

(c) $$t=\frac{\phi}{dW/dt}=\frac{2eV}{4.97\times10^{-3}eV/s}=403s=6.71\text{ min}$$

3-57. (a) The nonrelativistic expression for the kinetic energy pf the recoiling nucleus is

$$E_k=\frac{p^2}{2m}=\frac{(15MeV/c)^2}{2\times12u}\left(\frac{1u}{931.5MeV/c^2}\right)=1.10\times10^4eV$$

Internal energy $U=15MeV-0.0101MeV=14.9899MeV$

(Problem 3-57 continued)

(b) The nucleus must recoil with momentum equal to that of the emitted photon, about 14.98 MeV/c.

$$E_k = \frac{p^2}{2m} = \frac{(14.98MeV/c)^2}{2\times 12u}\left(\frac{1u}{931.5MeV/c^2}\right) = 1.00\times 10^{-2}eV$$

$$E_\gamma = U - E_k = 14.9899MeV - 0.0100MeV = 14.9799MeV$$

3-58. Derived in Problem 3-47, the electron's kinetic energy at the Compton edge is

$$E_k = \frac{hf}{1 + mc^2/2hf}$$

$$E = 520keV = \frac{hf}{1+(511keV)/2hf} \quad \therefore \quad 520keV = \frac{2(hf)^2}{2hf + 511keV}$$

Thus, $(hf)^2 - 520(hf) - (520)(511)/2 = 0$

Solving with the quadratic formula: $hf = \dfrac{520 \pm \left[(520)^2 + (2)(520)(511)\right]^2}{2} = 708keV$

(only the + sign is physically meaningful). Energy of the incident gamma ray

$hf = 708keV$.

$$\frac{hc}{\lambda} = 708keV \quad \Rightarrow \quad \lambda = \frac{(6.63\times 10^{-34}J\bullet s)(3.00\times 10^8 m/s)}{(708keV)(1.60\times 10^{-16}J/keV)} = 1.76\times 10^{-12}m = 1.76pm$$

3-59. (a) $E_k = 50keV$ and $\lambda_2 = \lambda_1 + 0.095nm$

$$\frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} = 5.0\times 10^4 eV \quad \therefore \quad \frac{1}{\lambda_1} + \frac{1}{\lambda_1 + 0.095} = \frac{5.0\times 10^4 eV}{hc}$$

$$\therefore \quad \frac{2\lambda_1 + 0.095}{\lambda_1^2 + 0.095\lambda_1} = \frac{5.0\times 10^4 eV}{hc}$$

$$\lambda_1^2 + \left(0.095nm - \frac{2hc}{5\times 10^4 eV}\right)\lambda_1 - \frac{(0.095nm)hc}{5\times 10^4 eV} = 0$$

$$\therefore \quad \lambda_1^2 + 0.04541\lambda_1 - 2.36\times 10^{-3} = 0$$

(Problem 3-59 continued)

Applying the quadratic formula,

$$\lambda_1 = \frac{-0.04541 \pm \left[\left(0.04541\right)^2 + 4\left(2.36\times 10^{-3}\right)\right]^{1/2}}{2}$$

$\lambda_1 = 0.03092nm$ and $\lambda_2 = 0.1259nm$

(b) $E_1 = \dfrac{hc}{\lambda_1} = \dfrac{1240eV\bullet nm}{0.03092nm} = 40.1keV \quad\rightarrow\quad E_{electron} = 9.90keV$

3-60. Let $x = \dfrac{\varepsilon}{kT} = \dfrac{hf}{kT}$ in Equation 3-15:

$$\sum_{n=0}^{\infty} f_n = A\sum_{n=0}^{\infty} e^{-nx} = A\left[e^0 + e^{-x} + \left(e^{-x}\right)^2 + \left(e^{-x}\right)^3 + \cdots\right] = A\left(1 + y + y^2 + y^3 + \cdots\right) = 1$$

Where $y = e^{-x}$. This sum is the series expansion of

$\left(1-y\right)^{-1}$, i.e., $\left(1-y\right)^{-1} = 1 + y + y^2 + y^3 + \cdots$. Then $\sum f_n = A\left(1-y\right)^{-1} = 1$ gives $A = 1-y$.

Writing Equation 3-16 in terms of x and y.

$$\overline{E} = \sum_{n=0}^{\infty} E_n A e^{-E_n/kT} = A\sum_{n=0}^{\infty} nhfe^{-nhf/kT} = Ahf\sum_{n=0}^{\infty} ne^{-nx}$$

Note that $\sum ne^{-nx} = -\left(d/dx\right)\sum e^{-nx}$. But $\sum e^{-nx} = \left(1-y\right)^{-1}$, so we have

$$\sum ne^{-nx} = -\frac{d}{dx}\sum e^{-nx} = -\frac{d}{dx}\left(1-y\right)^{-1} = \left(1-y\right)^{-2}\left(-\frac{dy}{dx}\right) = y\left(1-y\right)^{-2}$$

Since $\dfrac{dy}{dx} = \dfrac{d\left(e^{-x}\right)}{dx} = -e^{-x} = -y.$

Multiplying this sum by *hf* and by $A = \left(1-y\right)$, the average energy is

$$\overline{E} = hfA\sum_{n=0}^{\infty} ne^{-nx} = hf\left(1-y\right)y\left(1-y\right)^{-2} = \frac{hfy}{1-y} = \frac{hfe^{-x}}{1-e^{-x}}$$

Multiplying the numerator and the denominator by e^{-x} and substituting for *x*, we obtain

$\overline{E} = \dfrac{hf}{e^{hf/kT} - 1}$, which is Equation 3-17.

Chapter 4 – The Nuclear Atom

4-1. $$\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \text{ where } R = 1.097\times10^7 m^{-1} \quad \text{(Equation 4-2)}$$

The Lyman series ends on m =1, the Balmer series on m =2, and the Paschen series on m =3. The series limits all have $n = \infty$, so $\frac{1}{n} = 0$

$$\frac{1}{\lambda_L} = R\left(\frac{1}{1^2}\right) = 1.097\times10^7 m^{-1}$$

$$\lambda_L(\text{limit}) = 1.097\times10^7 m^{-1} = 91.16\times10^{-9} m = 91.16 nm$$

$$\frac{1}{\lambda_B} = R\left(\frac{1}{2^2}\right) = 1.097\times10^7 m^{-1}/4$$

$$\lambda_B(\text{limit}) = 4/1.097\times10^7 m^{-1} = 3.646\times10^{-7} m = 364.6 nm$$

$$\frac{1}{\lambda_P} = R\left(\frac{1}{3^2}\right) = 1.097\times10^7 m^{-1}/9$$

$$\lambda_P(\text{limit}) = 9/1.097\times10^7 m^{-1} = 8.204\times10^{-7} m = 820.4 nm$$

4-2. $$\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \text{ where } m = 2 \text{ for Balmer series} \quad \text{(Equation 4-2)}$$

$$\frac{1}{379.1nm} = \frac{1.097\times10^7 m^{-1}}{10^9 nm/m}\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{4} - \frac{1}{n^2} = \frac{10^9 nm/m}{379.1nm\left(1.097\times10^7 m^{-1}\right)} = 0.2405$$

$$\frac{1}{n^2} = 0.2500 - 0.2405 = 0.0095$$

$$n^2 = \frac{1}{0.0095} \quad\rightarrow\quad n = \left(1/0.0095\right)^{1/2} = 10.3 \quad\rightarrow\quad n = 10$$

$$n = 10 \rightarrow n = 2$$

4-3. $\dfrac{1}{\lambda_{mn}} = R\left(\dfrac{1}{m^2} - \dfrac{1}{n^2}\right)$ where $m = 1$ for Lyman series (Equation 4-2)

$$\frac{1}{164.1nm} = \frac{1.097\times 10^7 m^{-1}}{10^9 nm/m}\left(1 - \frac{1}{n^2}\right)$$

$$\frac{1}{n^2} = 1 - \frac{10^9 nm/m}{164.1nm\left(1.097\times 10^7 m^{-1}\right)} = 1 - 0.5555 = 0.4445$$

$$n = (1/0.4445)^{1/2} = 1.5$$

No, this is not a hydrogen Lyman series transition because n is not an integer.

4-4. $\dfrac{1}{\lambda_{mn}} = R\left(\dfrac{1}{m^2} - \dfrac{1}{n^2}\right)$ (Equation 4-2)

For the Brackett series m = 4 and the first four (i.e., longest wavelength lines have n = 5, 6, 7, and 8.

$$\frac{1}{\lambda_{45}} = 1.097\times 10^7 m^{-1}\left(\frac{1}{4^2} - \frac{1}{5^2}\right) = 2.468\times 10^5 m^{-1}$$

$\lambda_{45} = \dfrac{1}{2.68\times 10^5 m^{-1}} = 4.052\times 10^{-6} m = 4052nm.$ Similarly,

$$\lambda_{46} = \frac{1}{3.809\times 10^5 m^{-1}} = 2.625\times 10^{-6} m = 2625nm$$

$$\lambda_{47} = \frac{1}{4.617\times 10^5 m^{-1}} = 2.166\times 10^{-6} m = 2166nm$$

$$\lambda_{48} = \frac{1}{5.142\times 10^5 m^{-1}} = 1.945\times 10^{-6} m = 1945nm$$

These lines are all in the infrared.

4-5. None of these lines are in the Paschen series, whose limit is 820.4 nm (see Problem 4-1) and whose first line is given byL $\dfrac{1}{\lambda_{34}} = R\left(\dfrac{1}{3^2} - \dfrac{1}{4^2}\right) \rightarrow \lambda_{34} = 1875nm.$ Also, none are in the Brackett series, whose longest wavelength line is 4052 nm (see Problem 4-4). The Pfund series has m = 5. Its first three (i.e., longest wavelength) lines have n = 6, 7, and 8.

(Problem 4-5 continued)

$$\frac{1}{\lambda_{56}} = 1.097\times10^{7}m^{-1}\left(\frac{1}{5^2}-\frac{1}{5^2}\right) = 1.341\times10^{5}m^{-1}$$

$\lambda_{56} = \dfrac{1}{1.341\times10^{5}m^{-1}} = 7.458\times10^{-6}m = 7458nm.$ Similarly,

$$\lambda_{57} = \frac{1}{2.155\times10^{5}m^{-1}} = 4.653\times10^{-6}m = 4653nm$$

$$\lambda_{58} = \frac{1}{2.674\times10^{5}m^{-1}} = 3.740\times10^{-6}m = 3740nm$$

Thus, the line at 4103 *nm* is not a hydrogen spectral line.

4-6. (a) $f = \pi b^2 nt$ (Equation 4-5)

For Au, $n = 5.90\times10^{28}\,atoms/m^3$ (see Example 4-2) and for this foil $t = 2.0\mu m = 2.0\times10^{-6}m.$

$$b = \frac{kq_\alpha Q}{m_\alpha v^2}\cot\frac{\theta}{2} = \frac{(2)(79)ke^2}{2K_\alpha}\cot\frac{90}{2} = \frac{(2)(79)(1.44eV\bullet nm)}{2(7.0\times10^{6}eV)}$$

$$= 1.63\times10^{-5}nm = 1.63\times10^{-14}m$$

$$f = \pi\left(1.63\times10^{-14}m\right)^2\left(5.90\times10^{28}/m^3\right)\left(2.0\times10^{-6}m\right) = 9.8\times10^{-5}$$

(b) For $\theta = 45°$, $b(45°) = b(90°)(\cot 45°/2)/(\cot 90°/2)$

$$= b(90°)(\tan 90°/2)/(\tan 45°/2)$$

$$= 3.92\times10^{-5}nm = 3.92\times10^{-14}m$$

and $f(45°) = 5.7\times10^{-4}$

For $\theta = 75°$, $b(75°) = b(90°)(\tan 90°/2)/(\tan 75°/2)$

$$= 2.12\times10^{-5}nm = 2.12\times10^{-14}m$$

and $f(75°) = 1.66\times10^{-4}$

Therefore, $\Delta f(45° - 75°) = 5.7\times10^{-4} - 1.66\times10^{-4} = 4.05\times10^{-4}$

(Problem 4-6 continued)

(c) Assuming the Au atom to be a sphere of radius *r*,

$$\frac{4}{3}\pi r^3 = \frac{M}{N_A\rho} = \frac{197g/mole}{\left(6.02\times10^{23}atoms/mole\right)\left(19.3g/cm^3\right)}$$

$$r = \left[\frac{3}{4\pi}\frac{197g/mole}{\left(6.02\times10^{23}atoms/mole\right)\left(19.3g/cm^3\right)}\right]^{1/3}$$

$$r = 1.62\times10^{-3}cm = 1.62\times10^{-10}m = 16.2nm$$

4-7. $\Delta N \propto \frac{1}{\sin^4(\theta/2)} = \frac{A}{\sin^4(\theta/2)}$ (From Equation 4-6), where A is the product of the two quantities in parentheses in Equation 4-6.

(a) $\frac{\Delta N(10°)}{\Delta N(1°)} = \frac{A/\sin^4(10°/2)}{A/\sin^4(1°/2)} = \frac{\sin^4(0.5°)}{\sin^4(5°)} = 1.01\times10^{-4}$

(b) $\frac{\Delta N(30°)}{\Delta N(1°)} = \frac{\sin^4(0.5°)}{\sin^4(15°)} = 1.29\times10^{-6}$

4-8. $b = \frac{kq_\alpha Q}{m_\alpha v^2}\cot\frac{\theta}{2}$ (Equation 4-3)

$$= \frac{k\bullet 2e\bullet Ze}{m_\alpha v^2}\cot\frac{\theta}{2} = \frac{(1.44MeV\bullet fm)Z}{E_{k\alpha}}\cot\frac{\theta}{2}$$

$$= \frac{(1.44MeV\bullet fm)(79)}{7.7MeV}\cot\frac{2°}{2} = 8.5\times10^{-13}m$$

4-9. $r_d = \frac{kq_\alpha Q}{(1/2)m_\alpha v^2} = \frac{ke^2\bullet 2\bullet 79}{E_{k\alpha}}$ (Equation 4-11)

For $E_{k\alpha} = 5.0MeV$: $r_d = \frac{(1.44MeV\bullet fm)(2)(79)}{5.0MeV} = 45.5fm$

For $E_{k\alpha} = 7.7MeV$: $r_d = 29.5fm$

For $E_{k\alpha} = 12MeV$: $r_d = 19.0fm$

4-10. $r_d = \dfrac{kq_\alpha Q}{(1/2)m_\alpha v^2} = \dfrac{ke^2 \bullet 2 \bullet 79}{E_{k\alpha}}$ (Equation 4-11)

$$E_{k\alpha} = \frac{(1.44MeV \bullet fm)(2)(13)}{4\,fm} = 9.4MeV$$

4-11. $x_{rms} = \sqrt{N}(\delta)$ $10° = \sqrt{N}(0.01°) \rightarrow N = (10°/0.01°)^2 = 10^6$ collisions

$$n = \frac{t}{\Delta t} = \frac{10^{-6}m}{10^{-10}m} = 10^4 \text{ layers}$$

10^4 atomic layers is not enough to produce a deflection of $10°$, assuming 1 collision/layer.

4-12. (a) $f = \pi b^2 nt$ (Equation 4-5)

For $\theta = 25°$ (refer to Problem 4-6).

$$b = \frac{(2)(79)ke^2}{2K_\alpha}\cot\frac{25}{2} = \frac{(2)(79)(1.44eV \bullet nm)}{2(7.0\times10^6 eV)}\cot\left(\frac{25°}{2}\right)$$

$$= 7.33\times10^{-5}nm = 7.33\times10^{-14}m$$

$$f = \pi(7.33\times10^{-14}m)^2(5.90\times10^{28}/m^3)(2.0\times10^{-6}m) = 1.992\times10^{-3}$$

Because $\Delta N = f \times N = 1000 \quad \rightarrow \quad N = 1000/1.992\times10^{-3} = 5.02\times10^5$

For $\theta = 45°$, $b = \dfrac{(2)(79)(1.44eV \bullet nm)}{2(7.0\times10^6 eV)}\cot\left(\dfrac{45°}{2}\right) = 3.92\times10^{-14}m$

$$f = \pi(3.92\times10^{-14}m)^2(5.90\times10^{28}/m^3)(2.0\times10^{-6}m) = 5.70\times10^{-4}$$

Because $\Delta N(\theta > 45°) = f \times N = 5.70\times10^{-4}(5.02\times10^5) = 286$

(b) $\Delta N(25° \rightarrow 45°) = 1000 - 286 = 714$

(c) For $\theta = 75°$, $b = b(\theta > 25°)(\tan 25°/2)/(\tan 75°/2) = 2.12\times10^{-14}m$

$$f = 1.992\times10^{-3}(2.12\times10^{-14}m)^2/(7.33\times10^{-14}m)^2$$

$$= 1.992\times10^{-3}(2.12/7.33)^2 = 1.67\times10^{-4}$$

(Problem 4-12 continued)

For $\theta = 90°$, $b = b(\theta > 25°)(\tan 25°/2)/(\tan 90°/2) = 1.63\times10^{-14}m$

$$f = 1.992\times10^{-3}\left(1.63\times10^{-14}m\right)^2 / \left(7.33\times10^{-14}m\right)^2$$
$$= 1.992\times10^{-3}\left(1.63/7.33\right)^2 = 9.85\times10^{-5}$$
$$\Delta N = f\times N = 9.85\times10^{-5}\left(5.02\times10^5\right) = 49$$
$$\Delta N = \left(75° \to 90°\right) = 84 - 49 = 35$$

4-13. (a) $r_n = \dfrac{n^2 a_0}{Z}$ (Equation 4-18)

$$r_6 = \frac{6^2\left(0.053nm\right)}{1} = 1.91nm$$

(b) $r_6\left(He^+\right) = \dfrac{6^2\left(0.053nm\right)}{2} = 0.95nm$

4-14. $a_0 = \dfrac{\hbar^2}{mke^2}$ (Equation 4-19)

$$= \frac{\hbar\hbar c}{mcke^2} = \frac{\hbar c}{mc^2}\times\frac{1}{ke^2/\hbar c} = \frac{1}{2\pi}\times\frac{h}{mc}\times\frac{1}{ke^2/\hbar c} = \frac{\lambda_c}{2\pi\alpha}$$

$\left|E_1\right| = \dfrac{mk^2e^4}{2\hbar^2}$ (from Equation 4-20)

$$= \frac{mc^2\left(ke^2\right)^2}{2\left(\hbar c\right)^2} = \frac{mc^2}{2}\times\left(\frac{ke^2}{\hbar c}\right)^2 = \frac{1}{2}mc^2\alpha^2$$

$$a_0 = \frac{\lambda_c}{2\pi\alpha} = \frac{0.00243nm}{2\pi\left(1/137\right)} = 0.053nm \qquad \left|E_1\right| = \frac{1}{2}mc^2\alpha^2 = \frac{5.11\times10^5 eV}{2\left(137\right)^2} = 13.6eV$$

4-15. $\dfrac{1}{\lambda} = Z^2R\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right)$ (Equation 4-22)

(Problem 4-15 continued)

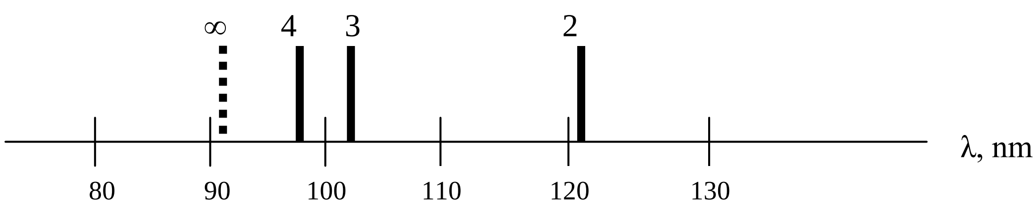
$$\frac{1}{\lambda_{ni}} = R\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right) = \mathrm{R}\left(\frac{\mathrm{n_i^2} - 1}{n_i^2}\right)$$

$$\lambda_{ni} = \frac{n_i^2}{R\left(n_i^2 - 1\right)} = \frac{\mathrm{n_i^2}}{\left(1.0968\times10^7 m\right)\left(\mathrm{n_i^2} - 1\right)} = \left(91.17nm\right)\left(\frac{\mathrm{n_i^2}}{\mathrm{n_i^2} - 1}\right)$$

$$\lambda_2 = \frac{4}{3}\left(91.17nm\right) = 121.57nm \qquad \lambda_3 = \frac{9}{8}\left(91.17nm\right) = 102.57nm$$

$$\lambda_4 = \frac{16}{15}\left(91.17nm\right) = 97.25nm \qquad \lambda_\infty = 91.17nm$$

None of these are in the visible; all are in the ultraviolet.



4-16. $L = mvr = n\hbar$ (Equation 4-17)

$$m_E = 5.98\times10^{24} kg \qquad v_E = 2\pi r/1y = 2\pi r/3.16\times10^7 s$$

$$n = m\left(2\pi r/3.16\times10^7 s\right)r/\hbar = 2\pi m r^2/\left(3.16\times10^7 s\right)\hbar$$

$$= \frac{2\pi\left(5.98\times10^{24} kg\right)\left(1.50\times10^{11} m\right)^2}{3.16\times10^7 s\left(1.055\times10^{-34} J\bullet s\right)} = 2.54\times10^{74}$$

$$mv = n\hbar/r \quad\rightarrow\quad E = \left(mv\right)^2/2m = \left(n\hbar/r\right)^2/2m \quad \text{(from Equation 4-17)}$$

$$\Delta E = \left(\frac{\hbar}{r}\right)^2\frac{1}{2m}\left(2n\Delta n\right) = \frac{\left(1.055\times10^{-34} J\bullet s\right)^2\left(2.54\times10^{74}\right)\left(1\right)}{\left(1.50\times10^{11} m\right)^2\left(5.98\times10^{24} kg\right)} = 0.210\times10^{-40} J$$

This would not be detectable.

$$\Delta E = \frac{\left(n\hbar\right)^2}{2m}\left(-\frac{2\Delta r}{r^3}\right) = \frac{\left(1.055\times10^{-34} J\bullet s\right)^2\left(2.54\times10^{74}\right)^2\left(-\Delta r\right)}{\left(1.50\times10^{11} m\right)^2\left(5.98\times10^{24} kg\right)} = 5.34\times10^{33}\left(-\Delta r\right)$$

or $-\Delta r = 0.210\times10^{-40} J/5.34\times10^{33} J/m = 3.93\times10^{-75} m$

The orbit radius r would still be $1.50\times10^{11} m$.

4-17. $f_{rev} = \frac{mk^2 Z^2 e^4}{2\pi\hbar^3 n^3}$ (Equation 4-29)

$$= \frac{mc^2 Z^2 \left(ke^2\right)^2}{2\pi\hbar n^3 \left(\hbar c\right)^2} = \frac{cZ^2}{\left(h/mc\right)n^3}\left(\frac{ke^2}{\hbar c}\right)^2 = \frac{cZ^2\alpha^2}{\lambda_c n^3}$$

$$= \frac{\left(3.00\times10^8 m/s\right)\left(1\right)^2}{\left(0.00243\times10^{-9}m\right)\left(2\right)^3}\left(\frac{1}{137}\right)^2 = 8.22\times10^{14}Hz$$

$$N = f_{rev}t = \left(8.22\times10^{14}Hz\right)\left(10^{-8}s\right) = 8.22\times10^6 \text{ revolutions}$$

4-18. The number of revolutions N in 10^{-8} s is:

$$N = 10^{-8}s/\left(\text{time/revolution}\right) = 10^{-8}s/\left(\text{circumference of orbit/speed}\right)$$

$$N = 10^{-8}s/\left(C/v\right) = 10^{-8}s/\left(2\pi r/v\right)$$

The radius of the orbit is given by:

$$r = \frac{n^2 a_0}{Z} = \frac{4^2\left(0.0529nm\right)}{3}$$

so the circumference of the orbit $C = 2\pi r$ is

$$C = 2\pi\left[4^2\left(0.0529nm\right)/3\right] = 1.77nm = 1.77\times10^{-9}m$$

The electron's speed in the orbit is given by

$$v^2 = \left(kZe^2/mr\right) = \frac{\left(8.99\times10^9 N\bullet m^2/C^2\right)\left(3\right)\left(1.60\times10^{-19}C\right)^2}{\left(9.11\times10^{-31}kg\right)\left(1.77\times10^{-9}m\right)}$$

$$v = 6.54\times10^5 m/s$$

Therefore, $N = 10^{-8}s/\left(C/v\right) = 3.70\times10^6$ revolutions

In the planetary analogy of Earth moving around the sun, this corresponds to 3.7 million "years".

4-19. (a) $a_u = \frac{\hbar^2}{\mu_\mu ke^2} = \frac{\mu_e}{\mu_\mu}\bullet\frac{\hbar^2}{\mu_e ke^2} = \frac{\mu_e}{\mu_\mu}a_0 = \frac{9.11\times10^{-31}kg}{1.69\times10^{-28}kg}\left(0.0529nm\right) = 2.56\times10^{-4}nm$

(b) $E_\mu = \frac{\mu_\mu k^2 e^4}{2\hbar^2} = \frac{\mu_\mu}{\mu_e}\bullet\frac{\mu_e k^2 e^4}{2\hbar^2} = \frac{\mu_\mu}{\mu_e}\bullet E_0 = \frac{1.69\times10^{-28}kg}{9.11\times10^{-31}kg}\left(13.6eV\right) = 2520eV$

(Problem 4-19 continued)

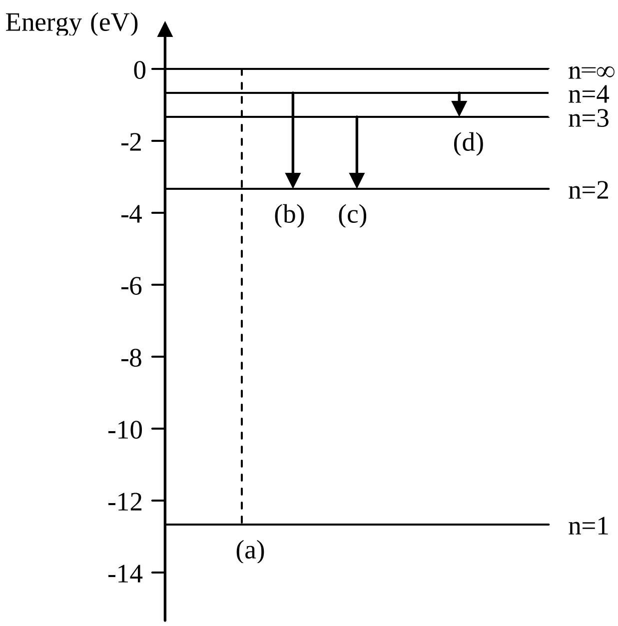
(c) The shortest wavelength in the Lyman series is the series limit ($n_i = \infty$, $n_f = 1$). The photon energy is equal in magnitude to the ground state energy $-E_\mu$.

$$\lambda_\infty = \frac{hc}{E_\mu} = \frac{1240eV \bullet nm}{2520eV} = 0.492nm$$

(The reduced masses have been used in this solution.)

4-20. $E = -Z'^2 E_0 / n^2$ $\quad Z' = \left[\frac{-n^2 E}{E_0}\right]^{1/2} = \left[\frac{-2^2(-5.39eV)}{13.6eV}\right]^{1/2} = 1.26$

4-21.



(a) Lyman limit, (b) H_β line, (c) H_α line, (d) longest wavelength line of Paschen series

4-22. (a) $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

For Lyman α: $\frac{1}{\lambda} = 1.097373 \times 10^7 m^{-1}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \quad \rightarrow \quad \lambda_L = 121.5023nm$

$$E_L = \frac{hc}{\lambda_L} = \frac{1240eV \bullet nm}{121.5023nm} = 10.2056eV \text{ and } p_L = \frac{E_L}{c} = 10.2056eV/c$$

(Problem 4-22 continued)

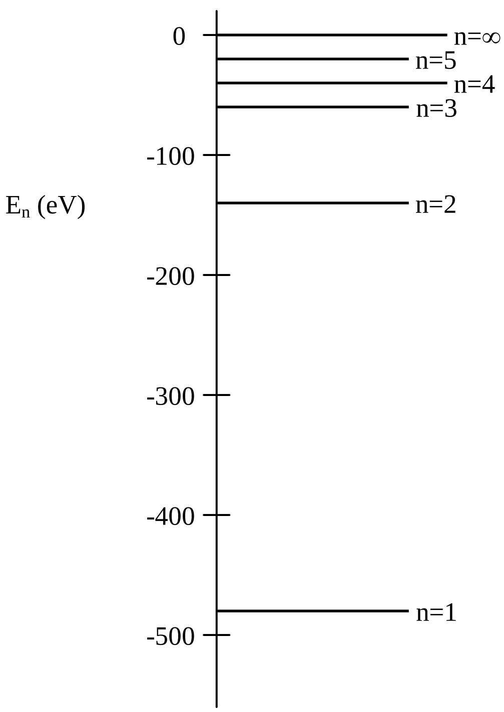
Conservation of momentum requires that the recoil momentum of the H atom $p_H = p_L$ and the recoil energy E_H is:

$$E_H = (p_H)^2/2m_H = (p_H c)^2/2m_H c^2 = \frac{(10.2056eV/c)^2}{2(1.007825uc^2)(931.50\times10^6 eV/uc^2)}$$

$$= 5.55\times10^{-8} eV$$

(b) $\dfrac{E_H}{(E_L + E_H)} \approx \dfrac{5.5\times10^{-8}eV}{10.21eV} = 5\times10^{-9}$

4-23. (a) For C^{5+} ($Z = 6$) $E_n = -13.6\dfrac{Z^2}{n^2} = -\dfrac{489.6}{n^2}$



E_∞ = 0 *eV*
E_5 = -19.6 *eV*
E_4 = -30.6 *eV*
E_3 = -54.4 *eV*
E_2 = -122.4 *eV*
E_1 = -489.6 *eV*

(b) $\lambda = \dfrac{hc}{\Delta E} = \dfrac{hc}{E_3 - E_2} = \dfrac{1240eV\bullet nm}{[-54.4-(-122.4)]eV} = 18.2nm$

(c) 18.2*nm* lies in the UV (ultraviolet) part of the EM spectrum.

4-24. (a) The reduced mass correction to the Rydberg constant is important in this case.

$$R = R_\infty\left(\frac{1}{1+m/M}\right) = R_\infty\left(\frac{1}{2}\right) = 5.4869\times10^6 m^{-1} \quad \text{(from Equation 4-26)}$$

$E_n = -hcR/n^2$ (from Equations 4-23 and 4-24)

$$E_1 = -(1240eV\bullet nm)(5.4869\times10^6 m^{-1})(10^{-9} m/nm)/(1)^2 = -6.804eV$$

Similarly, $E_2 = -1.701eV$ and $E_3 = -0.756eV$

(b) Lyman α is the $n = 2 \rightarrow n = 1$ transition.

$$\frac{hc}{\lambda} = E_2 - E_1 \quad \rightarrow \quad \lambda_\alpha = \frac{hc}{E_2 - E_1} = \frac{1240eV\bullet nm}{-1.701eV - (-6.804eV)} = 243nm$$

Lyman β is the $n = 3 \rightarrow n = 1$ transition.

$$\lambda_\beta = \frac{hc}{E_3 - E_1} = \frac{1240eV\bullet nm}{-0.756eV - (-6.804eV)} = 205nm$$

4-25. (a) The radii of the Bohr orbits are given by (see Equation 4-18)

$r = n^2 a_0 / Z$ where $a_0 = 0.0529nm$ and $Z = 1$ for hydrogen.

For $n = 600$, $r = (600)^2(0.0529nm) = 1.90\times10^4 nm = 19.0\mu m$

This is about the size of a tiny grain of sand.

(b) The electron's speed in a Bohr orbit is given by

$v^2 = ke^2/mr$ with $Z = 1$

Substituting r for the n = 600 orbit from (a), then taking the square root,

$$v^2 = (8.99\times10^9 N\bullet m^2)(1.609\times10^{-19}C)^2/(9.11\times10^{-31}kg)(19.0\times10^{-6}m)$$

$$v^2 = 1.33\times10^7 m^2/s^2 \quad \rightarrow \quad v = 3.65\times10^3 m/s$$

For comparison, in the n = 1 orbit, v is about $2\times10^6 m/s$

4-26. (a) $\frac{1}{\lambda}=R(Z-1)^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

$$\lambda_3=\left[\left(1.097\times10^7 m^{-1}\right)(42-1)^2\left(\frac{1}{1^2}-\frac{1}{3^2}\right)\right]^{-1}=6.10\times10^{-11}m=0.0610nm$$

$$\lambda_4=\left[\left(1.097\times10^7 m^{-1}\right)(42-1)^2\left(\frac{1}{1^2}-\frac{1}{4^2}\right)\right]^{-1}=5.78\times10^{-11}m=0.0578nm$$

(b) $\lambda_{\text{limit}}=\left[\left(1.097\times10^7 m^{-1}\right)(42-1)^2\left(\frac{1}{1^2}-0\right)\right]^{-1}=5.42\times10^{-11}m=0.0542nm$

4-27. $\frac{1}{\lambda}=R(Z-1)^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)=R(Z-1)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$ for K_α

$$Z-1=\left[\frac{1}{\lambda R\left(1-\frac{1}{4}\right)}\right]^{1/2}=\left[\frac{1}{(0.0794nm)\left(1.097\times10^{-2}/nm\right)(3/4)}\right]^{1/2}$$

$Z=1+39.1\approx40$ Zirconium

4-28. (a) $Z=43;\ f^{1/2}=21\times10^8 Hz^{1/2}\ \rightarrow\ f=4.4\times10^{18}Hz$

$Z=61;\ f^{1/2}=30\times10^8 Hz^{1/2}\ \rightarrow\ f=9.0\times10^{18}Hz$

$Z=75;\ f^{1/2}=37\times10^8 Hz^{1/2}\ \rightarrow\ f=1.4\times10^{19}Hz$

Note: $f^{1/2}$ for $Z=61$ and 75 are off the graph 4-19; however, the graph is linear and extrapolation is easy.

(b) For Z = 43 $\lambda=\frac{1}{R_\infty(Z-1)^2\left(1-\frac{1}{n^2}\right)}$ (Equation 4-37)

where $R_\infty=1.097\times10^7 m^{-1}$ and $n=2$

$$\lambda=\frac{1}{\left(1.097\times10^7 m^{-1}\right)(43-1)^2\left(1-\frac{1}{4}\right)}=6.89\times10^{-11}m=0.0689nm$$

(Problem 4-28 continued)

Similarly,

For $Z = 61$, $\lambda = 0.0327nm$

For $Z = 75$, $\lambda = 0.0216nm$

4-29. $r_n = \dfrac{n^2 a_0}{Z}$ (Equation 4-18)

The $n = 1$ electrons "see" a nuclear charge of approximately $Z - 1$, or 78 for Au.

$r_1 = 0.0529nm/78 = 6.8\times10^{-4}\,nm\left(10^{-9}\,m/nm\right)\left(10^{15}\,fm/m\right) = 680\,fm$, or about 100 times the radius of the Au nucleus.

4-30. $E_n = -13.6\dfrac{Z^2}{n^2}eV$ (Equation 4-20)

For Fe ($Z = 26$) $E_1 = -13.6\dfrac{(26)^2}{1^2} = -9.194keV$

The fact that E_1 computed this way (i.e., by Bohr theory) is approximate, is not a serious problem, since the K_α x-ray energy computed from Figure 4-19 provides the correct *spacing* between the levels.

The energy of the Fe K_α x-ray is:

$$E\left(Fe\ K_\alpha\right) = hf \text{ where } f^{1/2} = 12.2\times10^8\,Hz^{1/2}$$

$$E\left(Fe\ K_\alpha\right) = \left(6.626\times10^{-34}\,J\bullet s\right)\left(12.2\times10^8\,Hz^{1/2}\right)^2 = 9.862\times10^{-16}\,J = 6.156keV$$

Therefore, $E_2 = E_1 + E\left(K_\alpha\right) = \left(-9.194 + 6.156\right) = -3.038keV$

The Auger electron energy $E\left(K_\alpha\right) - \left|E_2\right| = 6.156 - 3.038 = 3.118keV$

4-31. $E = \gamma m_e c^2 = \dfrac{511keV}{\sqrt{1-\left(2.25\times10^8/3.00\times10^8\,m/s\right)^2}} = 772.6keV$

After emitting a 32.5 keV photon, the total energy is:

(Problem 4-31 continued)

$$E = 740.1keV = \frac{511keV}{\sqrt{1-\beta^2}} \quad \rightarrow \quad \beta^2 = v^2/c^2 = 1-(511/740)^2$$

$$v = \left[1-(511/740)^2\right]^{1/2} c = 2.17\times10^8 m/s$$

4-32. (a) $-E_1 = E_0 Z^2/n^2$ (Equation 4-20)

$$= 13.6eV(74-1)^2/(1)^2 = 7.25\times10^4 eV = 72.5keV$$

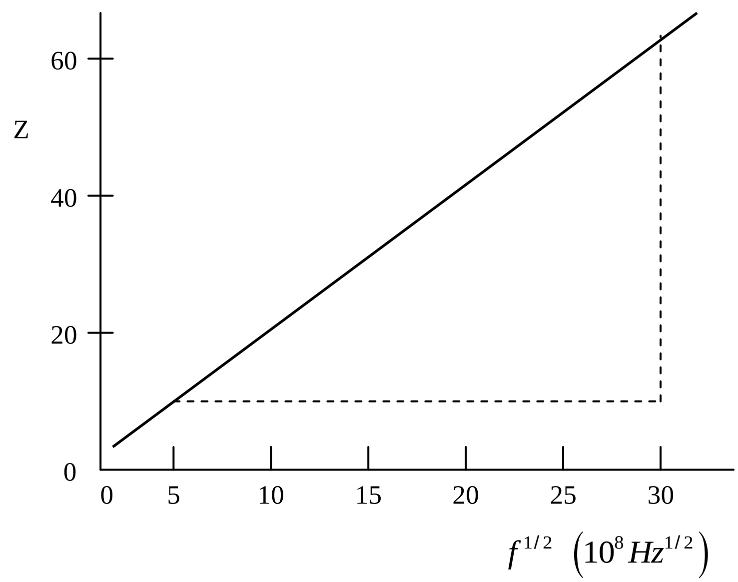
(b) $-E_1 = E_0(Z-\sigma)^2/n^2 = 69.5\times10^3 eV = 13.6eV(74-\sigma)^2/(1)^2$

$$(74-\sigma)^2 = 69.5\times10^3 eV/13.6eV$$

$$\sigma = 74-\left(69.5\times10^3 eV/13.6eV\right)^{1/2} = 2.5$$

4-33.

Element	Al	Ar	Sc	Fe	Ge	Kr	Zr	Ba
Z	13	18	21	26	32	36	40	56
E (keV)	1.56	3.19	4.46	7.06	10.98	14.10	17.66	36.35
$f^{1/2}\left(10^8 Hz^{1/2}\right)$	6.14	8.77	10.37	13.05	16.28	18.45	20.64	29.62



(Problem 4-33 continued)

$$\text{slope} = \frac{58-10}{(30-4.8)\times 10^8} = 1.90\times 10^{-8}\,Hz^{-1/2}$$

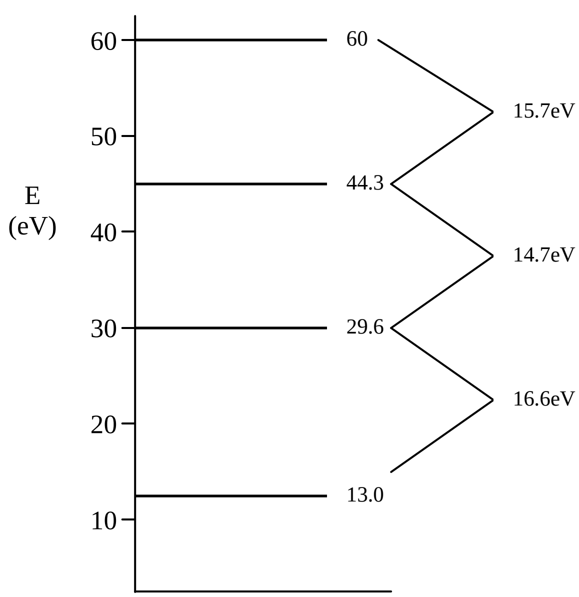
$$\text{slope (Figure 4-19)} = \frac{30-13}{(5-7)\times 10^8} = 2.13\times 10^{-8}\,Hz^{-1/2}$$

The two values are in good agreement.

4-34. (a) The available energy is not sufficient to raise ground state electrons to the n =5 level which requires $13.6 - 0.54 = 13.1eV$. The shortest wavelength (i.e., highest energy) spectral line that will be emitted is the 3rd line of the Lyman series, the $n = 4 \rightarrow n = 1$ transition. (See Figure 4-16.)

(b) The emitted lines will be for those transitions that begin on the n = 4, n = 3, or n = 2 levels. These are the first three lines of the Lyman series, the first two lines of the Balmer series, and the first line of the Paschen series.

4-35.



Average transition energy = 15.7 eV

4-36. $\Delta E = \frac{hc}{\lambda} = \frac{1240 eV \bullet nm}{790 nm} = 1.610 eV$. The first decrease in current will occur when the voltage reaches 1.61*V*.

4-37. Using the results from Problem 4-24, the energy of the positronium Lyman α line is $\Delta E = E_2 - E_1 = -1.701 eV - (-6.804 eV) = 5.10 eV$. The first Franck-Hertz current decrease would occur at 5.10*V*, the second at 10.2*V*.

4-38. In an elastic collision, both momentum and kinetic energy are conserved. Introductory physics texts derive the following expression when the second object (the Hg atom here) is initially at rest: $v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i}$. The fraction of the initial kinetic energy lost by the incident electron in a head-on collision is:

$$f = \frac{KE_{ei} - KE_{ef}}{KE_{ei}} = \frac{v_{1i}^2 - v_{1f}^2}{v_{1i}^2} = \frac{v_{1i}^2 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 v_{1i}^2}{v_{1i}^2}$$
$$= 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 = 1 - \left(\frac{0.511 MeV - 200 uc^2 (931.5 MeV / uc^2)}{0.511 MeV + 200 uc^2 (931.5 MeV / uc^2)}\right)^2$$
$$= 1.10 \times 10^{-5}$$

If the collision is not head-on, the fractional loss will be less.

4-39. (a) Equation 4-24: $E_n = -E_0 / n^2 = -13.6 / n^2 \text{ eV}$

$$E_{n+1} - E_n = -13.6\left[\frac{1}{(n+1)^2} - \frac{1}{n^2}\right]\text{eV} = -13.6\left[\frac{1}{46^2} - \frac{1}{45^2}\right]\text{eV} = 2.89 \times 10^{-4} \text{ eV}$$

(b) Ionization energy $= |E_n| = 13.6 / n^2 = 13.6 / 45^2 = 6.72 \times 10^{-3} \text{ eV}$

(c) $E = hf = hc / \lambda \;\Rightarrow\; f = E / h = (2.89 \times 10^{-4} \text{ eV})(1.60 \times 10^{-19} \text{ J/eV}) / 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$

$$f = 6.97 \times 10^{10} \text{ Hz}$$
$$\lambda = c / f = (3.00 \times 10^8 \text{ m/s}) / (6.96 \times 10^{10} \text{ Hz}) = 4.30 \times 10^{-3} \text{ m} = 4.30 \text{ mm}$$

(Problem 4-39 continued)

(d) Equation 4-18: $r_n = n^2 a_0 / Z$

For hydrogen: $r_{45} = 45^2 a_0 / 1 = 107\,\text{nm} = 1.07\times10^{-4}\,\text{mm}$, or 2025× the radius of the hydrogen atom ground state.

4-40. (*a*) Equation 4-26: $R = R_\infty\left(\dfrac{1}{1+m/M}\right)$ where $R_\infty = 1.0973732\times10^{7}\,\text{m}^{-1}$

$$R_d = R_\infty\left(\frac{1}{1+9.1094\times10^{-31}\,\text{kg}/3.3436\times10^{-27}\,\text{kg}}\right)$$
$$R_d = 1.0970743\times10^{7}\,\text{m}^{-1}$$
$$R_t = R_\infty\left(\frac{1}{1+9.1094\times10^{-31}\,\text{kg}/5.0074\times10^{-27}\,\text{kg}}\right)$$
$$R_t = 1.0971736\times10^{7}\,\text{m}^{-1}$$

(b) Equation 4-22: $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right)$ with $Z = 1$.

The Balmer α transition is $n = 3 \rightarrow n = 2$. $\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right) = \left(\dfrac{1}{2^2} - \dfrac{1}{3^2}\right) = \dfrac{5}{36}$

$$\lambda_d - \lambda_t = \frac{36}{5}\left(\frac{1}{R_d} - \frac{1}{R_t}\right) = 5.3978\times10^{-11}\,\text{m} = 5.3978\times10^{-2}\,\text{nm}$$

(c) Computed as in (*b*) above with $R_H = 1.096762\times10^{7}\,\text{m}^{-1}$,

$$\lambda_H - \lambda_t = \frac{36}{5}\left(\frac{1}{R_H} - \frac{1}{R_t}\right) = 2.4627\times10^{-10}\,\text{m} = 2.4627\times10^{-1}\,\text{nm}$$

4-41. $N = I_0\left(2\pi b\right)db$ where $b = \dfrac{kq_\alpha Q}{m_\alpha v^2}\cot\dfrac{\theta}{2}$ (Equation 4-3)

and $db = \dfrac{kq_\alpha Q}{2m_\alpha v^2}\left(-\csc\dfrac{\theta}{2}\right)d\theta$

$$N = I_0 2\pi\left(\frac{kq_\alpha Q}{m_\alpha v^2}\right)^2\left(\frac{1}{2}\cot\frac{\theta}{2}\right)\left(\csc^2\frac{\theta}{2}\right)d\theta$$

Using the trigonometric identities:

$$\csc^2 = \frac{1}{\sin^2\theta/2} \quad \text{and} \quad \cot\frac{\theta}{2} = \frac{\sin\theta}{1-\cos\theta} = \frac{\sin\theta}{1-\cos^2\left(\theta/2\right)+\sin^2\left(\theta/2\right)} = \frac{\sin\theta}{2\sin^2\left(\theta/2\right)}$$

$$N = I_0 2\pi\left(\frac{kq_\alpha Q}{m_\alpha v^2}\right)^2\left(\frac{1}{2}\right)\left(\frac{\sin\theta}{2\sin^2\left(\theta/2\right)}\right)\left(\frac{1}{\sin^2\left(\theta/2\right)}\right)d\theta$$

and inserting $2e = q_\alpha$ and $Ze = Q$,

$$N = I_0 2\pi\left(\frac{kZe^2}{m_\alpha v^2}\right)^2\frac{\sin\theta\, d\theta}{\sin^4\left(\theta/2\right)}$$

4-42. Those scattered at $\theta = 180°$ obeyed the Rutherford formula. This is a head-on collision where the α comes instantaneously to rest before reversing direction. At that point its kinetic energy has been converted entirely to electrostatic potential energy, so

$\dfrac{1}{2}m_\alpha v^2 = 7.7MeV = \dfrac{k\left(2e\right)\left(79e\right)}{r}$ where r = upper limit of the nuclear radius.

$$r = \frac{k\left(2\right)\left(79\right)e^2}{7.7MeV} = \frac{2\left(79\right)\left(1.440MeV\bullet fm\right)}{7.7MeV} = 29.5\,fm$$

4-43. (a) $i = qf_{rev} = e\dfrac{Z^2mk^2e^4}{2\pi\hbar^3n^3}$ (from Equation 4-28)

$$= e\frac{mc^2\left(ke^2\right)^2\left(1\right)^2}{2\pi\hbar\left(\hbar c\right)^2\left(1\right)^3} = \frac{ec}{\left(h/mc\right)}\left(\frac{ke^2}{\hbar c}\right)^2 = \frac{ec\alpha^2}{\lambda_c}$$

$$= \frac{\left(1.602\times10^{-19}C\right)\left(3.00\times10^{17}\,nm/s\right)}{0.00243nm}\left(\frac{1}{137}\right)^2 = 1.054\times10^{-3}A$$

(Problem 4-43 continued)

(b) $$\mu = iA = i\pi a_0^2 = \left(\frac{emk^2e^4}{2\pi\hbar^3}\right)\pi\left(\frac{\hbar^2}{mke^2}\right) = \frac{e\hbar}{2m}$$

$$= \frac{\left(1.602\times10^{-19}C\right)\left(1.055\times10^{-34}J\bullet s\right)}{2\left(9.11\times10^{-31}kg\right)} = 9.28\times10^{-24}A\bullet m^2$$

or

$$= \left(1.054\times10^{-3}A\right)\pi\left(0.529\times10^{-10}m\right)^2 = 9.27\times10^{-24}A\bullet m^2$$

4-44. Using the Rydberg-Ritz equation (Equation 4-2), set-up the columns of the spreadsheet to carry out the computation of λ as in this example (not all lines are included here).

m	*n*	*C=m²*	*D=n²*	*1/C−1/D*	1/λ	λ (nm)
1	5	1	25	0.96	10534572	94.92
1	4	1	16	0.9375	10287844	97.20
1	3	1	9	0.888889	9754400	102.52
1	2	1	4	0.75	8230275	121.50
2	6	4	36	0.222222	2438600	410.07
2	5	4	25	0.21	2304477	433.94
2	4	4	16	0.1875	2057569	486.01
2	3	4	9	0.138889	1524125	656.11
3	7	9	49	0.090703	995346.9	1004.67
3	6	9	36	0.083333	914475	1093.52
3	5	9	25	0.071111	780352	1281.47
3	4	9	16	0.048611	533443.8	1874.61

4-45. $$\lambda = \left[R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)\right]^{-1} \qquad \Delta\lambda = \frac{d\lambda}{d\mu}\Delta\mu = \left(-R^{-2}\right)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)^{-1}\frac{dR}{d\mu}\Delta\mu$$

Because $R \propto \mu$, $dR/d\mu = R/\mu$. $\Delta\lambda \approx \left(-R^{-2}\right)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)^{-1}\left(R/\mu\right)\Delta\mu = -\lambda\left(\Delta\mu/\mu\right)$

(Problem 4-45 continued)

$$\mu_H = \frac{m_e m_p}{m_e + m_p} \qquad \mu_D = \frac{m_e m_d}{m_e + m_d}$$

$$\frac{\Delta\mu}{\mu} = \frac{\mu_D - \mu_H}{\mu_H} = \frac{\mu_D}{\mu_H} - 1 = \frac{m_e m_d / \left(m_e + m_d\right)}{m_e m_p / \left(m_e + m_p\right)} - 1 = \frac{m_d / \left(m_e + m_d\right)}{m_p / \left(m_e + m_p\right)} - 1 = \frac{m_e\left(m_d - m_p\right)}{m_p\left(m_e + m_d\right)}$$

If we approximate $m_d = 2m_p$ and $m_e \ll m_d$, then $\frac{\Delta\mu}{\mu} \approx \frac{m_e}{2m_p}$ and

$$\Delta\lambda = -\lambda\left(\Delta\mu / \mu\right) = -\left(656.3nm\right)\frac{0.511MeV}{2\left(938.28MeV\right)} = -0.179nm$$

4-46. For maximum recoil energy for the Hg atoms, the collision is ‘head-on’.

(a)

	Before collision	After collision
kinetic energy	$E_k = \frac{1}{2}mv_{ei}^2$	$E_k' = \frac{1}{2}mv_{ef}^2$ $E_{Hg} = \frac{1}{2}Mv_{Hg}^2$
momentum	$p_{ei} = mv_{ei}$	$p_{ef} = mv_{ef}$ $p_{Hg} = Mv_{Hg}$

Conservation of momentum requires:

$$mv_{ei} = -mv_{ef} + Mv_{Hg} \rightarrow v_{Hg} = \frac{m}{M}\left(v_{ei} + v_{ef}\right)$$

Therefore, the maximum Hg recoil kinetic energy is given by:

$$\frac{1}{2}Mv_{Hg}^2 = \frac{1}{2}M\left(\frac{m}{M}\right)^2\left(v_{ei} + v_{ef}\right)^2 = \frac{m^2}{2M}\left(v_{ei}^2 + 2v_{ei}v_{ef} + v_{ef}^2\right)$$

$$\approx \frac{m^2}{2M}\left(4v_{ei}^2\right) \text{ since } m \ll M,\ v_{ei} \approx v_{ef}$$

$$\approx \frac{4m}{M}\left(\frac{1}{2}mv_{ei}^2\right) = \frac{4m}{M}E_k$$

(Problem 4-46 continued)

(b) Since the collision is elastic, kinetic energy is conserved, so the maximum kinetic energy gained by the Hg atom equals the maximum kinetic energy lost by the electron. If $E_k = 2.5eV$, then the maximum lost is equal to:

$$4\frac{m}{M}(2.5eV) = 4\frac{9.11\times10^{-31}kg(2.5eV)}{(201u)(1.66\times10^{-27}kg/u)} = 2.7\times10^{-5}eV$$

4-47. (a) $E_n = -E_0Z^2/n^2$ (Equation 4-20)

For Li^{++}, $Z = 3$ and $E_n = -13.6eV(9)/n^2 = -122.4/n^2eV$

The first three Li^{++} levels that have the same (nearly) energy as H are:

$n = 3,\ E_3 = -13.6eV \quad n = 6,\ E_6 = -3.4eV \quad n = 9,\ E_9 = -1.51eV$

Lyman α corresponds to the $n = 6 \to n = 3$ Li^{++} transitions. Lyman β corresponds To the $n = 9 \to n = 3$ Li^{++} transition.

(b) $R(H) = R_\infty\left(1/(1+0.511MeV/938.8MeV)\right) = 1.096776\times10^7 m^{-1}$

$R(Li) = R_\infty\left(1/(1+0.511MeV/6535MeV)\right) = 1.097287\times10^7 m^{-1}$

For Lyman α:

$$\frac{1}{\lambda} = R(H)\left(1-\frac{1}{2^2}\right) = 1.096776\times10^7 m^{-1}\left(10^{-9}m/nm\right)(3/4) \to 121.568nm$$

For Li^{++} equivalent:

$$\frac{1}{\lambda} = R(Li)\left(\frac{1}{3^2}-\frac{1}{6^2}\right)Z^2 = 1.097287\times10^7 m^{-1}\left(10^{-9}m/nm\right)\left(\frac{1}{9}-\frac{1}{36}\right)(3)^2$$

$\lambda = 121.512nm \quad \Delta\lambda = 0.056nm$

4-48. $$\Delta N = \left(\frac{I_0A_{SC}nt}{r^2}\right)\left(\frac{kZe^2}{2E_K}\right)^2\frac{1}{\sin^4\frac{\theta}{2}}$$ (Equation 4-6)

where $A_{SC} = 0.50cm^3 \quad r = 10cm \quad t = 10^{-6}m$

(Problem 4-48 continued)

$$n(Ag) = \frac{(10.5g/cm^3)(6.02\times10^{23}atoms/mol)}{107.5g/mol}$$

$$= 5.88\times10^{22}atoms/cm^3 = 5.88\times10^{28}atoms/m^3$$

$$E_K = 6.0MeV \qquad I_0 = 1.0nA = (10^{-9}C/s)\left(\frac{1}{2(1.60\times10^{-19}C)}\right) = 3.18\times10^9 alphas/s$$

(a) At θ = 60°

$$\Delta N = \left(\frac{(3.13\times10^9\alpha/s)(0.50cm^2)(5.88\times10^{28}/m^3)(10^{-6})}{10^2cm^2}\right)$$

$$= \left(\frac{(9\times10^9 N\bullet m^2/C^2)(1.60\times10^{-19}C)^2(47)}{2(6.0MeV)(1.60\times10^{-13}J/MeV)}\right)\left(\frac{1}{\sin^4\frac{60°}{2}}\right) = 468\alpha/s$$

(b) At θ = 120°: $\Delta N = \Delta N(60°)\left(\sin^4\frac{60°}{2}\right)/\left(\sin^4\frac{120°}{2}\right) = 52\alpha/s$

4-49. $E_n = -E_0Z^2/n^2$ (Equation 4-20)

For Ca, Z = 20 and $E_1 = -13.6eV(20)^2/(1)^2 = -5.440keV$

The fact that E_1 computed this way is only approximate is not a serious problem because the measured x-ray energies provide us the correct *spacings* between the levels.

$$E_2 = E_1 + 3.69keV = -5.440 + 3.69 = -1.750keV$$

$$E_3 = E_2 + 0.341keV = -1.750 + 0.341 = -1.409keV$$

$$E_4 = E_3 + 0.024keV = -1.409 + 0.024 = -1.385keV$$

These are the ionization energies for the levels. Auger electron energies $\Delta E = -|E_n|$ where $\Delta E = 3.69keV$.

Auger L electron: $3.69keV - 1.750keV = 1.94keV$
Auger M electron: $3.69keV - 1.409keV = 2.28keV$
Auger N electron: $3.69keV - 1.385keV = 2.31keV$

4-50. (a) $E_\alpha = hc/\lambda = 1240eV{\bullet}nm/0.071nm = 17.465keV$

$E_\beta = hc/\lambda = 1240eV{\bullet}nm/0.063nm = 19.683keV$

(b) Select Nb (Z = 41)

The Kβ Mo x-rays have enough energy to eject photoelectrons, producing 0.693 keV electrons. The Kα Mo x-rays could not produce photoelectrons in Nb.

4-51. (a) $b = R\sin\beta = R\sin\left(\frac{180° - \theta}{2}\right) = R\cos\frac{\theta}{2}$

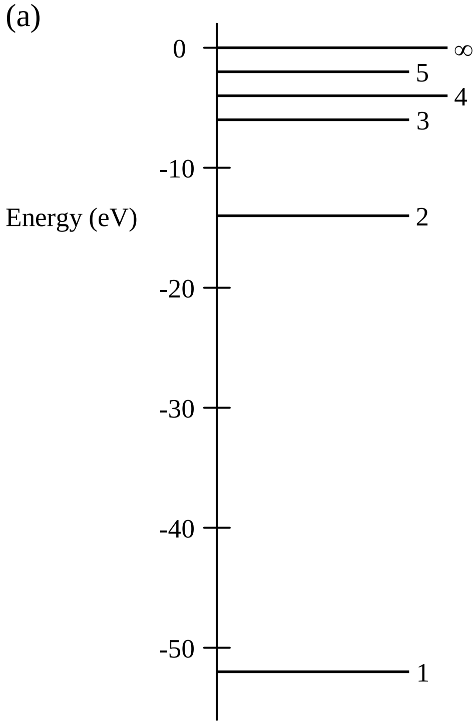
(b) Scattering through an angle larger than θ corresponds to an impact parameter smaller than *b*. Thus, the shot must hit within a circle of radius *b* and area πb^2. The rate at which this occurs is $I_0 \pi b^2 = I_0 R^2 \cos^2\frac{\theta}{2}$

(c) $\sigma = \pi b_0^2 = \pi\left(R\cos\frac{\theta}{2}\right)^2 = \pi R^2$

(d) An α particle with an arbitrarily large impact parameter still feels a force and is scattered.

4-52. For He: $E_n = -13.6eV\,Z^2/n^2 = -54.4eV/n^2$ (Equation 4-20)

(a)



$E_1 = -54.4\ eV$
$E_2 = -13.6\ eV$
$E_3 = -6.04\ eV$
$E_4 = -3.04\ eV$
$E_5 = -2.18\ eV$
$E_\infty = 0\ eV$

(Problem 4-52 continued)

(b) Ionization energy is 54.5eV.

(c) H Lyman α: $\lambda = hc/\Delta E = 1240eV{\bullet}nm/(13.6eV - 3.4eV) = 121.6nm$

H Lyman β: $\lambda = hc/\Delta E = 1240eV{\bullet}nm/(13.6eV - 1.41eV) = 102.6nm$

He^+ Balmer α: $\lambda = hc/\Delta E = 1240eV{\bullet}nm/(13.6eV - 6.04eV) = 164.0nm$

He^+ Balmer β: $\lambda = hc/\Delta E = 1240eV{\bullet}nm/(13.6eV - 3.40eV) = 121.6nm$

$\Delta\alpha = 42.4nm \quad \Delta\beta = 19.0nm$

(The reduced mass correction factor does not change the energies calculated above to three significant figures.)

(d) $E_n = -13.6eV\,Z^2/n^2$ because for He^+, $Z = 2$, then $Z^2 = 2^2$. Every time n is an even number a 2^2 can be factored out of n^2 and cancelled with the $Z^2 = 2^2$ in the numerator; e.g., for He^+,

$E_2 = -13.6eV{\bullet}2^2/2^2 = -13.6eV$ (H ground state)

$E_4 = -13.6eV{\bullet}2^2/4^2 = -13.6eV/2^2$ (H -1^{st} excited state)

$E_6 = -13.6eV{\bullet}2^2/6^2 = -13.6eV/3^2$ (H -2^{nd} excited state)

⋮

etc.

Thus, all of the H energy level values are to be found within the He^+ energy levels, so He^+ will have within its spectrum lines that match (nearly) a line in the H spectrum.

4-53.

Element	P	Ca	Co	Kr	Mo	I
Z	15	20	27	36	42	53
Lα λ(nm)	10.41	4.05	1.79	0.73	0.51	0.33
$f^{1/2}\,(10^8 Hz)$	1.70	2.72	4.09	6.41	7.67	9.53

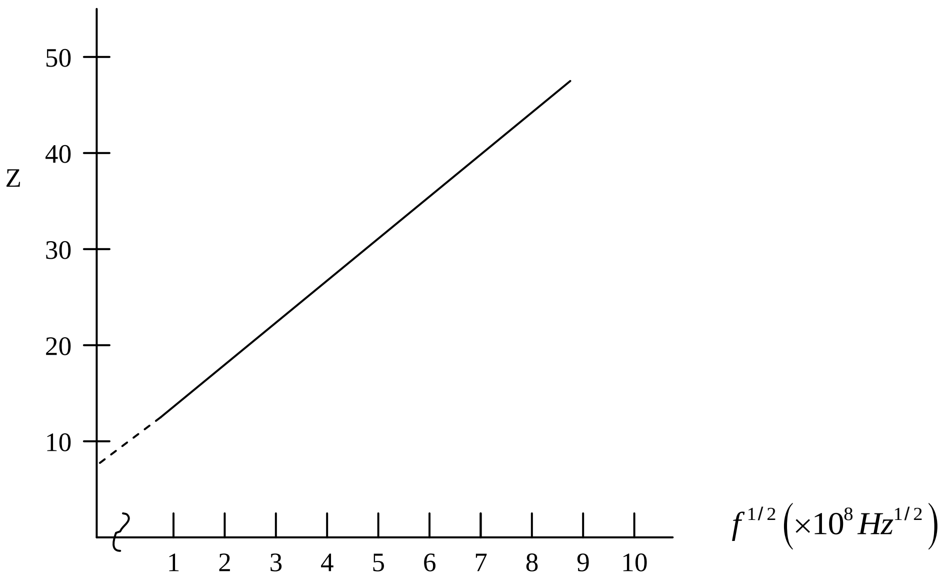
where $f^{1/2} = \left[(3.00\times10^8 m/s)(10^9 nm/m)/\lambda\right]^{1/2}$

$$\text{Slope} = \frac{50-15}{(9.15-1.58)\times10^8 Hz} = 4.62\times10^{-8} Hz^{-1}$$

$$\text{Slope (Figure 4-19)} = \frac{74-46}{(14-8)\times10^8 Hz} = 4.67\times10^{-8} Hz^{-1}$$

(Problem 4-53 continued)

The agreement is very good.



The $f^{1/2}=0$ intercept on the Z axis is the minimum Z for which an Lα X-ray could be emitted. It is about $Z = 8$.

4-54. (a) $E_n = -\dfrac{ke^2}{2r_n} = -\dfrac{ke^2}{2n^2 r_o}$ $\qquad E_{n-1} = -\dfrac{ke^2}{2\left(n-1\right)^2 r_o}$

$$hf = E_n - E_{n-1} = -\frac{ke^2}{2n^2 r_o} - \left(-\frac{ke^2}{2\left(n-1\right)^2 r_o}\right)$$

$$f = \frac{ke^2}{2hr_o}\left[\frac{1}{\left(n-1\right)^2} - \frac{1}{n^2}\right] = \frac{ke^2}{2hr_o}\frac{n^2-\left(n^2-2n+1\right)}{n^2\left(n-1\right)^2}$$

$$= \frac{ke^2}{2hr_o}\frac{2n-1}{n^2\left(n-1\right)^2} \approx \frac{ke^2}{r_o hn^3} \quad \text{for n} \gg 1$$

(b) $f_{rev} = \dfrac{v}{2\pi r} \quad \rightarrow \quad f_{rev}^2 = \dfrac{v^2}{4\pi^2 r^2} = \dfrac{1}{4\pi^2 mr}\dfrac{mv^2}{r} = \dfrac{1}{4\pi^2 mr}\dfrac{ke^2}{r^2} = \dfrac{ke^2}{4\pi^2 mr_o^3 n^6}$

(Problem 4-54 continued)

(c) The correspondence principle implies that the frequencies of radiation and revolution are equal.

$$f^2 = \left(\frac{ke^2}{r_o hn^3}\right)^2 = \frac{ke^2}{4\pi^2 m r_o^3 n^6} = f_{rev}^2 \qquad r_o = \frac{ke^2}{4\pi^2 mn^6}\left(\frac{hn^3}{ke^2}\right)^2 = \frac{h^2}{4\pi^2 mke^2} = \frac{\hbar^2}{mke^2}$$

which is the same as a_o in Equation 4-19.

4-55. $\dfrac{kZe^2}{r} = \dfrac{mv^2}{r} \quad \rightarrow \quad \dfrac{kZe^2}{r^2} = \dfrac{(\gamma mv)^2}{mr}$ (from Equation 4-12)

$$\gamma v = \left(\frac{kZe^2}{mr}\right)^{1/2} = \frac{v}{\sqrt{1-\beta^2}}$$

$\dfrac{c^2\beta^2}{1-\beta^2} = \left(\dfrac{kZe^2}{mr}\right)$ Therefore, $\beta^2\left[c^2 + \left(\dfrac{kZe^2}{mr}\right)\right] = \left(\dfrac{kZe^2}{mr}\right)$

$$\beta^2 \approx \frac{1}{c^2}\left(\frac{kZe^2}{ma_o}\right) \quad \rightarrow \quad \beta = 0.0075Z^{1/2} \quad \rightarrow \quad v = 0.0075cZ^{1/2} = 2.25\times10^6\, m/s \times Z^{1/2}$$

$$E = KE - kZe^2/r = mc^2(\gamma - 1) - \frac{kZe^2}{r} = mc^2\left[\frac{1}{\sqrt{1-\beta^2}} - 1\right] - \frac{kZe^2}{r}$$

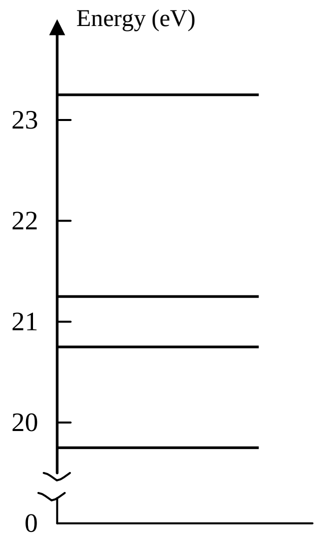
And substituting $\beta = 0.0075$ and $r = a_o$

$$E = 511\times10^3 eV\left[\frac{1}{\sqrt{1-(0.0075)^2}} - 1\right] - 28.8Z\ eV$$

$$= 14.4eV - 28.8Z\ eV = -14.4Z\ eV$$

4-56. (The solution to this problem depends on the kind of calculator or computer you use and the program you write.)

4-57.



Levels constructed from Figure 4-26.

4-58. Centripetal acceleration would be provided by the gravitational force:

$$F_G = G\frac{Mm}{r^2} = \frac{mv^2}{r} \qquad M = \text{proton mass and } m = \text{electron mass, so } v = \left(\frac{GM}{r}\right)^{1/2}$$

$$L = mvr = n\hbar \quad \rightarrow \quad r = n\hbar / mv \text{ or}$$

$$r_n = \frac{n\hbar}{m\left(GM/r_n\right)^{1/2}} \quad \rightarrow \quad r_n^2 = \frac{n^2\hbar^2 r_n}{m^2 GM} \text{ and, } r_n = \frac{n^2\hbar^2}{GMm^2} \quad \rightarrow \quad a_o = \frac{\hbar^2}{GMm^2}$$

The total energy is: $E = \frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right) = \frac{1}{2}m\left(\frac{GM}{r}\right) = -\frac{GMm}{2r}$

$$E_n = -\frac{GMm}{2r_n} = -\frac{(GMm)\left(GMm^2\right)}{2n^2\hbar^2} = -\frac{G^2M^2m^3}{2n^2\hbar^2}$$

The gravitational Hα line is: $\Delta E = E_2 - E_3 = \frac{G^2M^2m^3}{2\hbar^2}\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

$$\Delta E = \frac{\left(6.67\times10^{-11} N\bullet m^2/kg^2\right)^2 \left(1.67\times10^{-27} kg\right)^2 \left(9.11\times10^{-31} kg\right)^3 (0.1389)}{2\left(1.055\times10^{-34}\right)^2}$$

(Problem 4-58 continued)

$$= 5.85\times10^{-98}\,J = 3.66\times10^{-79}\,eV$$

$$f = \frac{\Delta E}{h} = \frac{5.85\times10^{-98}\,J}{6.63\times10^{-34}\,J\bullet s} = 8.28\times10^{-65}\,Hz$$

For the Balmer limit in each case,

$$\Delta E = 3.66\times10^{-79}\,eV\left(0.250/0.1389\right) = 6.58\times10^{-79}\,eV$$

$$f = 6.58\times10^{-79}\,eV/h = 1.59\times10^{-64}\,Hz$$

These values are immeasurably small. They do not compare with the actual H values.

4-59. Refer to Figure 4-16. All possible transitions starting at n = 5 occur.

n = 5 to n = 4, 3, 2, 1

n = 4 to n = 3, 2, 1

n = 3 to n = 2, 1

n = 2 to n = 1

Thus, there are 10 different photon energies emitted.

n_i	n_f	fraction	no. of photons
5	4	1/4	125
5	3	1/4	125
5	2	1/4	125
5	1	1/4	125
4	3	$1/4\times1/3$	42
4	2	$1/4\times1/3$	42
4	1	$1/4\times1/3$	42
3	2	$1/2\left[1/4+1/4\left(1/3\right)\right]$	83
3	1	$1/2\left[1/4+1/4\left(1/3\right)\right]$	83
2	1	$\left[\left(1/2\left(1/4+1/4\right)\left(1/3\right)\right)+1/4\left(1/3\right)+1/4\right]$	250

Total = 1,042

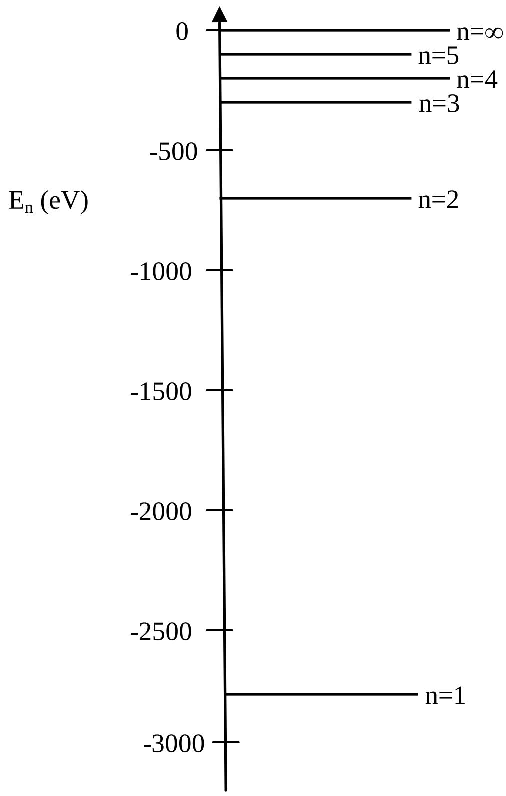
Note that the number of electrons arriving at the n = 1 level (125+42+83+250) is 500, as it should be.

4-60. $E_n = -E_o \frac{Z^2}{n^2}$ where $E_o = \frac{m_\mu k^2 e^4}{2\hbar^2}$

$m_\mu = 1.88\times10^{-28} kg \quad \rightarrow \quad E_o = 4.50\times10^{-16} J = 2.81\times10^3 eV$

Thus, for muonic hydrogen-like atom: $E_n = -\left(2.81\times10^3 eV\right)\frac{Z^2}{n^2}$

(a) muonic hydrogen



$E_\infty = 0\ eV$
$E_5 = -112\ eV$
$E_4 = -176\ eV$
$E_3 = -312\ eV$
$E_2 = -703\ eV$
$E_1 = -2.81\times10^3\ eV$

(b) $r_n = \frac{n^2\hbar^2}{mkZe^2} = \frac{\hbar^2}{mke^2}\frac{n^2}{Z} = 2.56\times10^{-4}\frac{n^2}{Z} nm$ (Equation 4-18)

For H, Z = 1: $r_1 = 2.56\times10^{-4} nm$

For He^{1+}, Z = 2: $r_1 = 1.28\times10^{-4} nm$

For Al^{12+}, Z = 13: $r_1 = 1.97\times10^{-5} nm$

For Au^{78+}, Z = 79: $r_1 = 3.2\times10^{-6} nm$

(Problem 4-60 continued)

(c) Nuclear radii are between about 1 and $8\times10^{-5}m$, or $1-8\times10^{-6}nm$. (See Chapter 11.) The muon $n = 1$ orbits in H, He^{1+}, and Al^{12+} are about roughly 10*nm* outside the nucleus. That for Au^{78+} is very near the nucleus' surface.

(d) $hf = hc/\lambda = E_2 - E_1 \quad \rightarrow \quad \lambda = hc/\left(E_2 - E_1\right)$ where $E_n = -\left(2.81\times10^3 eV\right)\dfrac{Z^2}{n^2}$

For H: $\lambda = \dfrac{hc}{\left(1^2\right)\left(2.81\times10^3\right)\left(1-1/4\right)} = 5.89\times10^{-10}m = 0.589nm$

Similarly,

For He^{1+}: $\lambda = 0.147nm$

For Al^{12+}: $\lambda = 0.00349nm$

For Au^{78+}: $\lambda = 9.44\times10^{-5}nm$

Chapter 5 – The Wavelike Properties of Particles

5-1. (a) $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{\left(6.63\times10^{-34}J\bullet s\right)\left(3.16\times10^{7}s/y\right)}{\left(10^{-3}kg\right)\left(1m/y\right)} = 2.1\times10^{-23}m$

(b) $v = \frac{h}{m\lambda} = \frac{6.63\times10^{-34}J\bullet s}{\left(10^{-3}kg\right)\left(10^{-2}m\right)} = 6.6\times10^{-29}m/s = 2.1\times10^{-21}m/y$

5-2. $\lambda = \frac{h}{p} \approx \frac{h}{E/c} = \frac{hc}{E} = \frac{1240MeV\bullet fm}{100MeV} = 12.4\,fm$

5-3. $E_k = eV_o = \frac{p^2}{2m} = \frac{\left(hc\right)^2}{2mc^2\lambda^2}$ $\qquad V_o = \frac{1}{e}\bullet\frac{\left(1240eV\bullet nm\right)^2}{2\left(5.11\times10^{5}eV\right)\left(0.04nm\right)^2} = 940V$

5-4. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}} = \frac{hc}{\sqrt{2mc^2E_k}}$ (from Equation 5-2)

(a) For an electron: $\lambda = \frac{1240eV\bullet nm}{\left[\left(2\right)\left(0.511\times10^{6}eV\right)\left(4.5\times10^{3}eV\right)\right]^{1/2}} = 0.0183nm$

(b) For a proton: $\lambda = \frac{1240eV\bullet nm}{\left[\left(2\right)\left(983.3\times10^{6}eV\right)\left(4.5\times10^{3}eV\right)\right]^{1/2}} = 4.27\times10^{-4}nm$

(c) For an alpha particle: $\lambda = \frac{1240eV\bullet nm}{\left[\left(2\right)\left(3.728\times10^{9}eV\right)\left(4.5\times10^{3}eV\right)\right]^{1/2}} = 2.14\times10^{-4}nm$

5-5. $\lambda = h/p = h/\sqrt{2mE_k} = hc/\left[2mc^2\left(1.5kT\right)\right]^{1/2}$ (from Equation 5-2)

Mass of N_2 molecule =

$$2\times14.0031u\left(931.5MeV/uc^2\right) = 2.609\times10^{4}MeV/c^2 = 2.609\times10^{10}eV/c^2$$

(Problem 5-5 continued)

$$\lambda = \frac{1240eV \bullet nm}{\left[(2)\left(2.609\times10^{10}eV\right)(1.5)\left(8.617\times10^{-5}eV/K\right)(300K)\right]^{1/2}} = 0.0276nm$$

5-6. $$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}} = \frac{hc}{\sqrt{2mc^2E_k}} = \frac{1240eV\bullet nm}{\left[2\left(939.57\times10^6eV\right)(0.02eV)\right]^{1/2}} = 0.202nm$$

5-7. (a) If there is a node at each wall, then $n(\lambda/2) = L$ where $n = 1, 2, 3,...$ or $\lambda = 2L/n$.

(b) $p = h/\lambda = hn/2L$ $\quad E = p^2/2m = (hn/2L)^2/2m = h^2n^2/8mL^2$

$$E_n = \frac{(hc)^2 n^2}{8mc^2L^2}$$

For n = 1: $E_1 = \dfrac{(1240eV\bullet nm)^2(1)^2}{8\left(938\times10^6eV\right)(0.01nm)^2} = 2.05eV$

For n = 2: $E_2 = 2.05eV(2)^2 = 8.20eV$

5-8. (a) $\lambda/\lambda_c = 10^2$ is a nonrelativistic situation, so

$$\lambda/\lambda_c = \left[\left(hc/\sqrt{2mc^2E_k}\right)\Big/\left(hc/mc^2\right)\right] = \left(mc^2/2E_k\right)^{1/2}$$

$$E_k = \frac{mc^2}{2(\lambda/\lambda_c)^2} = \frac{0.511\times10^6eV}{2\left(10^2\right)^2} = 25.6eV$$

(b) $\lambda/\lambda_c = 0.2$ is a relativistic for an electron, so $\lambda = h/\gamma mu \quad \rightarrow \quad \gamma u = h/\lambda m$.

$$\frac{u/c}{\sqrt{1-(u/c)^2}} = \frac{h}{mc\lambda} = \frac{\lambda_c}{\lambda}$$

$$\frac{(u/c)^2}{1-(u/c)^2} = \left(\frac{\lambda_c}{\lambda}\right)^2 \quad \rightarrow \quad u/c = \frac{\lambda_c/\lambda}{\left[1+(\lambda_c/\lambda)^2\right]^{1/2}}$$

(Problem 5-8 continued)

$$u/c = \frac{(1/0.2)}{\left[1+(1/0.2)^2\right]} = 0.981 \quad \rightarrow \quad \gamma = 5.10$$

$$E_k = mc^2(\gamma - 1) = 0.511MeV(\gamma - 1) = 2.10MeV$$

(c) $\lambda/\lambda_c = 10^{-3}$

$$u/c = \frac{(1/10^{-3})}{\left[1+(1/10^{-3})^2\right]^{1/2}} = 0.9999 \quad \rightarrow \quad \gamma = 1000$$

$$E_k = mc^2(\gamma - 1) = 0.511MeV(999) = 510MeV$$

5-9. $E_k = mc^2(\gamma - 1) \qquad p = \gamma mu$

(a) $E_k = 2GeV \qquad mc^2 = 0.938GeV$

$\gamma - 1 = E_k/mc^2 = 2GeV/0.938GeV = 2.132$ Thus, $\gamma = 3.132$

Because, $\gamma = 1\Big/\sqrt{1-(u/c)^2}$ where $u/c = 0.948$

$$\lambda = \frac{h}{p} = \frac{h}{\gamma mc(u/c)} = \frac{hc}{\gamma mc^2(u/c)}$$

$$= \frac{1240eV \bullet nm}{(3.132)(938\times 10^6 eV)(0.948)} = 4.45\times 10^{-7}\,nm = 0.445\,fm$$

(b) $E_k = 200GeV$

$\gamma - 1 = E_k/mc^2 = 200GeV/0.938GeV = 213$ Thus, $\gamma = 214$ and $u/c = 0.9999$

$$\lambda = \frac{1240eV \bullet nm}{(214)(938MeV)(0.9999)} = 6.18\times 10^{-3}\,fm$$

5-10. $n\lambda = D\sin\phi$ (Equation 5-5)

$$\sin\phi = \frac{n\lambda}{D} = \frac{n}{D}\frac{hc}{\sqrt{2mc^2E_k}}$$ (see Problem 5-6)

(Problem 5-10 continued)

$$\frac{1}{0.215nm}\times\frac{1240eV\bullet nm}{\left[2\left(5.11\times10^{5}eV\right)\right]^{1/2}\sqrt{E_k}}=\frac{\left(5.705eV\right)^{1/2}}{\sqrt{E_k}}$$

(a) $\sin\phi=\dfrac{\left(5.705eV\right)^{1/2}}{\sqrt{75eV}}=0.659 \qquad \phi=\sin^{-1}\left(0.659\right)=41.2°$

(b) $\sin\phi=\dfrac{\left(5.705eV\right)^{1/2}}{\sqrt{100eV}}=0.570 \qquad \phi=\sin^{-1}\left(0.570\right)=34.8°$

5-11. $\lambda=\dfrac{h}{p}=\dfrac{h}{\sqrt{2m_pE_k}}=0.25nm$

Squaring and rearranging,

$$E_k=\frac{h^2}{2m_p\lambda^2}=\frac{\left(hc\right)^2}{2\left(m_pc^2\right)\lambda^2}=\frac{\left(1240eV\bullet nm\right)^2}{2\left(938\times10^{6}eV\right)\left(0.25nm\right)^2}=0.013eV$$

$n\lambda=D\sin\phi \quad\rightarrow\quad \sin\phi=n\lambda/D=\left(1\right)\left(0.25nm\right)/\left(0.304nm\right)$

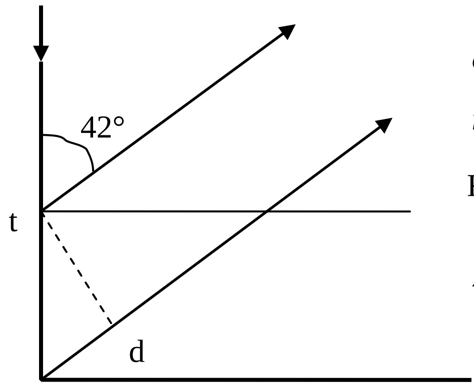
$\sin\phi=0.822 \quad\rightarrow\quad \phi=55°$

5-12. (a) $n\lambda=D\sin\phi \quad\therefore\quad D=\dfrac{n\lambda}{\sin\phi}=\dfrac{nhc}{\sin\phi\sqrt{2mc^2E_k}}$

$$=\frac{\left(1\right)\left(1240eV\bullet nm\right)}{\left(\sin55.6°\right)\left[2\left(5.11\times10^{5}eV\right)\left(50eV\right)\right]^{1/2}}=0.210nm$$

(b) $\sin\phi=\dfrac{n\lambda}{D}=\dfrac{\left(1\right)\left(1240eV\bullet nm\right)}{\left(0.210nm\right)\left[2\left(5.11\times10^{5}eV\right)\left(100eV\right)\right]^{1/2}}=0.584$

$\phi=\sin^{-1}\left(0.584\right)=35.7°$

5-13. 

$d = t\cos 42°$

$n\lambda = t + d = t(1+\cos 42°) = 0.30nm(1+\cos 42°)$

For the first maximum $n = 1$, so $\lambda = 0.523nm$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}} \quad \rightarrow \quad E_k = \frac{h^2}{2m\lambda^2} = \frac{(hc)^2}{2mc^2\lambda^2}$$

$$E_k = \frac{(1240eV \bullet nm)^2}{2(939\times10^6 eV)(0.523nm)} = 3.0\times10^{-3}eV$$

5-14. $\lambda = \dfrac{D\sin\phi}{n}$ (Equation 5-6)

For $54eV$ electrons $\lambda = 0.165nm$ and $\sin\phi = (0.165nm)n/0.215nm = 0.767n$

For $n = 2$ and larger $\sin\phi > 1$, so no values of n larger than one are possible.

5-15. $\sin\phi = n\lambda/D$ (Equation 5-6)

$$\lambda = h/p = h/\sqrt{2mE_k} = hc/\sqrt{2mc^2E_k} = \frac{1240eV\bullet nm}{\left[2(0.511\times10^6 eV)(350eV)\right]^{1/2}} = 0.0656nm$$

$\sin\phi = n(0.0656nm)/(0.315nm) = 0.208n$

For $n = 1$, $\phi = 12°$. For $n = 2$, $\phi = 24.6°$. For $n = 3$, $\phi = 38.6°$. For $n = 4$, $\phi = 56.4°$.

This is the largest possible ϕ. All larger n values have $\sin\phi > 1$.

5-16. (a) $\Delta t < \dfrac{1}{f} = \dfrac{1}{100,000s^{-1}} = 10^{-5}s = 10\mu s$

(b) $\Delta f\,\Delta t \approx \dfrac{1}{2\pi} \quad \therefore \quad \Delta f \approx \dfrac{1}{2\pi\Delta t} = \dfrac{1}{2\pi\times10^{-5}s} = 1.59\times10^4 Hz$

5-17. (a) $y = y_1 + y_2$

$$= 0.002m\cos(8.0x/m - 400t/s) + 0.002m\cos(7.6x/m - 380t/s)$$

$$= 2(0.002m)\cos\left[\frac{1}{2}(8.0x/m - 7.6x/m) - \frac{1}{2}(400t/s - 380t/s)\right]$$

$$\times\cos\left[\frac{1}{2}(8.0x/m + 7.6x/m) - \frac{1}{2}(400t/s + 380t/s)\right]$$

$$= 0.004m\cos(0.2x/m - 10t/s)\times\cos(7.8x/m - 390t/s)$$

(b) $v = \frac{\bar{\omega}}{\bar{k}} = \frac{390/s}{7.8/m} = 50m/s$

(c) $v_s = \frac{\Delta\omega}{\Delta k} = \frac{20/s}{0.4/m} = 50m/s$

(d) Successive zeros of the envelope requires that $0.2\Delta x/m = \pi$, thus

$\Delta x = \frac{\pi}{0.2} = 5\pi m$ with $\Delta k = k_1 - k_2 = 0.4m^{-1}$ and $\Delta x = \frac{2\pi}{\Delta k} = 5\pi m.$

5-18. (a) $v = f\lambda$ Thus, $\frac{dv}{d\lambda} = f + \lambda\frac{df}{d\lambda}$, multiplying by λ, $\lambda\frac{dv}{d\lambda} = \lambda f + \lambda^2\frac{df}{d\lambda} = v + \frac{\lambda^2}{2\pi}\frac{d\omega}{d\lambda}$

$-\frac{\lambda^2}{2\pi}\frac{d\omega}{d\lambda} = v - \lambda\frac{dv}{d\lambda}$ Because $k = 2\pi/\lambda$, $dk = -(2\pi/\lambda^2)d\lambda$ and

$$\frac{d\omega}{dk} = v_s = v - \lambda\frac{dv}{d\lambda}$$

(b) v decreases as λ decreases, $dv/d\lambda$ is positive.

5-19. (a) $c = f\lambda = \lambda/T \quad\rightarrow\quad T = \lambda/c = 2\times10^{-2}m/3\times10^{8}m/s = 6.7\times10^{-11}s/wave$

The number of waves = $0.25\mu s/(6.7\times10^{-11}s/wave) = 3.73\times10^{3}$

Length of the packet = $(\lambda)(\#\text{ of waves}) = 2\times10^{-2}m(3.73\times10^{3}) = 74.6m$

(b) $f = c/\lambda = (3\times10^{8}m/s)/2\times10^{-2}m = 1.50\times10^{10}Hz$

(c) $\Delta\omega\Delta t \approx 1 \quad\rightarrow\quad \Delta\omega \approx 1/\Delta t = 1/0.25\times10^{-6}s = 4.0\times10^{6}rad/s = 637kHz$

5-20. $\Delta\omega\Delta t \approx 1 \quad \rightarrow \quad \Delta\omega \approx 1/\Delta t = 1/0.25s = 4.0 rad/s$ or $\Delta f \approx 0.6Hz$

5-21. $\Delta\omega\Delta t \approx 1 \quad \rightarrow \quad (2\pi\Delta f)\Delta t = 1$ Thus, $\Delta t \approx 1/(2\pi \times 5000Hz) = 3.2\times 10^{-5}s$

5-22. (a) $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE_k}} = \dfrac{hc}{\sqrt{2mc^2E_k}} = \dfrac{1240eV\bullet nm}{\left[2\left(0.511\times 10^6 eV\right)\left(5eV\right)\right]^{1/2}} = 0.549nm$

$d\sin\theta = \lambda/2$ For first minimum (see Figure 5-17).

$d = \dfrac{\lambda}{2\sin\theta} = \dfrac{0.549nm}{2\sin 5^\circ} = 3.15nm$ slit separation

(b) $\sin 5^\circ = 0.5cm/L$ where L = distance to detector plane $L = \dfrac{0.5cm}{2\sin 5^\circ} = 5.74cm$

5-23. (a) The particle is found with equal probability in any interval in a force-free region. Therefore, the probability of finding the particle in any interval Δx is proportional to Δx. Thus, the probability of finding the sphere *exactly* in the middle, i.e., with $\Delta x = 0$ is zero.

(b) The probability of finding the sphere somewhere within 24.9*cm* to 25.1*cm* is proportional to Δx =0.2*cm*. Because there is a force free length L = 48*cm* available to the sphere and the probability of finding it somewhere in L is unity, then the probability that it will be found in Δx = 0.2*cm* between 24.9*cm* and 25.1*cm* (or any interval of equal size) is: $P\Delta x = (1/48)(0.2cm) = 0.00417cm$.

5-24. Because the particle must be in the box $\int_0^L \psi^*\psi dx = 1 = \int_0^L A^2\sin^2(\pi x/L)dx = 1$

Let $u = \pi x/L$; $x = 0 \rightarrow u = 0$; $x = L \rightarrow u = \pi$ and $dx = (L/\pi)du$, so we have

$$\int_0^\pi A^2(L/\pi)\sin^2 u\,du = A^2(L/\pi)\int_0^\pi \sin^2 u\,du = 1$$

$$(L/\pi)A^2\int_0^\pi \sin^2 u\,du = (L/\pi)A^2\left[\frac{u}{2} - \frac{\sin 2u}{4}\right]_0^\pi = (L/\pi)A^2(\pi/2) = (LA^2)/2 = 1$$

$\therefore \quad A^2 = 2/L \rightarrow A = (2/L)^{1/2}$

5-25. (a) At $x = 0$: $Pdx = |\psi(0,0)|^2 \, dx = |Ae^0|^2 \, dx = A^2 dx$

(b) At $x = \sigma$: $Pdx = \left|Ae^{-\sigma^2/4\sigma^2}\right|^2 dx = \left|Ae^{-1/4}\right|^2 dx = 0.61A^2 dx$

(c) At $x = 2\sigma$: $Pdx = \left|Ae^{-4\sigma^2/4\sigma^2}\right|^2 dx = \left|Ae^{-1}\right|^2 dx = 0.14A^2 dx$

(d) The electron will most likely be found at $x = 0$, where Pdx is largest.

5-26. (a) One does not know at which oscillation of small amplitude to start or stop counting.

$$f = \frac{N}{\Delta t} \qquad \Delta f = \frac{\Delta N}{\Delta t} \approx \frac{1}{\Delta t}$$

(b) $\lambda = \dfrac{\Delta x}{N}$ and $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi N}{\Delta x}$, so $\Delta k = \dfrac{2\pi\,\Delta n}{\Delta x} \approx \dfrac{2\pi}{\Delta x}$

5-27. $\Delta E \Delta t \approx \hbar \;\rightarrow\; \Delta E \approx \hbar/\Delta t = \dfrac{1.055 \times 10^{-34} J \bullet s}{10^{-7} s\left(1.609 \times 10^{-19} J/eV\right)} \approx 6.6 \times 10^{-9} eV$

5-28. $\Delta x \Delta p \approx \dfrac{\hbar}{2} \quad \rightarrow \quad \Delta p = m\Delta v = \dfrac{\hbar}{2\Delta x}$

$$\therefore \quad \Delta v = \frac{\hbar}{2m\Delta x} = \frac{1.055 \times 10^{-34} J \bullet s}{\left(2 \times 10^{-3} kg\right)\left(\times 10^{-2} \times 10^{-3} m\right)} = 5.3 \times 10^{-27} m/s$$

5-29. $\Delta E \Delta t \approx \hbar \quad \rightarrow \quad \Delta E \approx \hbar/\Delta t = \dfrac{6.58 \times 10^{-16} eV \bullet s}{3.823d\left(8.64 \times 10^4 s/d\right)} \approx 1.99 \times 10^{-21} eV$

The energy uncertainty of the excited state is ΔE, so the α energy can be no sharper than ΔE.

5-30. $\Delta x \Delta p \approx \hbar \quad \rightarrow \quad \lambda \Delta p \approx h \quad \rightarrow \quad \Delta p \approx h/\lambda$. Because $\lambda = h/p$, $p = h/\lambda$; thus, $\Delta p = p$.

5-31. For the cheetah $p = mv = 30kg\left(40m/s\right) = 1200kg \bullet m/s$. Because $\Delta p = p$

(see Problem 5-30), $\Delta x \approx \hbar/\Delta p = 50J \bullet s/1200kg \bullet m/s \approx 4.2 \times 10^{-2} m = 4.2cm$

5-32. Because $c = f\lambda$ for photon, $\lambda = c/f = hc/hf = hc/E$, so

$$E = \frac{hc}{\lambda} = \frac{1240eV \bullet nm}{5.0\times10^{-3}nm} = 2.48\times10^{5}eV$$

and $p = \frac{E}{c} = \frac{2.48\times10^{5}eV}{3\times10^{8}m/s} = 8.3\times10^{-7}eV \bullet s/m$

For electron:

$$\Delta p = \frac{h}{\Delta x} = \frac{4.14\times10^{-15}eV \bullet s}{5.0\times10^{-12}m} = 8.3\times10^{-4}eV \bullet s/m$$

Notice that Δp for the electron is 1000 times larger than p for the photon.

5-33. (a) For ^{48}Ti:

$$\Delta E\left(\text{upper state}\right) = \frac{\hbar}{\Delta t} = \frac{1.055\times10^{-34}J \bullet s}{1.4\times10^{-14}s\left(1.60\times10^{-13}J/MeV\right)} \approx 4.71\times10^{-10}MeV$$

$$\Delta E\left(\text{lower state}\right) = \frac{\hbar}{\Delta t} = \frac{1.055\times10^{-34}J \bullet s}{3.0\times10^{-12}s\left(1.60\times10^{-13}J/MeV\right)} \approx 2.20\times10^{-10}MeV$$

$$\Delta E\left(\text{total}\right) = \Delta E_U + \Delta E_L = 6.91\times10^{-10}MeV$$

$$\frac{\Delta E_T}{E} = \frac{6.91\times10^{-10}MeV}{1.312MeV} = 5.3\times10^{-10}$$

(b) For Hα: $\Delta E_U \approx \frac{1.055\times10^{-34}J \bullet s}{10^{-8}s\left(1.60\times10^{-19}J/eV\right)} \approx 6.59\times10^{-8}eV$

and $\Delta E_L \approx 6.59\times10^{-8}eV$ also.

$\Delta E_T = 1.32\times10^{-7}eV$ is the uncertainty in the Hα transition energy of $1.9eV$.

5-34. $\Delta\omega\Delta t \approx 1 \quad\rightarrow\quad 2\pi\Delta f\,\Delta t \approx 1$

For the visible spectrum the range of frequencies is $\Delta f = \left(7.5-4.0\right)\times10^{14} = 3.5\times10^{14}Hz$

The time duration of a pulse with a frequency uncertainty of Δf is then:

$$\Delta t = \frac{1}{2\pi\Delta f} = \frac{1}{2\pi\times3.5\times10^{14}Hz} = 4.5\times10^{-16}s = 0.45\,fs$$

5-35. The size of the object needs to be of the order of the wavelength of the $10MeV$ neutron.

$\lambda = h/p = h/\gamma mu$. γ and u are found from:

$E_k = m_n c^2(\gamma - 1)$ or $\gamma - 1 = 10MeV/939MeV$

$\gamma = 1 + 10/939 = 1.0106 = 1/\left(1 - u^2/c^2\right)^{1/2}$ or $u = 0.14c$

Then, $\lambda = \dfrac{h}{\gamma mu} = \dfrac{hc}{\left[\gamma mc^2(u/c)\right]} = \dfrac{1240eV \bullet nm}{\left[(1.0106)\left(939\times10^6 eV\right)(0.14)\right]} = 9.33\,fm$

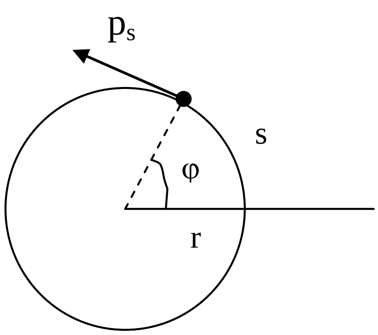
Nuclei are of this order of size and could be used to show the wave character of $10MeV$ neutrons.

5-36. (a) $\Delta E = 135MeV$, the rest energy of the pion.

(b) $\Delta E \Delta t \approx \dfrac{\hbar}{2}$

$$\Delta t = \frac{\hbar}{2\Delta E} = \frac{6.58\times10^{-16} eV \bullet s}{2\times135\times10^6 eV} = 2.44\times10^{-24} s$$

5-37.



$L = rp_s \qquad \phi = \dfrac{s}{r}$

$\Delta L = r\Delta p \qquad \Delta\phi = \dfrac{\Delta s}{r}$

$\Delta L \Delta\phi = (r\Delta p_s)(\Delta s/r) = \Delta p_s \Delta s > \hbar$

In the Bohr model, $L = n\hbar$ and may be known to within $\Delta L \approx 0.1\hbar$.

Then $\Delta\phi > \hbar/(0.1\hbar) = 10rad$. This exceeds one revolution, so that ϕ is completely unknown.

5-38. $E = hf \quad \rightarrow \quad \Delta E = h\Delta f$ E

$\Delta E \Delta t \approx h \quad \rightarrow \quad \Delta f\,\Delta t \approx 1$ where $\Delta t = 0.85ns$

(Problem 5-38 continued)

$\Delta f = 1/0.85ns = 1.18\times10^{9}\,Hz$

For $\lambda = 0.01nm$ $\quad f = c/\lambda = \dfrac{3.00\times10^{8}\,m/s\times10^{9}\,nm/m}{0.01nm} = 3.00\times10^{19}\,Hz$

$$\frac{\Delta f}{f} = \frac{1.18\times10^{9}\,Hz}{3.00\times10^{19}\,Hz} = 3.9\times10^{-11}$$

5-39. $\Delta E\Delta t \approx \dfrac{\hbar}{2} \quad \rightarrow \quad \Delta t = \dfrac{\hbar}{2\Delta E}$

$$\Delta t = \frac{6.58\times10^{-16}\,eV\bullet s}{2\times250\times10^{6}\,eV} = 1.32\times10^{-24}\,s$$

5-40. In order for diffraction to be observed, the aperture diameter must be of the same order of magnitude as the wavelength of the particle. In this case the latter is

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}\ \mathrm{J\cdot s}}{(4\times10^{-3}\ \mathrm{kg})(100\ \mathrm{m/s})} = 1.66\times10^{-33}\ \mathrm{m}$$

The diameter of the aperture would need to be of the order of 10^{-33}m. This is many, many orders of magnitude smaller than even the diameter of a proton or neutron. No such apertures are available.

5-41. The kinetic energy of the electron needed must be no larger than 0.1 nm. The minimum kinetic energy of the electrons needed is then given by:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e E}} \quad \Rightarrow \quad E = \frac{h^2}{2m_e\lambda^2}$$

$$E = \frac{(6.63^{-34}\ \mathrm{J\cdot s})^2}{2(9.11\times10^{-31}\ \mathrm{kg})(0.1\times10^{-9}\ \mathrm{nm})^2} = 2.41\times10^{-17}\ \mathrm{J} = 151\mathrm{eV}$$

5-42. (a) For a proton or neutron:

$\Delta x \Delta p \approx \dfrac{\hbar}{2}$ and $\Delta p = m\Delta v$ assuming the particle speed to be non-relativistic.

$$\Delta v = \frac{\hbar}{2m\Delta x} = \frac{1.055\times10^{-34}\,J\bullet s}{2\left(1.67\times10^{-27}\,kg\right)\left(10^{-15}\,m\right)} = 3.16\times10^{7}\,m/s \approx 0.1c \text{ (non-relativistic)}$$

(b) $$E_k \approx \frac{1}{2}mv^2 = \frac{\left(1.67\times10^{-27}\,kg\right)\left(3.16\times10^{7}\,m/s\right)^2}{2} = 8.34\times10^{-13}\,J = 5.21MeV$$

(c) Given the proton or neutron velocity in (a), we expect the electron to be relativistic, in which case, $E_k = mc^2\left(\gamma - 1\right)$ and

$$\Delta p = \frac{\hbar}{2\Delta x} \approx \gamma m v \quad \rightarrow \quad \gamma v \approx \frac{\hbar}{2m\Delta x}$$

For the relativistic electron we assume $v \approx c$

$$\gamma \approx \frac{\hbar}{2mc\Delta x} = \frac{1.055\times10^{-34}\,J\bullet s}{2\left(9.11\times10^{-31}kg\right)\left(3.00\times10^{8}\,m/s\right)\left(10^{-15}\,m\right)} = 193$$

$$E_k = mc^2\left(\gamma - 1\right) = \left(9.11\times10^{-31}kg\right)\left(3.00\times10^{8}\,m/s\right)^2\left(192\right) = 1.58\times10^{-11}\,J = 98MeV$$

5-43. (a) $E^2 = p^2c^2 + m^2c^4 \qquad E = hf = \hbar\omega \qquad p = h/\lambda = \hbar/k \qquad \hbar^2\omega^2 = \hbar^2k^2c^2 + m^2c^4$

$$v = \frac{\omega}{k} = \frac{\hbar\omega}{\hbar k} = \frac{\sqrt{\hbar^2k^2c^2 + m^2c^4}}{\hbar k} = c\sqrt{1 + m^2c^2/\hbar^2k^2} > c$$

(b) $$v_s = \frac{d\omega}{dk} = \frac{d}{dk}\sqrt{\frac{k^2c^2 + m^2c^4}{\hbar k}} = \frac{c^2k}{\sqrt{\dfrac{k^2c^2 + m^2c^4}{\hbar^2}}}$$

$$= \frac{c^2k}{\omega} = \frac{c^2\hbar k}{\hbar\omega} = \frac{c^2p}{E} = u \quad \text{(by Equation 2-41)}$$

5-44. $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$ (Equation 5-11) $\qquad y_3 = C_1y_1 + C_2y_2$

$$\frac{\partial^2 y_3}{\partial x^2} = C_1\frac{\partial^2 y_1}{\partial x^2} + C_2\frac{\partial^2 y_2}{\partial x^2} = C_1\left(\frac{1}{v^2}\frac{\partial^2 y_1}{\partial t^2}\right) + C_2\left(\frac{1}{v^2}\frac{\partial^2 y_2}{\partial t^2}\right)$$

(Problem 5-44 continued)

$$= \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\left(C_1 y_1 + C_2 y_2\right) = \frac{1}{v^2}\frac{\partial^2 y_3}{\partial t^2}$$

5-45. The classical uncertainty relations are

$\Delta\omega\Delta t \;\rightarrow\; 2\pi\Delta f\,\Delta t \approx 1$ (Equation 5-18)

and $\Delta x \Delta\lambda \approx \dfrac{\lambda^2}{2\pi}$ (Equation 5-20)

(a) $\Delta f = \dfrac{1}{2\pi\Delta t} = \dfrac{1}{2\pi(3.0s)} = 0.0541Hz$

(b) Length of the wave traing $L = v\Delta t$, where v = speed of sound in air = 330*m/s.*

$L = (330m/s)(3.0s) = 990m$

(c) $\Delta\lambda = \dfrac{\lambda^2}{2\pi\Delta x}$ where Δx = length of the wave train $= 990m$ from (b)

and $\lambda = 0.13m$ from (d). $\Delta\lambda = \dfrac{(0.13m)^2}{2\pi(990m)} = 2.72\times10^{-6} = 2.72\mu m$

(d) $v = f\lambda \quad\rightarrow\quad \lambda = \dfrac{v}{f} = \dfrac{(330m/s)}{(2500Hz)} = 0.13m = 13cm$

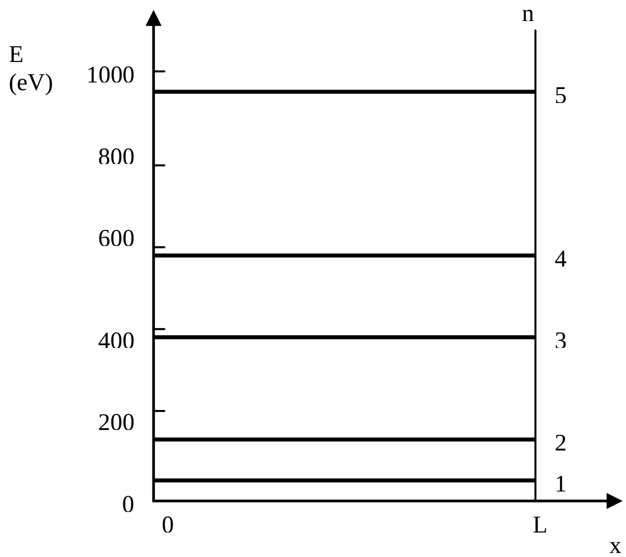
5-46. (a) $n(\lambda/2) = L \quad\rightarrow \lambda = 2L/n.$ Because $\lambda = h/p = h/\sqrt{2mE}$, then

$$E = \frac{h^2}{2m\lambda^2} = \frac{h^2}{2m(L/n)^2} = \frac{h^2n^2}{8mL^2} \quad \text{If } E_1 = h^2/8mL^2, \text{ then } E_n = \frac{h^2n^2}{8mL^2} = n^2E_1$$

(b) For $L = 0.1nm$, $E_1 = \dfrac{h^2}{8mL^2} = \dfrac{(hc)^2}{8mc^2L^2} = \dfrac{(1240eV\bullet nm)^2}{8(0.511\times10^6 eV)(0.1nm)^2}$

$E_1 = 37.6eV$ and $E_n = 37.6n^2eV$

(Problem 5-46 continued)



(c) $f = \Delta E / h \;\rightarrow\; c/\lambda = \Delta E / h \;\rightarrow\; \lambda = \dfrac{hc}{\Delta E}$

For $n = 2 \rightarrow n = 1$ transition, $\Delta E = 112.8eV$ and $\lambda = \dfrac{1240eV \bullet nm}{112.8eV} = 11.0nm$

(d) For $n = 3 \rightarrow n = 2$ transition, $\Delta E = 188eV$ and $\lambda = \dfrac{1240eV \bullet nm}{188eV} = 6.6nm$

(e) For $n = 5 \rightarrow n = 1$ transition, $\Delta E = 903eV$ and $\lambda = \dfrac{1240eV \bullet nm}{903eV} = 1.4nm$

5-47. (a) For proton: $E_1 = \dfrac{(hc)^2}{8m_p c^2 L^2}$ from Problem 5-46.

$$E_1 = \frac{(1240MeV \bullet fm)^2}{8(938MeV)(1\,fm)^2} = 205MeV \text{ and } E_n = 205n^2 MeV$$

$\therefore \quad E_2 = 820MeV$ and $E_3 = 1840MeV$

(b) For $n = 2 \rightarrow n = 1$ transition, $\lambda = \dfrac{hc}{\Delta E} = \dfrac{1240MeV \bullet fm}{615MeV} = 2.02\,fm$

(c) For $n = 3 \rightarrow n = 2$ transition, $\lambda = \dfrac{hc}{\Delta E} = \dfrac{1240MeV \bullet fm}{1020MeV} = 1.22\,fm$

(d) For $n = 3 \rightarrow n = 1$ transition, $\lambda = \dfrac{hc}{\Delta E} = \dfrac{1240MeV \bullet fm}{1635MeV} = 0.76\,fm$

5-48. (a) $E \geq \hbar^2 / 2mL^2$ (Equation 5-28) and $E = \hbar^2 / 2mA^2$

(b) For electron with $A = 10^{-10}\,m$:

$$E = \frac{(\hbar c)^2}{2mc^2 A^2} = \frac{(197.3eV{\bullet}nm)^2}{2(0.511\times10^6 eV)(10^{-1}nm)^2} = 3.81eV$$

For electron with $A = 1cm$ or $A = 10^{-2}\,m$:

$$E = 3.81eV\left(10^{-1}\right)^2 / \left(10^7\,nm\right)^2 = 3.81\times10^{-16}\,eV$$

(c) $$E = \frac{\hbar^2}{2mL^2} = \frac{\left(1.055\times10^{-34}\,J{\bullet}s\right)^2}{2\left(100\times10^{-3}\,g\times10^{-3}\,kg/g\right)\left(2\times10^{-2}\right)^2} = 1.39\times10^{-61}\,J = 8.7\times10^{-43}\,eV$$

5-49. $\Delta p = m\Delta v = m(0.0001)(500m/s) = 0.05m$

For proton: $\Delta x \Delta p \approx \hbar$

$$\Delta x \approx \hbar / \Delta p = \left(6.58\times10^{-16}\,eV{\bullet}s\right) / \left(0.05m/s\right)\left(938\times10^6\,eV\right)$$

$$\approx 1.40\times10^{-23}\,m = 1.40\times10^{-8}\,fm$$

For bullet: $\Delta x \approx \left(1.055\times10^{-34}\,J{\bullet}s\right) / \left(0.05m/s\right)\left(10\times10^{-3}\,kg\right) \approx 2.1\times10^{-31}\,m$

5-50. $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v}\dfrac{\partial^2 y}{\partial t^2}$ (Equation 5-11) where $y = f(\phi)$ and $\phi = x - vt$.

$$\frac{\partial y}{\partial x} = \frac{\partial f}{\partial \phi}\times\frac{\partial \phi}{\partial x} \quad \text{and} \quad \frac{\partial y^2}{\partial x^2} = \frac{\partial f}{\partial \phi}\times\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial \phi}{\partial x}\times\frac{\partial^2 f}{\partial \phi^2}\times\frac{\partial \phi}{\partial x}$$

$$\frac{\partial y}{\partial t} = \frac{\partial f}{\partial \phi}\times\frac{\partial \phi}{\partial t} \quad \text{and} \quad \frac{\partial y^2}{\partial t^2} = \frac{\partial f}{\partial \phi}\times\frac{\partial^2 \phi}{\partial t^2} + \frac{\partial \phi}{\partial t}\times\frac{\partial^2 f}{\partial \phi^2}\times\frac{\partial \phi}{\partial t}$$

Noting that $\dfrac{\partial^2 \phi}{\partial x^2} = 0,\quad \dfrac{\partial \phi}{\partial x} = 1,\quad \dfrac{\partial^2 \phi}{\partial t^2} = 0,$ and $\dfrac{\partial \phi}{\partial t} = -v,$ we then have:

$$\frac{\partial f}{\partial \phi}\times 0 + 1\times\frac{\partial^2 f}{\partial \phi^2}\times 1 = \frac{1}{v^2}\left(\frac{\partial f}{\partial \phi}\times 0 + (-v)\times\frac{\partial^2 f}{\partial \phi^2}\times(-v)\right)$$

$$\frac{\partial^2 f}{\partial \phi^2} = \frac{\partial^2 f}{\partial \phi^2}$$

5-51. (a) $\lambda = h/p$ The electrons are not moving at relativistic speeds, so

$$\lambda = h/mv = 6.63\times10^{-34} J\bullet s/\left(9.11\times10^{-31} kg\right)\left(3\times10^{6} m/s\right) = 2.43\times10^{-19} m = 0.243 nm$$

(b) The energy, momentum, and wavelength of the two photons are equal.

$$E = \frac{1}{2}mv^2 + mc^2 = \frac{1}{2}mc^2\left(v^2/c^2\right) + mc^2 = mc^2\left[\frac{1}{2}\left(v^2/c^2\right)+1\right]$$

$$= 0.511\times10^6 eV\left[\frac{1}{2}\left(3\times10^6\right)/\left(3\times10^8\right)^2 + 1\right] \approx 0.511 MeV$$

(c) $p = E/c = 0.511 MeV/c$

(d) $\lambda = hc/E = 1240 eV\bullet nm/0.511\times10^6 eV = 2.43\times10^{-3} nm$

5-52. (a) $Q = m_p c^2 - m_n c^2 - m_\pi c^2$

$$= 1.007825uc^2 - 1.008665uc^2 - 139.6 MeV$$

$$= 938.8 MeV - 939.6 MeV - 139.6 MeV = -140.4 MeV$$

$$\Delta E = -140.4 MeV$$

(b) $\Delta E \Delta t \approx \quad \rightarrow \quad \Delta t \approx \hbar/\Delta E = 6.58\times10^{-16} eV\bullet s/140.4\times10^6 eV \approx 4.7\times10^{-24} s$

(c) $d = c\Delta t = 3\times10^8 m/s\left(4.7\times10^{-24} s\right) = 1.4\times10^{-15} m = 1.4 fm$

5-53. $hf = \gamma mc^2 \quad \rightarrow \quad \gamma = \dfrac{hf}{mc^2} = \dfrac{1}{\sqrt{1 - v^2/c^2}}$

$$1 - v^2/c^2 = \left(\frac{mc^2}{hf}\right)^2 \rightarrow v/c = \left[1 - \left(\frac{mc^2}{hf}\right)^2\right]^{1/2}$$

Expanding the right side, assuming $mc^2 \ll hf$,

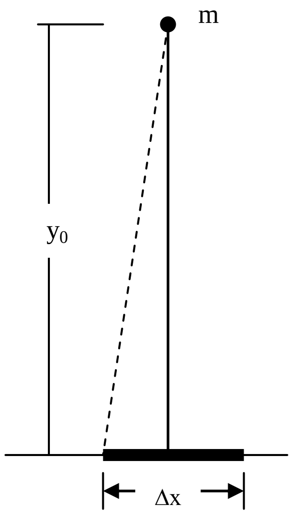
$\dfrac{v}{c} = 1 - \dfrac{1}{2}\left(\dfrac{mc^2}{hf}\right)^2 - \dfrac{1}{8}\left(\dfrac{mc^2}{hf}\right)^4 + \cdots$ and neglecting all but the first two terms,

$\dfrac{v}{c} = 1 - \dfrac{1}{2}\left(\dfrac{mc^2}{hf}\right)^2$ Solving this for m and inserting deBroglie's assumptions that

$\dfrac{v}{c} \geq 0.99$ and $\lambda = 30m$, m is then:

$$m = \frac{\left[\left(1-0.99\right)2\right]^{1/2}\left(6.63\times10^{-34} J\bullet s\right)}{\left(3.00\times10^8 m/s\right)\left(30m\right)} = 1.04\times10^{-44} kg$$

5-54. (a)



$$\Delta x \Delta p \approx \hbar \quad \rightarrow \quad m \Delta x \Delta v_x \approx \hbar \quad \rightarrow \quad \Delta v_x \approx \hbar / m \Delta x$$

$$y_0 = \frac{1}{2} g t^2 \;\rightarrow\; t = \left(\frac{2y_0}{g}\right)^{1/2} \qquad \frac{1}{2}\Delta X = \Delta v_x \bullet t = \Delta v_x \left(\frac{2y_0}{g}\right)^{1/2}$$

$$\Delta X = 2\Delta v_x \left(\frac{2y_0}{g}\right)^{1/2} = \frac{2\hbar\left(\frac{2y_0}{g}\right)^{1/2}}{m\Delta x}$$

(b) If also $\Delta y \Delta p_y \approx \hbar \;\rightarrow\; \Delta v_y \approx \hbar / my$ and $\frac{1}{2}\Delta X = \Delta x\left(t + \Delta t\right)$ where $\Delta v_y = g\Delta t$ or

$\Delta t = \Delta v_y / g = \hbar / mg\Delta y$ so, $\Delta X = \frac{2\hbar}{m\Delta x}\left[\left(2y_0 / g\right)^{1/2} + \hbar / mg\Delta y\right]$

5-55. $\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT$

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \left[\frac{3\left(1.381\times10^{-23} J/K\right)\left(300K\right)}{56u\left(1.66\times10^{-27} kg/u\right)}\right]^{1/2} = 366 m/s$$

$$f' = f_o\left(1 + v/c\right) \quad \rightarrow \quad hf' = hf_o\left(1 + v/c\right)$$

$$\Delta E = hf' - hf_o = hf_o v/c = \frac{\left(1eV\right)\left(366 m/s\right)}{3.0\times10^8 m/s} = 1.2\times10^{-6} eV$$

This is about 12 times the natural line width.

$$\Delta E = hf_o v/c = \frac{\left(10^6 eV\right)\left(366 m/s\right)}{3.0\times10^8 m/s} = 1.2 eV$$

This is over 10^7 times the natural line width.

5-56. $\rho_{recoil} = \rho_\gamma = E_\gamma / c$ $\qquad E_{recoil} = \dfrac{(\rho_{recoil})^2}{2m} = \dfrac{E_\gamma^2}{2mc^2}$

(a) $E_{recoil} = \dfrac{(1eV)^2}{2(56uc^2)} \dfrac{uc^2}{931.5\times10^6 eV} = 9.6\times10^{-12} eV$

This is about 10^{-4} times the natural line width estimated at $10^{-7}eV$.

(b) $E_{recoil} = \dfrac{(1MeV)^2}{2(56uc^2)} \dfrac{uc^2}{931.5\times10^6 eV} = 9.6eV$

This is about 10^{8} times the natural line width.

Chapter 6 – The Schrödinger Equation

6-1. $\frac{d\Psi}{dx} = kAe^{kx-\omega t} = k\Psi$ and $\frac{d^2\Psi}{dx^2} = k^2\Psi$

Also, $\frac{d\Psi}{dt} = -\omega\Psi$. The Schrödinger equation is then, with these substitutions,

$-\hbar^2 k^2\Psi / 2m + V\Psi = -i\hbar\omega\Psi$. Because the left side is real and the right side is a pure Imaginary number, the proposed Ψ does not satisfy Schrödinger's equation.

6-2. For the Schrödinger equation: $\frac{\partial\Psi}{\partial x} = ik\Psi$ and $\frac{\partial^2\Psi}{\partial x^2} = -k^2\Psi$. Also, $\frac{\partial\Psi}{\partial t} = -i\omega\Psi$.

Substituting these into the Schrödinger equation yields:

$\hbar^2 k^2\Psi / 2m + V\Psi = \hbar\omega\Psi$, which is true, provided $\hbar\omega = \hbar^2 k^2 / 2m + V$, i.e., if $E = E_k + V$.

For the classical wave equation: (from Equation 6-1)

From above: $\frac{\partial^2\Psi}{\partial x^2} = -k^2\Psi$ and also $\frac{\partial^2\Psi}{\partial t^2} = -\omega^2\Psi$. Substituting into Equation 6-1 (with Ψ replacing $\mathcal{E}$ and *v* replacing *c*) $-k^2\Psi = \left(1/v^2\right)\left(-\omega^2\Psi\right)$, which is true for $v = \omega / k$.

6-3. (a) $\frac{d\psi}{dx} = -\left(x / L^2\right)\psi$ and $\frac{d^2\psi}{dx^2}\left[\left(-\frac{x}{L^2}\right)\left(-\frac{x}{L^2}\right) - \frac{1}{L^2}\right]\psi = \frac{x^2}{L^4}\psi - \frac{1}{L^2}\psi$

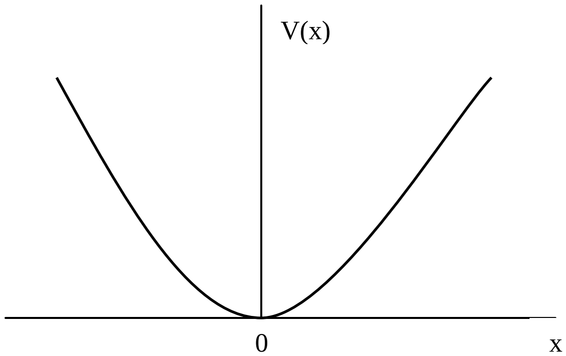
Substituting into the time-independent Schrödinger equation,

$$\left(-\frac{\hbar^2 x^2}{2mL^4} + \frac{\hbar^2}{2mL^2}\right)\psi + V\left(x\right) = E\psi = \frac{\hbar^2}{2mL^2}\psi$$

Solving for *V(x)*, $V\left(x\right) = \frac{\hbar^2}{2mL^2} - \left(-\frac{\hbar^2 x^2}{2mL^4} + \frac{\hbar^2}{2mL^2}\right) = \frac{\hbar^2 x^2}{2mL^4} = \frac{1}{2}kx^2$

(Problem 6-3 continued)

where $k = \hbar^2 / mL^4$. This is the equation of a parabola centered at $x = 0$.



(b) The classical system with this dependence is the harmonic oscillator.

6-4. (a) $E_k(x) = E - V(x) = \hbar^2 / 2mL^2 - \hbar^2 x^2 / 2mL^4 = \left(\hbar^2 / 2mL^2\right)\left(1 - x^2 / L^2\right)$

(b) The classical turning points are the points where $E = V(x)$ or $E_k(x) = 0$. That occurs when $x^2 / L^2 = 1$, or when $x = \pm L$.

(c) For a harmonic oscillator $V(x) = m\omega^2 x^2 / 2$, so

$$\frac{\hbar^2 x^2}{2mL^4} = \omega^2 x^2 / 2 \;\rightarrow\; \omega^2 = \hbar^2 / m^2 L^4 \;\rightarrow\; \omega = \hbar / mL^2$$

Thus, $E = \dfrac{\hbar^2}{2mL^2} = \left(\dfrac{\hbar}{mL^2}\right)\dfrac{\hbar}{2} = \dfrac{1}{2}\hbar\omega$

6-5. (a) $\Psi(x,t) = A\sin(kx - \omega t)$

$$\frac{\partial \Psi}{\partial t} = -\omega A\cos(kx - \omega t)$$

$$i\hbar\frac{\partial \Psi}{\partial t} = -i\hbar\omega A\cos(kx - \omega t)$$

$$\frac{\partial^2 \Psi}{\partial x^2} = -k^2 A\sin(kx - \omega t)$$

$$\frac{-\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} = \frac{-\hbar k^2 A}{2m}\sin(kx - \omega t) \neq i\hbar\frac{\partial \Psi}{\partial t}$$

(Problem 6-5 continued)

(b) $\Psi(x,t) = A\cos(kx-\omega t) + iA\sin(kx-\omega t)$

$$i\hbar\frac{\partial\Psi}{\partial t} = i\hbar\omega A\sin(kx-\omega t) - i^2\hbar\omega A\cos(kx-\omega t)$$

$$= \hbar\omega A\cos(kx-\omega t) + i\hbar\omega A\sin(kx-\omega t)$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} = \frac{\hbar^2 k^2 A}{2m}\cos(kx-\omega t) + \frac{\hbar^2 ik^2 A}{2m}\sin(kx-\omega t)$$

$$= \frac{\hbar^2 k^2}{2m}\left[A\cos(kx-\omega t) + iA\sin(kx-\omega t)\right]$$

$$= i\hbar\frac{\partial\Psi}{\partial t} \quad \text{if } \frac{\hbar^2 k^2}{2m} = \hbar\omega \text{ it does. (Equation 6-5 with } V = 0\text{)}$$

6-6. (a) For a free electron $V(x) = 0$, so $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \rightarrow \frac{d^2\psi}{dx^2} = -\left(2.5\times10^{10}\right)^2\psi$

Substituting into the Schrödinger equation gives: $\left(2.5\times10^{10}\right)^2\left(\hbar^2/2m\right)\psi = E\psi$

and, since $E = E_k = p^2/2m$ for a free particle, $p^2 = 2m\left(2.5\times10^{10}\right)^2\left(\hbar^2/2m\right)$ and

$p = \left(2.5\times10^{10}\right)\hbar = 2.64\times10^{-24}\,kg\bullet m/s$

(b) $E = p^2/2m = \left(2.64\times10^{-24}\,kg\bullet m/s\right)^2/(2)\left(9.11\times10^{-31}\,kg\right) = 3.82\times10^{-18}\,J$

$= \left(3.82\times10^{-18}\,J\right)\left(1/1.60\times10^{-19}\,J/eV\right) = 23.9eV$

(c) $\lambda = h/p = 6.63\times10^{-34}\,J\bullet s/2.64\times10^{-24}\,kg\bullet m/s = 2.51\times10^{-10}\,m = 0.251nm$

6-7. $\psi(x) = Ce^{-x^2/L^2}$ and $E = 0$

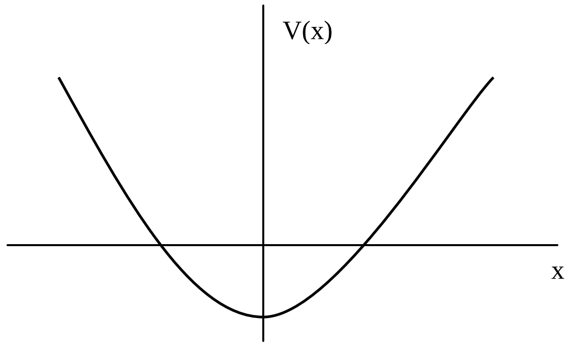
(a) $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = 0$

$\frac{d\psi}{dx} = -\frac{2x}{L^2}\psi(x)$ and $\frac{d^2\psi}{dx^2} = \left(\frac{4x^2}{L^4} - \frac{2}{L^2}\right)\psi$

And $-\frac{\hbar^2}{2m}\left(\frac{4x^2}{L^4} - \frac{2}{L^2}\right)\psi + V(x)\psi = 0$ so $V(x) = \frac{\hbar^2}{mL^2}\left(\frac{2x^2}{L^2} - 1\right)$

(Problem 6-7 continued)

(b)



6-8. $\int_{-a}^{+a} \psi * \psi \, dx = A^2 \int_{-a}^{+a} e^{-i(kx-\omega t)} \times e^{i(kx-\omega t)} dx = 1$

$$= A^2 \int_{-a}^{+a} dx = A^2 x \Big|_{-a}^{+a} = A^2 (2a) = 1$$

$$\therefore \; A = \frac{1}{(2a)^{1/2}}$$

Normalization between $-\infty$ and $+\infty$ is not possible because the value of the integral is infinite.

6-9. (a) The ground state of an infinite well is $E_1 = h^2 / 8mL^2 = (hc)^2 / 8mc^2L^2$

For $m = m_p$, $L = 0.1nm$: $E_1 = \dfrac{(1240MeV \bullet fm)^2}{8(938.3 \times 10^6 eV)(0.1nm)^2} = 0.021eV$

(b) For $m = m_p$, $L = 1fm$: $E_1 = \dfrac{(1240MeV \bullet fm)^2}{8(938.3 \times 10^6 eV)(1fm)^2} = 205MeV$

6-10. The ground state wave function is (n = 1) $\psi_1(x) = \sqrt{2/L}\,\sin(\pi x/L)$ (Equation 6-32)

The probability of finding the particle in Δx is approximately:

$$P(x)\Delta x = \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right)\Delta x = \frac{2\Delta x}{L}\sin^2\left(\frac{\pi x}{L}\right)$$

(Problem 6-10 continued)

(a) For $x = \frac{L}{2}$ and $\Delta x = 0.002L$, $P(x)\Delta x = \frac{2(0.002L)}{L}\sin^2\left(\frac{\pi L}{2L}\right) = 0.004\sin^2\frac{\pi}{2} = 0.004$

(b) For $x = \frac{2L}{3}$ and $\mathrm{P}(\mathrm{x})\Delta x = \frac{2(0.002L)}{L}\sin^2\left(\frac{2\pi L}{3L}\right) = 0.004\sin^2\frac{2\pi}{3} = 0.0030$

(c) For $x = L$ and $\mathrm{P}(\mathrm{x})\Delta x = 0.004\sin^2\pi = 0$

6-11. The second excited state wave function is (n = 3) $\psi_3(x) = \sqrt{2/L}\sin(3\pi x/L)$ (Equation 6-32). The probability of finding the particle in Δx is approximately:

$$P(x)\Delta x = \frac{2}{L}\sin^2\left(\frac{3\pi x}{L}\right)\Delta x$$

(a) For

$x = \frac{L}{2}$ and $\Delta x = 0.002L$, $P(x)\Delta x = \frac{2(0.002L)}{L}\sin^2\left(\frac{3\pi L}{2L}\right) = 0.004\sin^2\frac{3\pi}{2} = 0.004$

(b) For $x = \frac{2L}{3}$ and $\mathrm{P}(\mathrm{x})\Delta x = 0.004\sin^2\left(\frac{6\pi L}{3L}\right) = 0.004\sin^2 2\pi = 0$

(c) For $x = L$ and $\mathrm{P}(\mathrm{x})\Delta x = 0.004\sin^2\left(\frac{3\pi L}{L}\right) = 0.004\sin^2 3\pi = 0$

6-12. $E = \frac{1}{2}mv^2 = \frac{n^2\pi^2\hbar^2}{2mL^2}$ (Equation 6-24) $n^2 = \left(\frac{1}{2}mv^2\right)\left(\frac{2mL^2}{\pi^2\hbar^2}\right) = \left(\frac{mvL}{\pi\hbar}\right)^2$

$$n = \frac{mvL}{\pi\hbar} = \frac{(10^{-9}kg)(10^{-3}m/s)(10^{-2}m)}{\pi(1.055\times10^{-34}J\bullet s)} = 3\times10^{19}$$

6-13. (a) $\Delta x = 0.0001L = (0.0001)(10^{-2}m) = 10^{-6}m$

$\Delta p = 0.0001p = (0.0001)(10^{-9}kg)(10^{-3}m/s) = 10^{-16}kg\bullet m/s$

(b) $\frac{\Delta x\Delta p}{\hbar} = \frac{(10^{-6}m)(10^{-16}kg\bullet m/s)}{1.055\times10^{-34}J\bullet s} = 9\times10^{11}$

6-14 (a) This is an infinite square well with width L. $V(x) = 0$, and $E = E_k = p^2/2m$. From uncertainty principle: $E_{k_{\min}} \rightarrow p_{\min} \approx \Delta p = \hbar/\Delta x = \hbar/L$ and

$$E_{\min} = p_{\min}^2/2m \approx \hbar^2/2mL^2 = h^2/8\pi^2 mL^2$$

(b) The solution to the Schrödinger equation for the ground state is:

$$\psi_1(x) = (2/L)^{1/2}\sin(\pi x/L)$$

and $\dfrac{d^2\psi_1}{dx^2} = -\left(\dfrac{\pi}{L}\right)^2\left(\dfrac{2}{L}\right)^{1/2}\sin(\pi x/L) = -\left(\dfrac{\pi}{L}\right)^2\psi_1$

So, $\dfrac{\hbar^2}{2m}\left(\dfrac{\pi}{L}\right)^2\psi_1 = E\psi_1$ or $E_1 = \dfrac{h^2}{8mL^2}$

The result in (a) is about 1/10 of the computed value, but has the correct dependencies on h, m, and L.

6-15. (a) For the ground state, $L = \lambda/2$, so $\lambda = 2L$.

(b) Recall that state n has n half-wavelengths between $x = 0$ and $x = L$, so for $n = 3$, $L = 3\lambda/2$, or $\lambda = 2L/3$.

(c) $p = h/\lambda = h/2L$ in the ground state.

(d) $p^2/2m = \left(h^2/4L^2\right)/2m = h^2/8mL^2$, which is the ground state energy.

6-16. $E_n = \dfrac{h^2 n^2}{8mL^2}$ and $\Delta E_n = E_{n+1} - E_n = \dfrac{h^2}{8mL^2}\left(n^2 + 2n + 1\right)$

or, $\Delta E_n = (2n+1)\dfrac{h^2}{8mL^2} = \dfrac{hc}{\lambda}$

so, $L = \left(\dfrac{3\lambda h}{8mc}\right)^{1/2} = \left(\dfrac{3\lambda hc}{8mc^2}\right)^{1/2} = \left(\dfrac{3(694.3nm)(1240eV\bullet nm)}{8\left(0.511\times 10^6 eV\right)}\right)^{1/2} = 0.795nm$

6-17. The uncertainty principle requires that $\overline{E} \geq \dfrac{\hbar^2}{2mL^2}$ for any particle in any one-dimensional box of width L (Equation 5-28). For a particle in an infinite one-dimensional square well:

$$E_n = \frac{n^2 h^2}{8mL^2}$$

(Problem 6-17 continued)

For $n = 0$, then E_0 must be 0 since $\dfrac{h^2}{8mL^2} > 0$. This violates Equation 5-28 and, hence, the exclusion principle.

6-18. (a) Using Equation 6-24 with $L = 0.05$ nm and $n = 92$, the energy of the 92nd electron in the model atom is E_{92} given by:

$$E_n = n^2 \frac{\pi^2\hbar^2}{2mL^2} \quad \Rightarrow \quad E_{92} = (92)^2 \frac{(6.63\times10^{-34}\ \mathrm{J\cdot s})^2}{8(9.11\times10^{-31}\ \mathrm{kg})(0.05\times10^{-9}\ \mathrm{m})^2}$$

$$E_{92} = 2.04\times10^{-13}\ \mathrm{J}\times\frac{1\mathrm{eV}}{1.60\times10^{-19}\ \mathrm{J}} = 1.28\times10^{6}\ \mathrm{eV} = 1.28\,\mathrm{MeV}$$

(b) The rest energy of the electron is 0.511 MeV. $E_{92} = 2.5\times$ the electron's rest energy.

6-19. This is an infinite square well with $L = 10cm$.

$$E_n = \frac{h^2n^2}{8mL^2} = \frac{1}{2}mv^2 = \frac{\left(2.0\times10^{-3}kg\right)\left(20nm/y\right)^2}{2\left(3.16\times10^{7}s/y\right)^2}$$

$$n^2 = \frac{8\left(2.0\times10^{-3}kg\right)^2\left(20\times10^{-9}m\right)^2\left(0.1m\right)^2}{2\left(3.16\times10^{7}s\right)^2\left(6.63\times10^{-34}J\bullet s\right)^2}$$

$$n = \frac{2\left(2.0\times10^{-3}kg\right)\left(20\times10^{-9}m\right)\left(0.1m\right)}{\left(3.16\times10^{7}s\right)\left(6.63\times10^{-34}J\bullet s\right)} = 3.8\times10^{14}$$

6-20. (a) $\psi_5\left(x\right) = \left(2/L\right)^{1/2}\sin\left(5\pi x/L\right)dx$

$$P = \int_{0.2L}^{0.4L}\left(2/L\right)\sin^2\left(5\pi x/L\right)dx$$

Letting $5\pi x/L = u$, then $5\pi dx/L = du$ and $x = 0.2L \rightarrow u = \pi$ and $x = 0.4L \rightarrow u = 2\pi$, so

(Problem 6-20 continued)

$$P = \left(\frac{2}{L}\right)\left(\frac{L}{5\pi}\right)\int_{\pi}^{2\pi}\sin^2 u\,du = \left(\frac{2}{L}\right)\left(\frac{L}{5\pi}\right)\left(\frac{\frac{x}{2} - \sin 2x}{4}\right)\Bigg|_{\pi}^{2\pi} = \frac{1}{5}$$

(b) $P = (2/L)\sin^2\dfrac{5\pi(L/2)}{L}(0.01L) = 0.02$ where $0.01L = \Delta x$

6-21. (a) For an electron: $E_1 = \dfrac{(1240 MeV \bullet fm)^2}{8(0.511 MeV)(10\,fm)^2} = 3.76\times 10^3\,MeV$

(b) For a proton: $E_1 = \dfrac{(1240 MeV \bullet fm)^2}{8(938.3 Mev)(10\,fm)^2} = 2.05 MeV$

(c) $\Delta E_{21} = 3E_1$ (See Problem 6-16)

For the electron: $\Delta E_{21} = 3E_1 = 1.13\times 10^4\,MeV$

For the proton: $\Delta E_{21} = 3E_1 = 6.15 MeV$

6-22. $F = -dE/dL$ comes from the impulse-momentum theorem $F\Delta t = 2mv$ where $\Delta t \approx L/v$. So, $F \sim mv^2/L \sim E/L$. Because $E_1 = h^2/8mL^2$, $dE/dL = -h^2/4mL^3$ where the minus sign means "on the wall". So $F = h^2/4mL^3 = \dfrac{(6.63\times 10^{-34}\,J\bullet s)^2}{4(9.11\times 10^{-31}\,kg)(10^{-10}\,m)^3} = 1.21\times 10^{-7}\,N$

The weight of an electron is $mg = 9.11\times 10^{-31}\,kg\,(9.8\,m/s^2) = 8.9\times 10^{-30}\,N$ which is minuscule by comparison.

6-23. $\psi_n(x) = \sqrt{\dfrac{2}{L}}\sin\dfrac{n\pi x}{L}$

To show that $\displaystyle\int_0^L \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)dx = 0$

Using the identity $2\sin A\sin B = \cos(A-B) - \cos(A+B)$, the integrand becomes

$\dfrac{1}{2}\left\{\cos\left[(n-m)\pi x/L\right] - \cos\left[(n+m)\pi x/L\right]\right\}$

(Problem 6-23 continued)

The integral part of the first term is $\frac{L}{\pi}\frac{\sin(n-m)\pi x/L}{(n-m)}$ and similarly for the second Term with (n + m) replacing (n – m). Since n and m are integers and $n \neq m$, the sines both vanish at the limits x = 0 and x = L.

$\therefore \int_0^L \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)dx = 0$ for $n \neq m$.

6-24. (a)

(b)

6-25. Refer to MORE section "Graphical Solution of the Finite Square Well". If there are only two allowed energies within the well, the highest energy $E_2 = V_0$, the depth of the well. From Figure 6-14, $ka = \pi/2$, i.e., $ka = \frac{\sqrt{2mE_2}}{\hbar} \times a = \pi/2$

where $a = 1/2(1.0\,fm) = 0.5\,fm$ and $m = 939.6MeV/c^2$ for the neutron.

Substituting above, squaring, and re-arranging, we have:

$$E_2 = V_0 = \left(\frac{\pi}{2}\right)^2 \frac{\hbar^2}{2(939.6MeV/c^2)(0.5\,fm)^2}$$

$$V_0 = \frac{(\pi)^2(\hbar c)^2}{8(939.6MeV)(0.5\,fm \times 10^{-6}nm/\,fm)^2} = \frac{(\pi)^2(197.3eV\bullet nm)^2}{8(939.6\times10^6 eV)(0.5\times10^{-6}nm)^2}$$

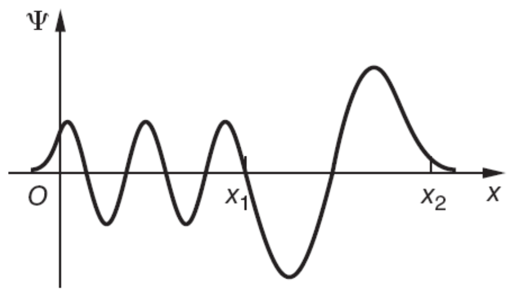
$$V_0 = 2.04\times10^8 eV = 204MeV$$

6-26. Because $E_1 = 0.5eV$ and for a finite well also $E_n \approx n^2 E_1$, then n = 4 is at about 8eV, i.e., near the top of the well. Referring to Figure 6-14, $ka \approx 2\pi$.

$$ka = \frac{\sqrt{2mE}}{\hbar} \times \frac{L}{2} = 7.24 \times 10^9 m^{-1} \times L = 2\pi$$

$$L = 2\pi / 7.24 \times 10^9 = 8.7 \times 10^{-10} m = 0.87 nm$$

6-27. For $V_2 > E > V_1$: x_1 is where $V = 0 \rightarrow V_1$ and x_2 and x_2 is where $V_1 \rightarrow V_2$

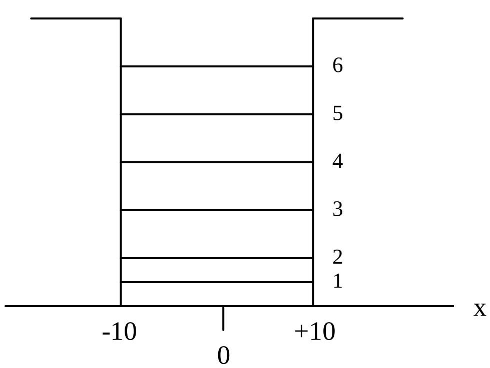


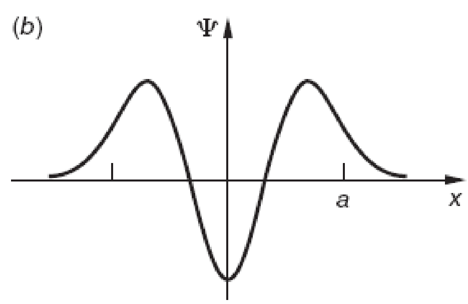
From $-\infty$ to 0 and x_2 to $+\infty$: ψ is exponential

0 to x_1: ψ is oscillatory; E_k is large so p is large and λ is small; amplitude is small because E_k is large, hence v is large.

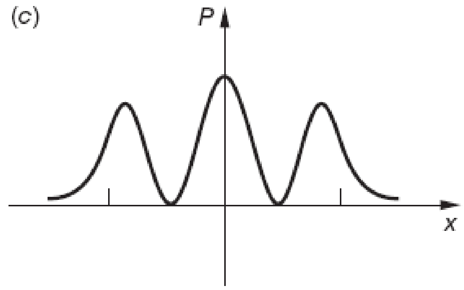
x_1 to x_2: ψ is oscillatory; E_k is small so p is small and λ is large; amplitude is large because E_k is small, hence v is small.

6-28. (a)





(Problem 6-28 continued)



6-29. $\langle p_x \rangle = \int_{-\infty}^{+\infty} \psi_3^* \;\; x \;\; \left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\psi_3 \;\; s \;\; dx$ (Equation 6-48)

$$= \int_0^L \sqrt{\frac{2}{L}}\sin\frac{3\pi x}{L}\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\sqrt{\frac{2}{L}}\sin\frac{3\pi x}{L}dx$$

$$= \frac{2}{L}\frac{\hbar}{i}\int_0^L\left(\sin\frac{3\pi x}{L}\right)\left(\cos\frac{3\pi x}{L}\right)\left(\frac{3\pi}{L}\right)dx$$

Let $\frac{3\pi x}{L} = y$ Then $x = 0 \rightarrow y = 0,\quad x = L \rightarrow y = 3\pi,$ and $\frac{3\pi}{L}dx = dy \rightarrow dx = \frac{L}{3\pi}dy$

Substituting above gives:

$$\langle p_x \rangle = \frac{2}{L}\frac{\hbar}{i}\frac{L}{3\pi}\int_0^{3\pi}\sin y\cos y dy \times \left(\frac{3\pi}{L}\right)$$

$$= \frac{2}{L}\frac{\hbar}{i}\int_0^{3\pi}\sin y\cos y dy$$

$$= \frac{2}{L}\frac{\hbar}{i}\left(\frac{\sin^2 y}{2}\right)\Bigg|_0^{3\pi} = \frac{2}{L}\frac{\hbar}{i}\;\; 0 - 0 \;\; = 0$$

Reconiliation: p_x is a vector pointing half the time in the $+x$ direction, half in the $-x$ direction. E_k is a scalar proportional to v^2, hence always positive.

6-30. For $n = 3$, $\psi_3 = \;\; 2/L \;\;^{1/2}\sin\;\; 3\pi x/L$

(a) $\langle x \rangle = \int_0^L x \;\; 2/L \;\; \sin^2 \;\; 3\pi x/L \;\; dx$

(Problem 6-30 continued)

Substituting $u = 3\pi x/L$, then $x = Lu/3\pi$ and $dx = (L/3\pi)\,du$. The limits become:

$x = 0 \to u = 0$ and $x = L \to u = 3\pi$

$$\langle x \rangle = (2/L)(L/3\pi)(1/3\pi)\int_0^{3\pi} u \sin^2 u\,du$$

$$= (2/L)(L/3\pi)^2\left[\frac{u^2}{4} - \frac{u\sin 2u}{4} - \frac{\cos 2u}{8}\right]_0^{3\pi}$$

$$= (2/L)(1/3\pi)^2(3\pi)^2/4 = L/2$$

(b) $\langle x^2 \rangle = \int_0^L x^2 (2/L)\sin^2(3\pi x/L)\,dx$

Changing the variable exactly as in (a) and noting that:

$$\int_0^{3\pi} u^2 \sin^2 u\,du = \left[\frac{u^3}{6} - \left(\frac{u^2}{4} - \frac{1}{8}\right)\sin 2u - \frac{u\cos 2u}{4}\right]_0^{3\pi}$$

We obtain $\langle x^2 \rangle = \left(\frac{1}{3} - \frac{1}{18\pi^2}\right)L^2 = 0.328L^2$

6-31. (a) Classically, the particle is equally likely to be found anywhere in the box, so

P(x) = constant. In addition, $\int_0^L P(x)\,dx = 1$ so $P(x) = 1/L$.

(b) $\langle x \rangle = \int_0^L (x/L)\,dx = L/2$ and $\langle x^2 \rangle = \int_0^L (x^2/L)\,dx = L^2/3$

6-32. $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\psi(x)$ (Equation 6-18)

$$\frac{1}{2m}\left(\frac{\hbar}{i}\frac{d}{dx}\right)\left(\frac{\hbar}{i}\frac{d}{dx}\right)\psi(x) = \left[E - V(x)\right]\psi(x)$$

$$\frac{1}{2m}p_{op}p_{op}\psi = \left[E - V(x)\right]\psi$$

Multiplying by ψ^* and integrating over the range of *x*,

(Problem 6-32 continued)

$$\int_{-\infty}^{+\infty} \psi^* \frac{p_{op}^2}{2m} \psi dx = \int_{-\infty}^{+\infty} \psi^* \left[E - V(x) \right] \psi dx$$

$$\left\langle \frac{p^2}{2m} \right\rangle = \left\langle \left[E - V(x) \right] \right\rangle \text{ or } \left\langle p^2 \right\rangle = \left\langle 2m\left[E - V(x) \right] \right\rangle$$

For the infinite square well *V(x)* = 0 wherever $\psi(x)$ does not vanish and vice versa.

Thus, $\left\langle V(x) \right\rangle = 0$ and $\left\langle p^2 \right\rangle = \left\langle 2mE \right\rangle = \left\langle 2m \dfrac{n^2\pi^2\hbar^2}{2mL^2} \right\rangle = \dfrac{\pi^2\hbar^2}{L^2}$ for $n = 1$

6-33. $\left\langle x^2 \right\rangle = \dfrac{L^2}{3} - \dfrac{L^2}{2\pi^2}$ (See Problem 6-30.) And $\left\langle x \right\rangle = \dfrac{L}{2}$

$$\sigma_x = \sqrt{\left\langle x^2 \right\rangle - \left\langle x \right\rangle^2} = \left[\frac{L^2}{3} - \frac{L^2}{2\pi^2} - \frac{L^2}{4} \right]^{1/2} = L\left[\frac{1}{12} - \frac{1}{2\pi^2} \right]^{1/2} = 0.181L$$

$\left\langle p_x^2 \right\rangle = \dfrac{\pi^2\hbar^2}{L^2}$ and $\left\langle p \right\rangle = 0$ (See Problem 6-32)

$\sigma_P = \sqrt{\left\langle p^2 \right\rangle - \left\langle p \right\rangle^2} = \left[\dfrac{\pi^2\hbar^2}{L^2} - 0 \right]^{1/2} = \dfrac{\pi\hbar}{L}$. And $\sigma_x\sigma_p = (0.181L)(\pi\hbar / L) = 0.568\hbar$

6-34. $\psi_0(x) = A_0 e^{-m\omega x^2/2\hbar}$ where $A_0 = (m\omega/\hbar\pi)^{1/4}$

$\left\langle x \right\rangle = \int_{-\infty}^{+\infty} A_0^2 x e^{-m\omega x^2/\hbar} dx$ Letting $u^2 = m\omega x^2/\hbar$ and $x = (\hbar/m\omega)^{1/2} u$

$2udu = (m\omega/\hbar)(2xdx)$. And thus, $(m\omega/\hbar)^{-1} udu = xdx$; limits are unchanged.

$\left\langle x \right\rangle = A_0^2 (\hbar/m\omega) \int_{-\infty}^{+\infty} ue^{-u^2} du = 0$ (Note that the symmetry of *V(x)* would also tell us that

$\left\langle x \right\rangle = 0$.)

$$\left\langle x^2 \right\rangle = \int_{-\infty}^{+\infty} A_0^2 x^2 e^{-m\omega x^2/\hbar} dx$$

$$= A_0^2 (\hbar/m\omega)^{3/2} \int_{-\infty}^{+\infty} u^2 e^{-u^2} du = 2A_0^2 (\hbar/m\omega)^{3/2} \int_{-\infty}^{+\infty} u^2 e^{-u^2} du$$

$$= 2A_0^2 (\hbar/m\omega)^{3/2} \sqrt{\pi}/4 = (m\omega/\hbar\pi)^{1/2} (\hbar/m\omega)^{3/2} \sqrt{\pi}/2 = \hbar/(2m\omega)$$

6-35. $\dfrac{p^2}{2m}+\dfrac{1}{2}m\omega^2x^2=(n+1/2)\hbar\omega$. For the ground state ($n$ = 0),

$$\langle x^2\rangle=\frac{2}{m\omega^2}\left(\hbar\omega/2-p^2/2m\right)\quad\text{and}\quad\langle x^2\rangle=\left\langle\frac{\hbar}{m\omega}-\frac{p^2}{m^2\omega^2}\right\rangle=\hbar/2m\omega\quad\text{(See Problem 6-34)}$$

$$\frac{\hbar}{m\omega}\left\langle 1-\frac{p^2}{m\hbar\omega}\right\rangle=\frac{\hbar}{2m\omega}\quad\text{or}\quad\left\langle 1-\frac{p^2}{m\hbar\omega}\right\rangle=\frac{1}{2}\rightarrow\langle p^2\rangle=\frac{1}{2}m\hbar\omega$$

6-36. (a) $\Psi_0(x,t)=(m\omega/\hbar\pi)^{1/4}e^{-m\omega x^2/2\hbar}e^{-i\omega t/2}$

(b) $p_{x\,op}=\dfrac{\hbar}{i}\dfrac{\partial}{\partial x}\qquad\langle p^2\rangle=\displaystyle\int_{-\infty}^{+\infty}\Psi_0^*(x,t)\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)^2\Psi_0(x,t)\,dx$

$$\frac{\partial\Psi_0}{\partial x}=A_0(m\omega x/\hbar\pi)^{1/4}e^{-m\omega x^2/2\hbar}e^{-i\omega t/2}$$

$$\frac{\partial^2\Psi_0}{\partial x^2}=A_0\left[(-m\omega x/\hbar)(-m\omega x/\hbar)-(m\omega/\hbar)\right]e^{-m\omega x^2/2\hbar}e^{-i\omega t/2}$$

$$\langle p^2\rangle=-\hbar^2A_0^2(m\omega/\hbar)\int_{-\infty}^{+\infty}(m\omega x^2/\hbar-1)e^{-m\omega x^2/\hbar}dx$$

$$=-\hbar^2A_0^2(m\omega/\hbar)\left[\int_{-\infty}^{+\infty}(m\omega x^2/\hbar)e^{-m\omega x^2/\hbar}dx-\int_{-\infty}^{+\infty}e^{-m\omega x^2/2\hbar}dx\right]$$

Letting $u=(m\omega x/\hbar)^{1/2}x$, then

$$\langle p^2\rangle=-\hbar^2A_0^2(m\omega/\hbar)(m\omega\hbar)^{-1/2}\left[\int_{-\infty}^{+\infty}u^2e^{-u^2}du-\int_{-\infty}^{+\infty}e^{-u^2}du\right]$$

$$=-\hbar^2A_0^2(m\omega/\hbar)^{1/2}2\left[\int_0^{\infty}u^2e^{-u^2}du-\int_0^{\infty}e^{-u^2}du\right]$$

$$=-\hbar^2(m\omega/\hbar\pi)^{1/2}(m\omega/\hbar)^{1/2}2\left(\frac{\sqrt{\pi}}{4}-\frac{\sqrt{\pi}}{2}\right)$$

$$=\hbar^2(m\omega/\hbar)(1/2)=m\hbar\omega/2$$

6-37. $\psi_0 \;\; x \;\; = C_0 e^{-m\omega x^2/2\hbar}$ (Equation 6-58)

(a) $\int_{-\infty}^{+\infty} \left|\psi_0 \;\; x \;\;\right|^2 dx = 1 = \int_{-\infty}^{+\infty} \left|C_0\right|^2 e^{-m\omega x^2/\hbar} dx$

$$= \left|C_0\right|^2 \times 2I_0 = \left|C_0\right|^2 \times 2 \times \frac{1}{2}\sqrt{\frac{\pi}{x}} \text{ with } \lambda = m\omega/\hbar$$

$$= \left|C_0\right|^2 \sqrt{\frac{\pi\hbar}{m\omega}}$$

$$C_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}$$

(b) $\left\langle x^2 \right\rangle = \int_{-\infty}^{+\infty} x^2 \left|\psi_0\right|^2 dx = \int_{-\infty}^{+\infty} x^2 \sqrt{\frac{m\omega}{\pi\hbar}} e^{-m\omega x^2/\hbar} dx$

$$= \sqrt{\frac{m\omega}{\pi\hbar}} \times 2I_2 = \sqrt{\frac{m\omega}{\pi\hbar}} \times 2 \times \frac{1}{4}\sqrt{\frac{\pi}{\lambda^3}} \text{ with } \lambda = m\omega/\hbar$$

$$= \frac{1}{2}\sqrt{\frac{m\omega}{\pi\hbar}}\sqrt{\frac{\pi\hbar^3}{m^3\omega^3}} = \frac{1}{2}\frac{\hbar}{\mathrm{m}\omega}$$

(c) $\left\langle V \;\; x \;\; \right\rangle = \left\langle \frac{1}{2} m\omega^2 x^2 \right\rangle = \frac{1}{2} m\omega^2 \left\langle x^2 \right\rangle = \frac{1}{2} m\omega \times \frac{1}{2}\frac{\hbar}{m\omega} = \frac{1}{4}\hbar\omega$

6-38. $\psi_1 \;\; x \;\; = C_1 x e^{-m\omega x^2/2\hbar}$ (Equation 6-58)

(a) $\int_{-\infty}^{+\infty} \left|\psi_1 \;\; x \;\;\right|^2 dx = 1 = \int_{-\infty}^{+\infty} \left|C_1\right|^2 x^2 e^{-m\omega x^2/\hbar} dx = \left|C_1\right|^2 \times 2I_2$

$$= \left|C_1\right|^2 \times 2 \times \frac{1}{4}\sqrt{\frac{\pi}{\lambda^3}} \text{ with } \lambda = m\omega/\hbar$$

$$= \left|C_1\right|^2 \times \frac{1}{2}\sqrt{\frac{\pi\hbar^3}{m^3\omega^3}}$$

$$C_1 = \left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/4}$$

(b) $\left\langle x \right\rangle = \int_{-\infty}^{+\infty} x \left|\psi_1\right|^2 dx = \int_{-\infty}^{+\infty} x^3 \left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/2} e^{-m\omega x^2/\hbar} dx = 0$

(Problem 6-38 continued)

(c) $$\left\langle x^2 \right\rangle = \int_{-\infty}^{+\infty} x^2 \left|\psi_1\right|^2 dx = \int_{-\infty}^{+\infty} x^2 \left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/2} e^{-m\omega x^2/\hbar} x^2 dx$$

$$= \left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/2} \times 2I_4 = \left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/2} \times 2 \times \frac{3}{8}\sqrt{\frac{\pi}{\lambda^5}} \quad \text{where } \lambda = m\omega/\hbar$$

$$= \frac{3}{2}\sqrt{\frac{m^3\omega^3}{\pi\hbar^3}}\sqrt{\frac{\pi\hbar^5}{m^5\omega^5}} = \frac{3}{2}\frac{\hbar}{m\omega}$$

(d) $$\left\langle V \;\; x \;\right\rangle = \left\langle \frac{1}{2}m\omega^2 x^2 \right\rangle = \frac{1}{2}m\omega^2 \left\langle x^2 \right\rangle = \frac{1}{2}m\omega^2 \times \frac{3}{2}\frac{\hbar}{m\omega} = \frac{3}{4}\hbar\omega$$

6-39. (a) $\Delta x \Delta p \approx \hbar \rightarrow \Delta p \approx \hbar / \Delta x \approx \hbar / 2A$

(b) $E_k = p^2/2m \approx \;\; h/2A \;\; ^2/2m \approx \hbar^2/8mA^2$

(1) $$E = \frac{1}{2}m\omega^2 A^2 \left(\frac{\hbar^2}{\hbar^2}\right)\left(\frac{2}{2}\right) = \frac{E_0^2\, 2mA^2}{\hbar^2}\left(\frac{4}{4}\right) = \frac{E_0^2}{4E_k}$$

Because $E_0 = \hbar\omega/2$ also $E_0 = 4E_k$

(2) $\partial^2\Psi_0/\partial x^2$ is computed in Problem 6-36(b). Using that quantity,

$$\left\langle E_k \right\rangle = -\frac{\hbar^2}{2m}\left(-\frac{m\omega}{2\hbar}\right) = \hbar\omega/4 = 2E_k$$

6-40. $E_n = \;\; n+1/2 \;\; \hbar\omega \qquad E_{n+1} = \;\; n+3/2 \;\; \hbar\omega$

$E_{n+1} - E_n = \Delta E_n \;\; n+3/2-n-1 \;\; \hbar\omega = \hbar\omega$

$\Delta E_n \big/ E_n = \hbar\omega \;\; n+1/2 \;\; \hbar\omega = 1\big/ \;\; n+1/2$

$$\lim_{n\to\infty}\frac{\Delta E_n}{E_n} = \lim_{n\to\infty}\left(\frac{1}{n+1/2}\right) = 0.$$ In agreement with the correspondence principle.

6-41. (a) Harmonic oscillator ground state is $n = 0$.

$$\psi_0(x) = A_0 e^{-m\omega x^2/2\hbar} \quad \text{(Equation 6-58)}$$

Therefore, for x: $\langle x \rangle = A_0^2 \int_{-\infty}^{+\infty} x e^{-m\omega x^2/\hbar} dx$

Let $m\omega x^2/\hbar = y^2 \rightarrow x = \sqrt{\hbar/m\omega}\, y$

$$dx = \sqrt{\hbar/m\omega}\, dy$$

Substituting above yields: $\langle x \rangle = A_0^2 (\hbar/m\omega) \int_{-\infty}^{+\infty} y e^{-y^2} dy = 0$

by inspection of Figure 6-18, integral tables, or integration by parts.

For x^2: $\langle x^2 \rangle = A_0^2 \int_{-\infty}^{+\infty} x^2 e^{-m\omega x^2/\hbar} dx$

Substituting as above yields: $\langle x^2 \rangle = A_0^2 (\hbar/m\omega)^{3/2} \int_{-\infty}^{+\infty} y^2 e^{-y^2} dy$

The value of the integral from tables is $\sqrt{\pi}/2$.

Therefore, $\langle x^2 \rangle = A_0^2 \left(\sqrt{\pi}/2\right)(\hbar/m\omega)^{3/2}$

(b) For the 1st excited state, $n = 1$,

$$\psi_1(x) = A_1 \sqrt{m\omega/\hbar}\, x e^{-m\omega x^2/2\hbar} \quad \text{(Equation 6-58)}$$

$$\langle x \rangle = A_1^2 \int_{-\infty}^{+\infty} x^3 (m\omega/\hbar) e^{-m\omega x^2/\hbar} dx$$

Changing the variable as in (a),

$$\langle x \rangle = A_1^2 (m\omega/\hbar) \int_{-\infty}^{+\infty} (\hbar/m\omega)^{3/2} y^3 e^{-y^2} (\hbar/m\omega)^{1/2} dy$$

$$= A_1^2 (\hbar/m\omega) \int_{-\infty}^{+\infty} y^3 e^{-y^2} dy = 0$$

by inspection of Figure 6-18, integral tables, or integration by parts.

$$\langle x^2 \rangle = A_1^2 \int_{-\infty}^{+\infty} x^4 (m\omega/\hbar) e^{-m\omega x^2/\hbar} dx$$

changing variables as above yields:

(Problem 6-41 continued)

$$\langle x^2 \rangle = A_1^2 \left(m\omega/\hbar\right) \int_{-\infty}^{+\infty} \left(\hbar/m\omega\right)^2 y^4 e^{-y^2} \left(\hbar/m\omega\right)^{1/2} dy$$

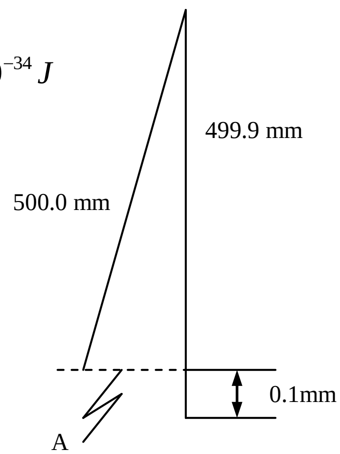
$$= A_1^2 \left(\hbar/m\omega\right)^{3/2} \int_{-\infty}^{+\infty} y^4 e^{-y^2} dy$$

The value of the integral from tables is $3\sqrt{\pi}/4$.

Therefore, $\langle x^2 \rangle = A_1^2 \left(3\sqrt{\pi}/4\right) \left(\hbar/m\omega\right)^{3/2}$

6-42. (a) $\omega = 2\pi f = 2\pi/T = 2\pi/1.42s = 4.42 rad/s$

$$E_0 = \frac{1}{2}\hbar\omega = \left(1.055\times10^{-34} J\bullet s\right)\left(4.42 rad/s\right)/2 = 2.33\times10^{-34} J$$



(b) $A = \sqrt{\left(500.0\right)^2 - \left(499.9\right)^2} = 10mm$

$$E = \left(n + 1/2\right)\hbar\omega = 1/2 m\omega^2 A^2$$

$$n + 1/2 = 1/2\left(0.010kg\right)\left(4.42 rad/s\right)\left(10^{-2} m\right)^2 / 1.055\times10^{-34} J\bullet s$$

$$= 2.1\times10^{28} \text{ or } n = 2.1\times10^{28}$$

(c) $f = \omega/2\pi = 0.70 Hz$

6-43. $\psi_0\left(x\right) = A_0 e^{-\omega x^2/2\hbar}$ $\quad \psi_1\left(x\right) = A_1\sqrt{\frac{m\omega}{\hbar}}\, x e^{-m\omega x^2/2\hbar}$

From Equation 6-58.

Note that ψ_0 is an even function of *x* and ψ_1 is an odd function of *x*.

It follows that $\int_{-\infty}^{+\infty} \psi_0 \psi_1 dx = 0$

6-44. (a) For $x > 0$, $\hbar^2 k_2^2 / 2m + V_0 = E = \hbar^2 k_1^2 / 2m = 2V_0$

So, $k_2 = (2mV_0)^{1/2}/\hbar$. Because $k_1 = (4mV_0)^{1/2}/\hbar$, then $k_2 = k_1/\sqrt{2}$

(b) $R = (k_1 - k_2)^2/(k_1 + k_2)^2$ (Equation 6-68)

$= (1 - 1/\sqrt{2})^2/(1 + 1/\sqrt{2})^2 = 0.0294$, or 2.94% of the incident particles are reflected.

(c) $T = 1 - R = 1 - 0.0294 = 0.971$

(d) 97.1% of the particles, or $0.971 \times 10^6 = 9.71 \times 10^5$, continue past the step in the +*x* direction. Classically, 100% would continue on.

6-45 (a) Equation 6-76: $T \approx 16\dfrac{E}{V_0}\left(1 - \dfrac{E}{V_0}\right)e^{-2\alpha a}$ where $\alpha = 2\sqrt{2m_p(V_0 - E)}/\hbar$

and $a =$ barrier width.

$$-2\alpha a = -2\left[\sqrt{2(938\,\text{MeV/c}^2)(50-44)\text{MeV}} / 6.58\times10^{-22}\ \text{MeV}\cdot\text{s}\right]\times10^{-15} = -1.075$$

$$T \approx 16\frac{44\,\text{MeV}}{50\,\text{MeV}}\left(1 - \frac{44\,\text{MeV}}{50\,\text{MeV}}\right)e^{-1.075}$$

$$T \approx 0.577$$

(b) decay rate $\approx N \times T$ where

$$N = \frac{v_{proton}}{2R} = \left[\frac{2\times 44\,\text{MeV}\times1.60\times10^{-13}\ \text{J/MeV}}{1.67\times10^{-27}\ \text{kg}}\right]^{1/2}\times\frac{1}{2\times10^{-15}\ \text{m}} = 4.59\times10^{22}\ \text{s}^{-1}$$

$$\text{decay rate} \approx 0.577\times4.59\times10^{22}\ \text{s}^{-1} = 2.65\times10^{22}\ \text{s}^{-1}$$

(c) In the expression for *T*, $e^{-1.075} \Rightarrow e^{-2.150}$, and so $T \approx 0.577 \Rightarrow T \approx 0.197$. The decay rate then becomes $9.05\times10^{21}\ \text{s}^{-1}$, a factor of 0.34× the original value.

6-46. (a) For $x > 0$, $\hbar^2 k_2^2 / 2m - V_0 = E = \hbar^2 k_1^2 / 2m = 2V_0$

So, $k_2 = (6mV_0)^{1/2}/\hbar$. Because $k_1 = (4mV_0)^{1/2}/\hbar$, then $k_2 = \sqrt{3/2}\,k_1$

(Problem 6-46 continued)

(b) $R = (k_1 - k_2)^2 / (k_1 + k_2)^2$

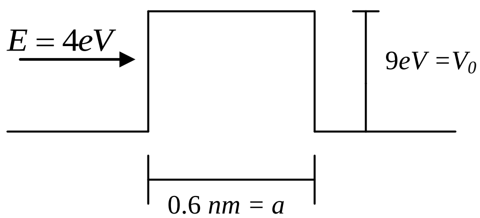
$$R = (k_1 - k_2)^2 / (k_1 + k_2)^2 = \left(1 - \sqrt{3/2}\right)^2 / \left(1 + \sqrt{3/2}\right)^2 = 0.0102$$

Or 1.02% are reflected at $x = 0$.

(c) $T = 1 - R = 1 - 0.0102 = 0.99$

(d) 99% of the particles, or $0.99 \times 10^6 = 9.9 \times 10^5$, continue in the $+x$ direction. Classically, 100% would continue on.

6-47. (a)



$$\alpha = \sqrt{2m(V_0 - E)}/\hbar$$

$$= \sqrt{2(0.511 \times 10^6 eV/c^2)(5eV)}/\hbar$$

$$= \sqrt{5.11 \times 10^6 eV}\left(\frac{eV}{c}\right)/\hbar$$

$$= \frac{2260eV}{197.3eV \bullet nm} = 11.46nm^{-1}$$

and $\alpha a = 0.6nm \times 11.46nm^{-1} = 6.87$

Since αa is not $\ll 1$, use Equation 6-75:

The transmitted fraction

$$T = \left[1 + \frac{\sinh^2 \alpha a}{4\left(E/V_0\right)\left(1 - E/V_0\right)}\right]^{-1} = \left[1 + \left(\frac{81}{80}\right)\sinh^2(6.87)\right]^{-1}$$

Recall that $\sinh x = \left(e^x - e^{-x}\right)/2$,

$$T = \left[1 + \frac{81}{80}\left(\frac{e^{6.87} - e^{-6.87}}{2}\right)^2\right]^{-1} = 4.3 \times 10^{-6}$$ is the transmitted fraction.

(b) Noting that the size of T is controlled by αa through the $\sinh^2 \alpha a$ and increasing T implies increasing E. Trying a few values, selecting $E = 4.5eV$ yields $T = 8.7 \times 10^{-6}$ or approximately twice the value in part (a).

6-48. $B = \dfrac{E^{1/2} - (E - V_0)^{1/2}}{E^{1/2} + (E - V_0)^{1/2}} A$ For $E < V_0$, $(E - V_0)^{1/2}$ is imaginary and the numerator and denominator are complex conjugates. Thus, $|B|^2 = |A|^2$ and therefore $R = |B|^2 / |A|^2 = 1$, hence $T = 1 - R = 0$.

6-49. $A + B = C$ and $k_1 A - k_1 B = k_2 C$ (Equations 6-65a & b)

Substituting for C, $k_1 A - k_1 B = k_2 (A + B) = k_2 A + k_2 B$ and solving for B,

$B = \dfrac{k_1 - k_2}{k_1 + k_2} A$, which is Equation 6-66. Substituting this value of B into Equation 6-65(a),

$A + \dfrac{k_1 - k_2}{k_1 + k_2} A = C = A\left[\dfrac{k_1 + k_2 + k_1 - k_2}{k_1 + k_2}\right]$ or $C = \dfrac{2k_1}{k_1 + k_2}$, which is Equation 6-67.

6-50. Using Equation 6-76,

$T \approx 16 \dfrac{E}{V_0}\left(1 - \dfrac{E}{V_0}\right) e^{-2\alpha a}$ where $E = 2.0eV$, $V_0 = 6.5eV$, and $a = 0.5nm$.

$T \approx 16\left(\dfrac{2.0}{6.5}\right)\left(1 - \dfrac{2.0}{6.5}\right) e^{-2(10.87)(0.5)} \approx 6.5 \times 10^{-5}$ (Equation 6-75 yields $T = 6.6 \times 10^{-5}$.)

6-51. $R = \dfrac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$ and $T = 1 - R$ (Equations 6-68 and 6-70)

(a) For protons:

$$k_1 = \sqrt{2mc^2 E} / \hbar c = \sqrt{2(938MeV)(40MeV)} / 197.3MeV \bullet fm = 1.388$$

$$k_2 = \sqrt{2mc^2 (E - V_0)} / \hbar c = \sqrt{2(938MeV)(10MeV)} / 197.3MeV \bullet fm = 0.694$$

$$R = \left(\frac{1.388 - 0.694}{1.388 + 0.694}\right)^2 = \left(\frac{0.694}{2.082}\right)^2 = 0.111 \quad \text{And } T = 1 - R = 0.889$$

(Problem 6-51 continued)

(b) For electrons:

$$k_1 = 1.388\left(\frac{0.511}{938}\right)^{1/2} = 0.0324 \qquad k_2 = 0.694\left(\frac{0.511}{938}\right)^{1/2} = 0.0162$$

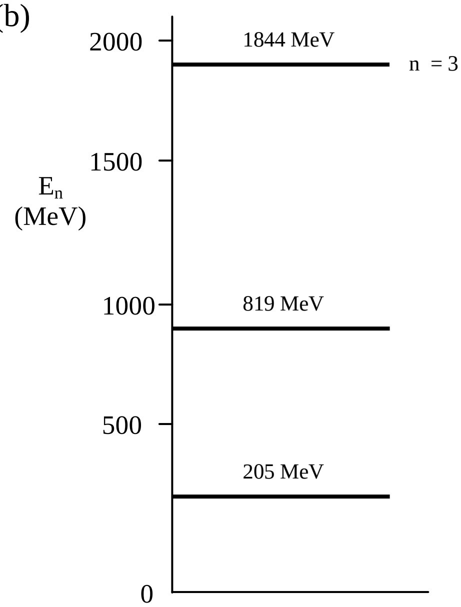
$$R = \left(\frac{0.0324 - 0.0162}{0.0324 + 0.0162}\right)^2 = 0.111 \quad \text{And } T = 1 - R = 0.889$$

No, the mass of the particle is not a factor. (We might have noticed that $\sqrt{m}$ could be canceled from each term.

6-52. (a) $E_n = \dfrac{n^2 h^2}{8mL^2}$ The ground state is $n = 1$, so

$$E_1 = \frac{(hc)^2}{8(mc^2)L^2} = \frac{(1240 MeV \bullet fm)^2}{8(938.3 MeV)(1 fm)^2} = 204.8 MeV$$

(b)



(c) $\Delta E_{21} = hc / \lambda_{21}$

$$\lambda_{21} = \frac{1240 MeV \bullet fm}{(819 - 205) MeV} = 2.02 fm$$

(d) $$\lambda_{32} = \frac{1240 MeV \bullet fm}{(1844 - 819) MeV} = 1.21 fm$$

(e) $$\lambda_{31} = \frac{1240 MeV \bullet fm}{(1844 - 205) MeV} = 0.73 fm$$

6-53. (a) The probability density for the ground state is $P(x) = \psi^2(x) = (2/L)\sin^2 \pi x/L$. The probability of finding the particle in the range $0 < x < L/2$ is:

$$P = \int_0^{L/2} P(x)\,dx = \frac{2}{L}\frac{L}{\pi}\int_0^{\pi/2} \sin^2 u\,du = \frac{2}{\pi}\left(\frac{\pi}{4} - 0\right) = \frac{1}{2} \quad \text{where } u = \pi x/L$$

(b) $$P = \int_0^{L/3} P(x)\,dx = \frac{2}{L}\frac{L}{\pi}\int_0^{\pi/3} \sin^2 u\,du = \frac{2}{\pi}\left(\frac{\pi}{6} - \frac{\sin 2\pi/3}{4}\right) = \frac{1}{3} - \frac{\sqrt{3}}{4\pi} = 0.195$$

(Note: 1/3 is the classical result.)

(c) $$P = \int_0^{3L/4} P(x)\,dx = \frac{2}{L}\frac{L}{\pi}\int_0^{3\pi/4} \sin^2 u\,du = \frac{2}{\pi}\left(\frac{3\pi}{8} - \frac{\sin 3\pi/2}{4}\right) = \frac{3}{4} + \frac{1}{2\pi} = 0.909$$

(Note: 3/4 is the classical result.)

6-54. (a) $E_n = \dfrac{n^2h^2}{8mL^2}$ and $E_{n+1} = \dfrac{(n+1)^2 h^2}{8mL^2}$

So, $$\frac{E_{n+1} - E_n}{E_n} = \frac{n^2 + 2n - 1 - n^2}{n^2} = \frac{2n-1}{n^2} = \frac{2 - 1/n}{n}$$ For large $n, 1/n \ll 2$ and

$$\frac{E_{n+1} - E_n}{E_n} \approx \frac{2}{n}$$

(b) For $n = 1000$ the fractional energy difference is $\dfrac{2}{1000} = 0.002 = 0.2\%$

(c) It means that the energy difference between adjacent levels per unit energy for large n is getting smaller, as the correspondence principle requires.

6-55. The $n = 2$ wave function is $\psi_2(x) = \sqrt{\dfrac{2}{L}}\sin\dfrac{2\pi x}{L}$ and the kinetic energy operator

$(E_k)_{op} = -\dfrac{\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2}$ Therefore,

$$\langle E_k \rangle = \int_{-\infty}^{+\infty} \psi_2^*(x)\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right)\psi_2(x)\,dx$$

$$= \int_{-\infty}^{+\infty} \sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right)\sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}\,dx$$

(Problem 6-55 continued)

$$= \frac{2}{L}\left(-\frac{\hbar^2}{2m}\right)\int_0^L \sin\frac{2\pi x}{L}\left(\frac{2\pi}{L}\right)^2\left(-\sin\frac{2\pi x}{L}\right)dx$$

$$= \frac{2}{L}\left(\frac{\hbar^2}{2m}\right)\left(\frac{2\pi}{L}\right)^2\int_0^L \sin^2\frac{2\pi x}{L}\,dx$$

Let $\frac{2\pi x}{L} = y$, then $x = 0 \rightarrow y = 0$ and $x = L \rightarrow y = 2\pi$ and $\frac{2\pi dx}{L} = dy \rightarrow dx = \frac{L}{2\pi}dy$

Substituting above gives: $\langle E_k \rangle = \frac{2}{L}\left(\frac{\hbar^2}{2m}\right)\left(\frac{2\pi}{L}\right)^2\left(\frac{L}{2\pi}\right)\int_0^{2\pi}\sin^2 y\,dy$

$$\int_0^{2\pi}\sin^2 y\,dy = \left(\frac{y}{2} - \frac{\sin 2y}{4}\right)\Bigg|_0^{2\pi} = \left[\left(\frac{2\pi}{2} - 0\right) - (0 - 0)\right] = \pi$$

Therefore, $\langle E_k \rangle = \frac{2}{L}\left(\frac{\hbar^2}{2m}\right)\left(\frac{2\pi}{L}\right)^2\left(\frac{L}{2\pi}\right)\pi = \frac{4\pi^2\hbar^2}{2mL^2} = \frac{h^2}{2mL^2}$

6-56. (a) The requirement is that $\psi^2(x) = \psi^2(-x) = \psi(-x)\psi(-x)$. This can only be true if:

$\psi(-x) = \psi(x)$ or $\psi(-x) = -\psi(x)$.

(b) Writing the Schrödinger equation in the form $\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi$, the general solutions of this 2nd order differential equation are: $\psi(x) = A\sin kx$ and $\psi(x) = A\cos kx$ where $k = \sqrt{2mE}/\hbar$. Because the boundaries of the box are at $x = \pm L/2$, both solutions are allowed (unlike the treatment in the text where one boundary was at $x = 0$). Still, the solutions are all zero at $x = \pm L/2$ provided that an integral number of half wavelengths fit between $x = -L/2$ and $x = +L/2$. This will occur for:

$\psi_n(x) = (2/L)^{1/2}\cos n\pi x/L$ when $n = 1,3,5,\cdots$. And for

$\psi_n(x) = (2/L)^{1/2}\sin n\pi x/L$ when $n = 2,4,6,\cdots$.

The solutions are alternately even and odd.

(c) The allowed energies are: $E = \hbar^2k^2/2m = \hbar^2(n\pi/L)^2/2m = n^2h^2/8mL^2$.

6-57. $\psi_0 = Ae^{-x^2/2L^2}$

(a) $\dfrac{d\psi_0}{dx} = (-x/L^2)\,Ae^{-x^2/2L^2}$ and $\psi_1 = L\dfrac{d\psi_0}{dx} = L(-x/L^2)\,Ae^{-x^2/2L^2} = (-x/L)\,\psi_0$

So, $\dfrac{d\psi_1}{dx} = -(1/L)\,\psi_0 - (x/L)\,d\psi_0/dx$

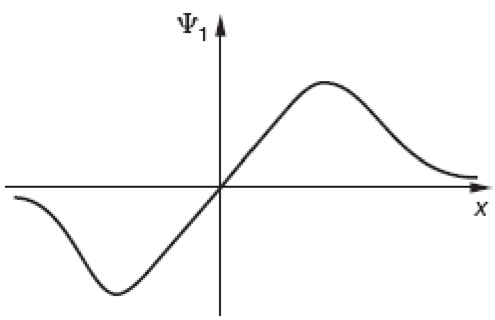
And $\dfrac{d^2\psi_1}{dx^2} = -(1/L)\,d\psi_0/dx - (1/L)\,d\psi_0/dx - (x/L)\,d^2\psi_0/dx^2$

$$= (2x/L^3)\,\psi_0 + (x/L^3)\,\psi_0 + (x^3/L^5)\,\psi_0$$

Recalling from Problem 6-3 that $V(x) = \hbar^2x^2/2mL^4$, the Schrödinger equation becomes $(-\hbar^2/2m)(3m/L^3 + x^3/L^5)\,\psi_0 + (\hbar^2x^3/2mL^5)\,\psi_0 = E(-x/L)\,\psi_0$ or, simplifying: $(-3\hbar^2x/2mL^3)\,\psi_0 = E(-x/L)\,\psi_0$. Thus, choosing E appropriately will make ψ_1 a solution.

(b) We see from (a) that $E = 3\hbar^2/2mL^2$, or three times the ground state energy.

(c) ψ_1 plotted looks as below. The single node indicates that ψ_1 is the first excited state. (The energy value in [b] would also tell us that.)



6-58. $\langle x^2 \rangle = \int_0^L \dfrac{2}{L} x^2 \sin^2 \dfrac{n\pi x}{L}\,dx$ Letting $u = n\pi x/L$, $du = (n\pi/L)\,dx$

$$\langle x^2 \rangle = \frac{2}{L}\left(\frac{L}{n\pi}\right)^2\left(\frac{L}{n\pi}\right)\int_0^{n\pi} u^2 \sin^2 u\,du$$

$$= \frac{2}{L}\left(\frac{L}{n\pi}\right)^3\left[\frac{u^3}{6} - \left(\frac{u^2}{4} - \frac{1}{8}\right)\sin 2u - \frac{u\cos 2u}{4}\right]_0^{n\pi}$$

(Problem 6-58 continued)

$$= \frac{2}{L}\left(\frac{L}{n\pi}\right)^3\left[\frac{n\pi^{\;3}}{6} - 0 - \frac{n\pi}{4} - 0\right] = \frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}$$

6-59. $T \approx 16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)e^{-2\alpha a}$ where $E = 10eV$, $V_0 = 25eV$, and $\alpha = 1nm$.

(a) $\alpha = \sqrt{2m\ V_0 - E}\ \big/\hbar = \sqrt{2\ m_0c^2\ \ V_0 - E}\ \big/\hbar c$

$$= \sqrt{2\ 0.511\times10^6 eV\ \ 15eV}\ \big/197.3eV\bullet nm = 19.84nm^{-1}$$

And $\alpha a =\ 19.84nm^{-1}\ \ 1nm\ = 19.84;\quad 2\alpha a = 29.68$

$$T \approx 16\left(\frac{10}{25}\right)\left(1-\frac{10}{25}\right)e^{-29.68} \approx 4.95\times10^{-13}$$

(b) For $a = 0.1nm$: $\alpha a =\ 19.84nm^{-1}\ \ 0.1nm\ = 1.984$

$$T \approx 16\left(\frac{10}{25}\right)\left(1-\frac{10}{25}\right)e^{-2.968} \approx 0.197$$

6-60. (a) For $\Psi\ x,t\ = A\sin\ kx - \omega t$

$\frac{d^2\Psi}{dx^2} = -k^2\Psi$ and $\frac{\partial\Psi}{\partial t} = -\omega A\cos\ kx - \omega t$ so the Schrödinger equation becomes:

$$-\frac{\hbar^2k^2}{2m}A\sin\ kx - \omega t\ + V\ x\ A\sin\ kx - \omega t\ = -i\hbar\omega\cos\ kx - \omega t$$

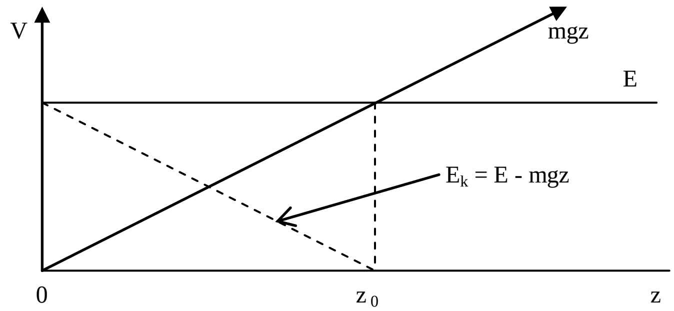
Because the *sin* and *cos* are not proportional, this Ψ cannot be a solution. Similarly, for $\Psi\ x,t\ = A\cos\ kx - \omega t$, there are no solutions.

(b) For $\Psi\ x,t\ = A\left[\cos\ kx - \omega t\ + i\sin\ kx - \omega t\ \right] = Ae^{i\ kx - \omega t}$, we have that

$\frac{d^2\Psi}{dx^2} = -k^2\Psi$ and $\frac{\partial\Psi}{\partial t} = -i\omega\Psi$. And the Schrödinger equation becomes:

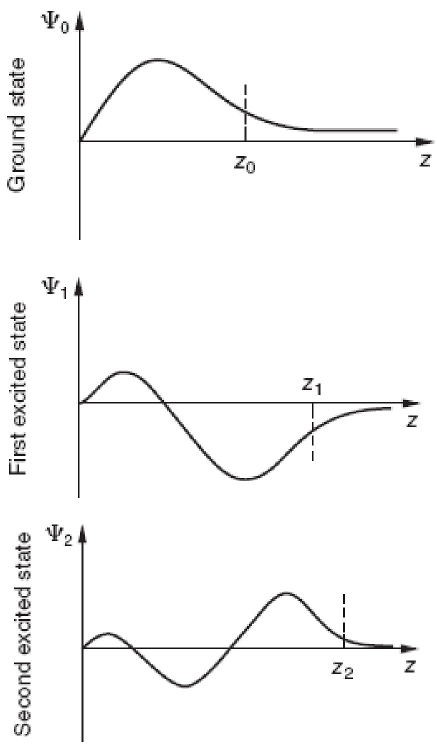
$$-\frac{\hbar^2k^2}{2m}\Psi + V\ x\ \Psi = -\hbar\omega\Psi \text{ for } \hbar\omega = \hbar^2k^2/2m + V.$$

6-61.



Classically allowed:
$0 < z < z_0$

The wave function will be concaved toward the *z* axis in the classically allowed region and away from the *z* axis elsewhere. Each wave function vanishes at $z = 0$ and as $z \rightarrow \infty$. The smaller amplitudes are in the regions where the kinetic energy is larger.



6-62. Writing the Schrödinger equation as: $E_k\psi(x) + V(x)\psi(x) = E\psi(x)$ from which we have: $E_k\psi(x) = \left[E - V(x)\right]\psi(x) = \left(-\hbar^2/2m\right)\left(d^2\psi/dx^2\right)$. The expectation value of

(Problem 6-62 continued)

E_k is $\langle E_k \rangle = \int_{-\infty}^{+\infty} E_k \psi(x)\psi(x)\,dx$. Substituting $E_k\psi(x)$ from above and reordering

multiplied quantities gives: $\langle E_k \rangle = \int_{-\infty}^{+\infty} \psi(x)\left(-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}\right)\psi(x)\,dx$.

6-63. (a) $\Delta p \Delta x \approx \hbar \rightarrow m\Delta v \Delta x \approx \hbar$

$\Delta v \approx \hbar / m\Delta a = (1.055\times10^{-34} J\bullet s) \big/ (9.11\times10^{-31})(10^{-12} m)$

$\Delta v \approx 1.6\times10^{8} m/s = 0.39c$

(b) The width of the well L is still an integer number of half wavelengths, $L = n(\lambda/2)$,

and deBroglie's relation still gives: $L = nh/2p$. However, p is *not* given by:

$p = \sqrt{2mE_k}$, but by the relativistic expression: $p = \left[E^2 - (mc^2)^2\right]^{1/2} \big/ c$.

Substituting this yields: $L = \frac{nhc}{2\left[E^2 - (mc^2)^2\right]^{1/2}} \rightarrow E^2 - (mc^2)^2 = (nhc/2L)^2$

$$E_n = \left[\left(\frac{nhc}{2L}\right)^2 + (mc^2)^2\right]^{1/2}$$

(c) $E_1 = \left[\left(\frac{hc}{4L^2}\right)^2 + (mc^2)^2\right]^{1/2} = \left[\frac{(1240eV\bullet nm)^2}{4(10^{-3} nm)^2} + (0.511\times10^{6} eV)^2\right]^{1/2} = 8.03\times10^{5} eV$

(d) Nonrelativistic:

$$E_1 = \frac{h^2}{8mL^2} = \frac{(hc)^2}{8(mc^2)L^2} = \frac{(1240eV\bullet nm)^2}{8(0.511\times10^{6} eV)(10^{-3} nm)^2} = 3.76\times10^{5} eV$$

E_1 computed in (c) is 2.14 times the nonrelativistic value.

6-64. (a) Applying the boundary conditions of continuity to ψ and $d\psi/dx$ at $x = 0$ and $x = a$, where the various wave functions are given by Equation 6-74, results in the two pairs of equation below:

(Problem 6-64 continued)

At $x = 0$: $A + B = C + D$ and $ikA - ikB = -\alpha C + \alpha D$

At $x = a$: $Fe^{ika} = Ce^{-\alpha a} + De^{\alpha a}$ and $ikFe^{ika} = -\alpha Ce^{-\alpha a} + \alpha De^{\alpha a}$

Eliminating the coefficients C and D from these four equations, a straightforward but lengthy task, yields: $4ik\alpha A = \left[(\alpha + ik)^2 e^{-\alpha a} - (\alpha - ik)^2 e^{\alpha a}\right] Fe^{ika}$

The transmission coefficient T is then:

$$T = \frac{|F|^2}{|A|^2} = \left\{ \frac{4ik\alpha}{e^{ika}\left[(\alpha + ik)^2 e^{-\alpha a} - (\alpha - ik)^2 e^{\alpha a}\right]} \right\}^2$$

Recalling that $\sinh\theta = \frac{1}{2}(e^{\theta} - e^{-\theta})$ and noting that $(\alpha + ik)$ and $(\alpha - ik)$ are complex conjugates, substituting $k = \sqrt{2mE}/\hbar$ and $\alpha = \sqrt{2m(V_0 - E)}/\hbar$, T then can be written as

$$T = \left[1 + \frac{\sinh^2 \alpha a}{4\frac{E}{V_0}\left(1 - \frac{E}{V_0}\right)}\right]^{-1}$$

(b) If $\alpha a \gg 1$, then the first term in the bracket on the right side of the * equation in part (a) is much smaller than the second and we can write:

$$\frac{F}{A} \approx \frac{4ik\alpha e^{-(\alpha + ik)a}}{(\alpha - ik)^2} \quad \text{and} \quad T = \left|\frac{F}{A}\right|^2 \approx \frac{6\alpha^2 k^2 e^{-2\alpha a}}{(\alpha^2 + k^2)^2}$$

Or $T \approx 16\left(\frac{E}{V_0}\right)\left(1 - \frac{E}{V_0}\right)e^{-2\alpha a}$

6-65. $|\psi_{II}|^2 = |C|^2 e^{-2\alpha a}$ (Equation 6-72)

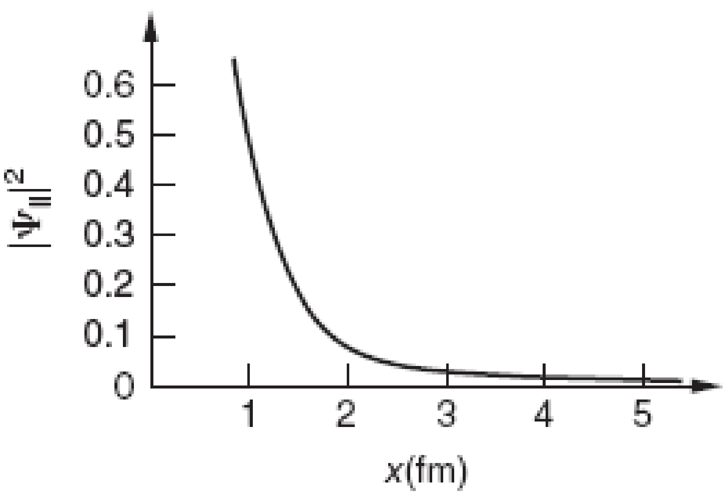
Where $|C|^2 = \left|\frac{2E^{1/2}}{E^{1/2} + (E - V_0)^{1/2}}\right|^2 |A|^2 = \left|\frac{2(0.5V_0)^{1/2}}{(0.5V_0)^{1/2} + (-0.5V_0)^{1/2}}\right|^2 = 2.000$

$\alpha = \sqrt{2m(V_0 - E)}/\hbar = \sqrt{2(m_p c^2)(20MeV)}/\hbar$

(Problem 6-65 continued)

$$= \sqrt{2\ \ 938.3MeV \quad 20MeV}\Big/197.3MeV \bullet fm = 0.982\, fm^{-1}$$

$x\ (fm)$	$e^{-2\alpha x}$	$\lvert\psi_{II}\rvert^2 = \lvert C\rvert^2 e^{-2\alpha x}$
1	0.1403	0.5612
2	0.0197	0.0788
3	2.76×10^{-3}	1.10×10^{-2}
4	3.87×10^{-4}	1.55×10^{-3}
5	5.4×10^{-5}	2.2×10^{-4}



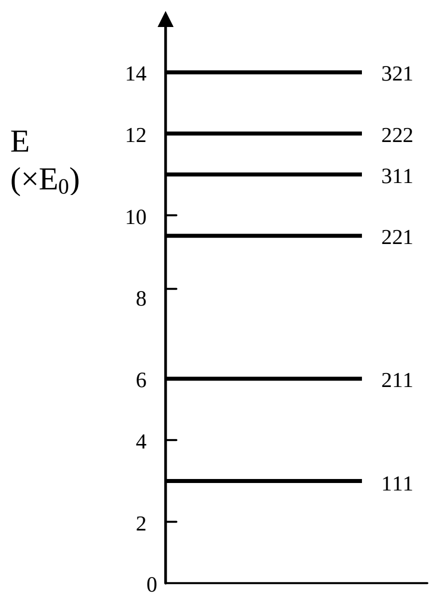
Chapter 7 – Atomic Physics

7-1. $E_{n_1 n_2 n_3} = \dfrac{\hbar^2 \pi^2}{2mL^2}\left(n_1^2 + n_2^2 + n_3^2\right)$ (Equation 7-4)

$E_{311} = \dfrac{\hbar^2 \pi^2}{2mL^2}\left(3^2 + 1^2 + 1^2\right) = 11E_0$ where $E_0 = \dfrac{\hbar^2 \pi^2}{2mL^2}$

$E_{222} = E_0\left(2^2 + 2^2 + 2^2\right) = 12E_0$ and $E_{321} = E_0\left(3^2 + 2^2 + 1^2\right) = 14E_0$

The 1st, 2nd, 3rd, and 5th excited states are degenerate.



7-2. $E_{n_1 n_2 n_3} = \dfrac{\hbar^2 \pi^2}{2m}\left(\dfrac{n_1^2}{L_1^2} + \dfrac{n_2^2}{L_2^2} + \dfrac{n_3^2}{L_3^2}\right) = \dfrac{\hbar^2 \pi^2}{2mL_1^2}\left(n_1^2 + \dfrac{n_2^2}{4} + \dfrac{n_3^2}{9}\right)$ (Equation 7-5)

$n_1 = n_2 = n_3 = 1$ is the lowest energy level.

$E_{111} = E_0\left(1 + 1/4 + 1/9\right) = 1.361E_0$ where $E_0 = \dfrac{\hbar^2 \pi^2}{2mL_1^2}$

The next nine levels are, increasing order,

(Problem 7-2 continued)

n_1	n_2	n_3	$E\left(\times E_0\right)$
1	1	2	1.694
1	2	1	2.111
1	1	3	2.250
1	2	2	2.444
1	2	3	3.000
1	1	4	3.028
1	3	1	3.360
1	3	2	3.472
1	2	4	3.778

7-3. (a) $\psi_{n_1 n_2 n_3}\left(x,y,z\right) = A\cos\frac{n_1\pi x}{L}\sin\frac{n_2\pi y}{L}\sin\frac{n_3\pi z}{L}$

(b) They are identical. The location of the coordinate origin does not affect the energy level structure.

7-4.

$$\psi_{111}\left(x,y,z\right) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{\pi z}{3L_1} \qquad \psi_{112}\left(x,y,z\right) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{2\pi z}{3L_1}$$

$$\psi_{121}\left(x,y,z\right) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{L_1}\sin\frac{\pi z}{3L_1} \qquad \psi_{122}\left(x,y,z\right) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{L_1}\sin\frac{2\pi z}{3L_1}$$

$$\psi_{113}\left(x,y,z\right) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{\pi z}{L_1}$$

7-5.

$$E_{n_1 n_2 n_3} = \frac{\hbar^2\pi^2}{2m}\left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{\left(2L_1\right)^2} + \frac{n_3^2}{\left(4L_1\right)^2}\right) = \frac{\hbar^2\pi^2}{2mL_1^2}\left(n_1^2 + \frac{n_2^2}{4} + \frac{n_3^2}{16}\right) \quad \text{(from Equation 7-5)}$$

$$E_{n_1 n_2 n_3} = \left(n_1^2 + \frac{n_2^2}{4} + \frac{n_3^2}{16}\right) \quad \text{where } E_0 = \frac{\hbar^2\pi^2}{2mL_1^2}$$

(Problem 7-5 continued)

(a)

n_1	n_2	n_3	$E\left(\times E_0\right)$
1	1	1	1.313
1	1	2	1.500
1	1	3	1.813
1	2	1	2.063
1	1	4	2.250
1	2	2	2.250
1	2	3	2.563
1	1	5	2.813
1	2	4	3.000
1	3	1	3.313

(b) 1,1,4 and 1,2,2

7-6. $\psi_{111}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{\pi z}{4L_1}$ $\qquad \psi_{112}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{\pi z}{2L_1}$

$\psi_{113}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{3\pi z}{4L_1}$ $\qquad \psi_{121}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{L_1}\sin\frac{\pi z}{4L_1}$

$\psi_{114}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{\pi z}{L_1}$ $\qquad \psi_{122}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{L_1}\sin\frac{\pi z}{2L_1}$

$\psi_{123}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{L_1}\sin\frac{3\pi z}{4L_1}$ $\qquad \psi_{115}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{5\pi z}{4L_1}$

$\psi_{124}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{L_1}\sin\frac{\pi z}{L_1}$ $\qquad \psi_{116}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{3\pi z}{2L_1}$

7-7. $$E_0 = \frac{\hbar^2\pi^2}{2mL^2} = \frac{\left(1.055\times10^{-34}J\bullet s\right)^2\pi^2}{2\left(9.11\times10^{-31}kg\right)\left(0.10\times10^{-9}m\right)^2\left(1.609\times10^{-19}J/eV\right)} = 37.68eV$$

$$E_{311} - E_{111} = \Delta E = 11E_0 - 3E_0 = 8E_0 = 301eV$$

(Problem 7-7 continued)

$$E_{222} - E_{111} = \Delta E = 12E_0 - 3E_0 = 9E_0 = 339eV$$

$$E_{321} - E_{111} = \Delta E = 14E_0 - 3E_0 = 11E_0 = 415eV$$

7-8. (a) Adapting Equation 7-3 to two dimensions (i.e., setting $k_3 = 0$), we have

$$\psi_{n_1 n_2} = A\sin\frac{n_1\pi x}{L}\sin\frac{n_2\pi y}{L}$$

(b) From Equation 7-5, $E_{n_1 n_2} = \frac{\hbar^2\pi^2}{2mL^2}\left(n_1^2 + n_2^2\right)$

(c) The lowest energy degenerate states have quantum numbers $n_1 = 1$, $n_2 = 2$, and $n_1 = 2$, $n_2 = 1$.

7-9. (a) For $n = 3$, $\ell = 0, 1, 2$

(b) For $\ell = 0, m = 0$. For $\ell = 1, m = -1, 0, +1$. For $\ell = 2, m = -2, -1, 0, +1, +2$.

(c) There are nine different m-states, each with two spin states, for a total of 18 states for $n = 3$.

7-10. (a) For $\ell = 4$

$$L = \sqrt{\ell(\ell+1)}\,\hbar = \sqrt{4(5)}\,\hbar = \sqrt{20}\,\hbar$$

$$m_\ell = 4\hbar$$

$$\theta_{\min} = \cos^{-1}\frac{4}{\sqrt{20}} \quad\rightarrow\quad \theta_{\min} = 26.6°$$

(b) For $\ell = 2$

$$L = \sqrt{6}\,\hbar \qquad m_\ell = 2\hbar$$

$$\theta_{\min} = \cos^{-1}\frac{2}{\sqrt{6}} \quad\rightarrow\quad \theta_{\min} = 35.3°$$

7-11. (a) $L = I\omega = \left(10^{-5}kg\bullet m^2\right)\left(2\pi\right)\left(735min^{-1}\right)\left(1min/60s\right) = 7.7\times10^{-4}kg\bullet m^2/s$

(b) $L = \sqrt{\ell(\ell+1)}\,\hbar = 7.7\times10^{-4}kg\bullet m^2/s$

(Problem 7-11 continued)

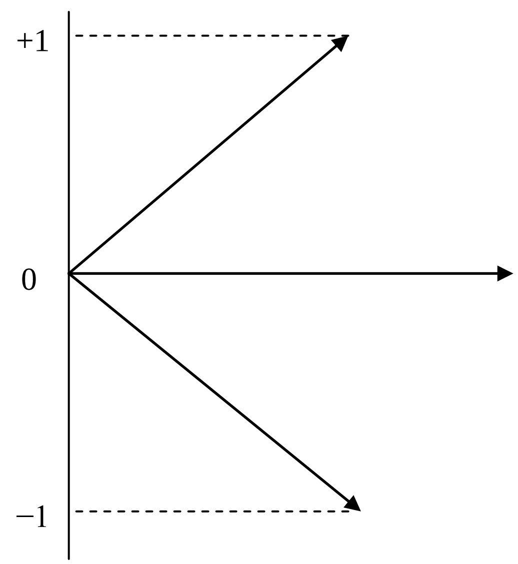
$$\ell(\ell+1) = \frac{\left(7.7\times10^{-4}\,kg\bullet m^2/s\right)^2}{\left(1.055\times10^{-34}\,J\bullet s\right)^2}$$

$$\ell \approx \frac{7.7\times10^{-4}\,kg\bullet m^2/s}{1.055\times10^{-34}\,J\bullet s} \approx 7.3\times10^{30}$$

7-12. (a)

$\ell = 1$

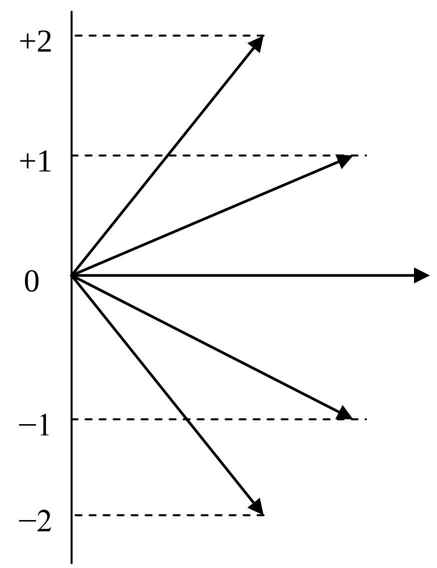
$|\mathbf{L}| = \sqrt{2}\hbar$



(b)

$\ell = 2$

$|\mathbf{L}| = \sqrt{6}\hbar$

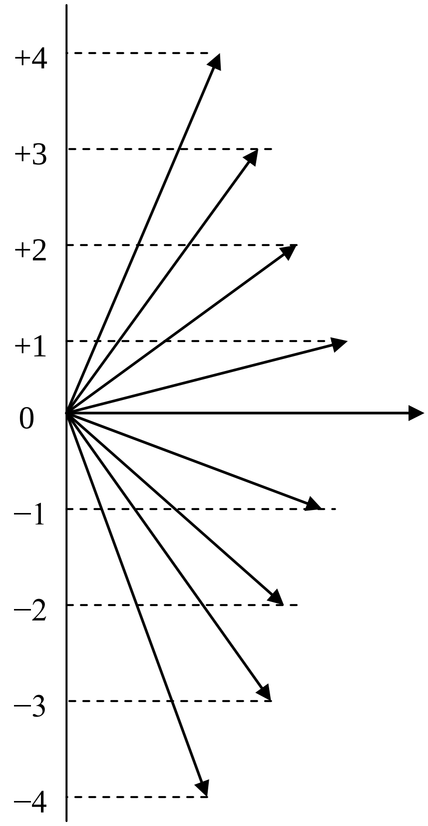


(Problem 7-12 continued)

(c)

$\ell = 4$

$|\mathbf{L}| = \sqrt{20}\hbar$



(d) $|\boldsymbol{L}| = \sqrt{\ell(\ell+1)}\,\hbar$ (See diagrams above.)

7-13. $L^2 = L_x^2 + L_y^2 + L_z^2 \rightarrow L_x^2 + L_y^2 = L^2 - L_z^2 = \ell(\ell+1)\hbar^2 - (m\hbar)^2 = (6 - m^2)\hbar^2$

(a) $(L_x^2 + L_y^2)_{\text{min}} = (6 - 2^2)\hbar^2 = 2\hbar^2$

(b) $(L_x^2 + L_y^2)_{\text{max}} = (6 - 0^2)\hbar^2 = 6\hbar^2$

(c) $L_x^2 + L_y^2 = (6-1)\hbar^2 = 5\hbar^2$ L_x and L_y cannot be determined separately.

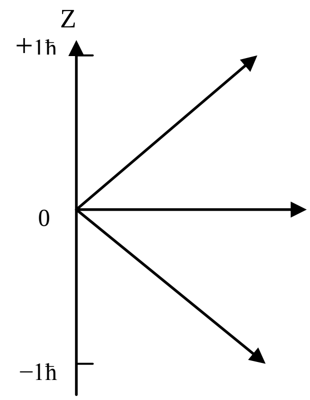
(d) $n = 3$

7-14 (a) For $\ell = 1$, $L = \sqrt{\ell(\ell+1)}\,\hbar = \sqrt{2}\,\hbar = 1.49 \times 10^{-34}\,J \bullet s$

(b) For $\ell = 1$, $m = -1, 0, +1$

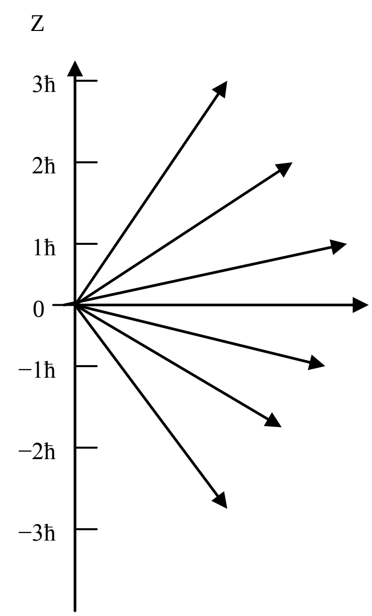
(Problem 7-14 continued)

(c)



$|\mathbf{L}| = \sqrt{2}\hbar$

(d) For $\ell = 3$, $L = \sqrt{\ell(\ell+1)}\,\hbar = \sqrt{12}\,\hbar = 3.65\times10^{-34}\,J\bullet s$ and $m = -3, -2, -1, 0, 1, 2, 3$



7-15. $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$ $\qquad \dfrac{d\boldsymbol{L}}{dt} = \dfrac{d\boldsymbol{r}}{dt} \times \boldsymbol{p} + \boldsymbol{r} \times \dfrac{d\boldsymbol{p}}{dt}$

$\dfrac{d\boldsymbol{r}}{dt} \times \boldsymbol{p} = \boldsymbol{v} \times m\boldsymbol{v} = m\boldsymbol{v} \times \boldsymbol{v} = 0$ and $\boldsymbol{r} \times \dfrac{d\boldsymbol{p}}{dt} = \boldsymbol{r} \times \boldsymbol{F}$. Since for $V = V(r)$, i.e., central forces,

$\boldsymbol{F}$ is parallel to $\boldsymbol{r}$, then $\boldsymbol{r} \times \boldsymbol{F} = 0$ and $\dfrac{d\boldsymbol{L}}{dt} = 0$

7-16. (a) For $\ell = 3$, $n = 4, 5, 6, \ldots$ and $m = -3, -2, -1, 0, 1, 2, 3$

(b) For $\ell = 4$, $n = 5, 6, 7, \ldots$ and $m = -4, -3, -2, -1, 0, 1, 2, ,3 ,4$

(Problem 7-16 continued)

(c) For $\ell = 0$, $n = 1$ and $m = 0$

(d) The energy depends only on n. The minimum in each case is:

$$E_4 = -13.6eV/n^2 = -13.6eV/4^2 = -0.85eV$$

$$E_5 = -13.6eV/5^2 = -0.54eV$$

$$E_1 = -13.6eV$$

7-17. (a) $6f$ state: $n = 6, \ell = 3$

(b) $E_6 = -13.6eV/n^2 = -13.6eV/6^2 = -0.38eV$

(c) $L = \sqrt{\ell(\ell+1)}\,\hbar = \sqrt{3(3+1)}\,\hbar = \sqrt{12}\,\hbar = 3.65\times10^{-34}\,J\bullet s$

(d) $L_z = m\hbar \qquad L_z = -3\hbar, -2\hbar, -1\hbar, 0, 1\hbar, 2\hbar, 3\hbar$

7-18. Referring to Table 7-2, $R_{30} = 0$ when

$$\left(1 - \frac{2r}{3a_0} + \frac{2r^2}{27a_0^2}\right) = 0$$

Letting $r/a_0 = x$, this condition becomes $x^2 - 9x + 13.5 = 0$

Solving for x (quadratic formula or completing the square), x = 1.90, 7.10.

Therefore, $r/a_0 = 1.90, 7.10$. Compare with Figure 7-10(a).

7-19. Equation 7-25: $E_n = -\left(\frac{kZe^2}{\hbar}\right)^2 \frac{\mu}{2n^2}$

Using SI units and noting that both Z and n are unitless, we have:

$$\mathrm{J} = \left(\frac{(\mathrm{N\cdot m^2/C^2})\times \mathrm{C^2}}{\mathrm{J\cdot s}}\right)^2 \times \mathrm{kg}$$

Cancelling the C^2 and substituting $\mathrm{J = N\cdot m}$ on the right yields $\mathrm{J} = \left(\frac{\mathrm{N\cdot m^2}}{\mathrm{N\cdot m\cdot s}}\right)^2 \times \mathrm{kg}$.

(Problem 7-19 continued)

Cancelling the N and m gives $\mathrm{J} = \left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 \times \mathrm{kg} = \mathrm{J}$, since $\mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^2$ are the units of kinetic energy.

7-20. (a) For the ground state $n = 1$, $\ell = 0$, and $m = 0$.

$$\psi_{100} = R_{10}Y_{00} = \frac{2}{\sqrt{a_0^3}} e^{-r/a_0} \frac{1}{\sqrt{4\pi}} = \frac{2}{\sqrt{4\pi a_0^2}} e^{-r/a_0} = \frac{2e^{-1}}{\sqrt{4\pi a_0^3}} \quad \text{at } r = a_0$$

(b) $\psi^2 = \dfrac{1}{\pi a_0^3} e^{-2r/a_0} = \dfrac{1}{\pi a_0^3} e^{-2} \quad \text{at } r = a_0$

(c) $P(r) = \psi^2 \times 4\pi r^2 = \dfrac{4}{a_0} e^{-2} \quad \text{at } r = a_0$

7-21. (a) For the ground state, $P(r)\Delta r = \psi^2 \left(4\pi r^2\right)\Delta r = \dfrac{4r^2}{a_0^3} e^{-2r/a_0} \Delta r$

For $\Delta r = 0.03a_0$, at $r = a_0$ we have $P(r)\Delta r = \dfrac{4a_0^2}{a_0^3} e^{-2}\left(0.03a_0\right) = 0.0162$

(b) For $\Delta r = 0.03a_0$, at $r = 2a_0$ we have $P(r)\Delta r = \dfrac{4\left(2a_0\right)^2}{a_0^3} e^{-4}\left(0.03a_0\right) = 0.0088$

7-22. $P(r) = Cr^2 e^{-2Zr/a_0}$ For $P(r)$ to be a maximum,

$$\frac{dP}{dt} = C\left[r^2\left(-\frac{2Z}{a_0}\right)e^{-2Zr/a_0} + 2re^{-2Zr/a_0}\right] = 0 \;\rightarrow\; C \times \frac{2Zr}{a_0}\left(\frac{a_0}{Z} - r\right)e^{-2Zr/a_0} = 0$$

This condition is satisfied with $r = 0$ or $r = a_0/Z$. For $r = 0$, $P(r) = 0$ so the maximum $P(r)$ occurs for $r = a_0/Z$.

7-23. $\int \psi^2 d\tau = \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi^2 r^2 \sin\theta dr\, d\theta\, d\phi = 1$

$$= 4\pi \int_0^\infty \psi^2 r^2 dr = 4\pi C_{210}^2 \int_0^\infty \left(\frac{Zr}{a_0}\right)^2 r^2 e^{-Zr/a_0} dr = 1$$

$$= 4\pi C_{210}^2 \int_0^\infty \left(\frac{Z^2 r^4}{a_0^2}\right) e^{-Zr/a_0} dr = 1$$

Letting $x = Zr/a_0$, we have that $r = a_0 x/Z$ and $dr = a_0 dx/Z$ and substituting these above,

$$\int \psi^2 d\tau = \frac{4\pi a_0^3 C_{210}^2}{Z^3} \int_0^\infty x^4 e^{-x} dx$$

Integrating on the right side

$$\int_0^\infty x^4 e^{-x} dx = 6$$

Solving for C_{210}^2 yields: $C_{210}^2 = \dfrac{Z^3}{24\pi a_0^3} \rightarrow C_{210} = \left(\dfrac{Z^3}{24\pi a_0^3}\right)^{1/2}$

7-24. $\psi_{200} = \dfrac{1}{\sqrt{32\pi}} \left(\dfrac{Z}{a_0}\right)^{3/2} \left(1 - \dfrac{r}{a_0}\right) e^{-r/2a_0}$ (Z = 1 for hyrdogen)

$$P(r)\Delta r = |\psi_{200}|^2 \left(4\pi r^2\right)\Delta r = \frac{1}{32\pi}\frac{1}{a_0^3}\left(1 - \frac{r}{a_0}\right)^2 e^{-r/a_0}\left(4\pi r^2\right)\Delta r$$

(a) For $\Delta r = 0.02 a_0$, at $r = a_0$ we have

$$P(r)\Delta r = \frac{4\pi}{32\pi}\frac{1}{a_0^3}(1-1)^2 e^{-1} a_0^2 \left(0.02 a_0\right) = \frac{1}{8}(0) e^{-1}(0.02) = 0$$

(b) For $\Delta r = 0.02 a_0$, at $r = 2a_0$ we have

$$P(r)\Delta r = \frac{4\pi}{32\pi}\frac{1}{a_0^3}(-1)^2 e^{-2} a_0^2 \left(0.02 a_0\right) = \frac{1}{8}(1) e^{-2}(0.02) = 3.4\times 10^{-4}$$

7-25. $\psi_{210} = C_{210}\dfrac{Zr}{a_0}e^{-Zr/2a_0}\cos\theta$ (Equation 7-34)

$$P(r) = |\psi_{210}|^2(4\pi r^2) = 4\pi r^2|C_{210}|^2\frac{Z^2r^2}{a_0^2}e^{-r/a_0}\cos^2\theta$$

$$= 4\pi|C_{210}|^2(Z^2/a_0^2)r^4e^{-r/a_0}\cos^2\theta$$

$$= Ar^4e^{-r/a_0}\cos^2\theta$$

where $A = 4\pi|C_{210}|^2(Z^2/a_0^2)$, a constant.

7-26. $\psi_{200} = \dfrac{1}{\sqrt{32\pi}}\left(\dfrac{1}{a_0}\right)^{3/2}\left(2-\dfrac{r}{2a_0}\right)e^{-r/2a_0}$ (Z = 1 for hyrdogen)

(a) At $r = a_0$, $\psi_{200} = \dfrac{1}{\sqrt{32\pi}}\left(\dfrac{1}{a_0^3}\right)(2-1)e^{-1/2} = \dfrac{0.606}{\sqrt{32\pi}}\left(\dfrac{1}{a_0}\right)^{3/2}$

(b) At $r = a_0$, $|\psi_{200}|^2 = \dfrac{1}{\sqrt{32\pi}}\left(\dfrac{1}{a_0^3}\right)e^{-1} = \dfrac{0.368}{32\pi}\dfrac{1}{a_0^3}$

(c) At $r = a_0$, $P(r) = |\psi_{200}|^2(4\pi r^2) = \dfrac{4}{32\pi}\dfrac{0.368a_0^2}{a_0^3} = \dfrac{0.368}{8a_0}$

7-27. For the most likely value of r, $P(r)$ is a maximum, which requires that (see Problem 7-25)

$$\frac{dP}{dr} = A\cos^2\theta\left[r^4\left(-\frac{Z}{a_0}\right)e^{-Zr/a_0} + 4r^3e^{-Zr/a_0}\right] = 0$$

For hydrogen $Z = 1$ and $A\cos^2\theta(r^3/a_0)(4a_0 - r)e^{-r/a_0} = 0$. This is satisfied for $r = 0$ and $r = 4a_0$. For $r = 0$, $P(r) = 0$ so the maximum $P(r)$ occurs for $r = 4a_0$.

7-28. From Table 7-1, $Y_{2-1}(\theta,\varphi) = \sqrt{\dfrac{15}{8\pi}}\sin\theta\cos\theta e^{-i\varphi}$

From Table 7-2, $R_{32}(r) = \dfrac{4}{81\sqrt{30a_0^2}}\dfrac{r^2}{a_0^2}e^{-r/3a_0}$

(Problem 7-28 continued)

$$\psi_{32-1}(r,\theta,\varphi)=\frac{4}{81\sqrt{30a_0^3}}\frac{1}{a_0^2}\sqrt{\frac{15}{8\pi}}\,r^2e^{-r/3a_0}\sin\theta\cos\theta e^{-i\varphi}$$

To be sure that ψ is normalized, we do the usual normalization as follows, where C is the normalization constant to be compared with the coefficient above.

$$C_{32-1}^2=\int_0^\infty\int_0^\pi\int_0^{2\pi}\psi^*\psi d\vec{r}=C_{32-1}^2\int_0^\infty\int_0^\pi\int_0^{2\pi}r^4e^{-2r/3a_0}\sin^2\theta\cos^2\theta r^2\sin\theta dr\,d\theta\,d\varphi=1$$

Noting that $\int_0^{2\pi}d\varphi=2\pi$, we have

$$2\pi C_{32-1}^2\int_0^\infty r^6e^{-2r/3a_0}dr\int_0^\pi\sin^3\theta\cos^2\theta d\theta=1$$

Evaluating the integrals (with the aid of a table of integrals) yields:

$$2\pi C_{32-1}^2\left[\left(1.23\times10^4\right)a_0^7\right]\left[\frac{4}{15}\right]=1$$

$$C_{32-1}=0.00697\Big/\sqrt{a_0^7}$$

This value agrees with the coefficient of ψ_{32-1} above.

7-29. $\psi_{100}=\dfrac{2}{\sqrt{4\pi a_0^3}}e^{-r/a_0}$

Because ψ_{100} is only a function of r, the angle derivatives in Equation 7-9 are all zero.

$$\frac{d\psi}{dr}=\frac{2}{\sqrt{4\pi a_0^3}}e^{-r/a_0}$$

$$r^2\frac{d\psi}{dr}=\frac{2}{\sqrt{4\pi a_0^3}}\left(-\frac{1}{a_0}\right)r^2e^{-r/a_0}$$

$$\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right)=\frac{2}{\sqrt{4\pi a_0^3}}\left(-\frac{1}{a_0}\right)\left[r^2\left(-\frac{1}{a_0}\right)+2r\right]e^{-r/a_0}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right)=\frac{2}{\sqrt{4\pi a_0^3}}\left(-\frac{1}{a_0}\right)\left[\frac{2}{r}-\frac{1}{a_0}\right]e^{-r/a_0}$$ Substituting into Equation 7-9,

(Problem 7-29 continued)

$$-\frac{\hbar^2}{2\mu}\left(\frac{1}{a_0^2}-\frac{2}{a_0 r}\right)\psi_{100}+V\psi_{100}=E\psi_{100}$$

For the 100 state $r = a_0$ and $2\pi a_0 = \lambda = 2\pi / k$ or $a_0 = 1/k$, so

$$\left(\frac{1}{a_0^2}-\frac{2}{a_0 r}\right)=\left(\frac{1}{a_0^2}-\frac{2}{a_0^2}\right)=-\frac{1}{a_0^2}=-k^2$$

Thus, $-\frac{\hbar^2}{2\mu}\left(\frac{1}{a_0^2}-\frac{2}{a_0 r}\right)=\frac{\hbar^2 k^2}{2\mu}$ and we have that

$\frac{\hbar^2 k^2}{2\mu}+V=E,$ satisfying the Schrödinger equation.

7-30. (a) Every increment of charge follows a circular path of radius R and encloses an area πR^2, so the magnetic moment is the total current times this area. The entire charge Q rotates with frequency $f = \omega / 2\pi$, so the current is

$$i = Qf = q\omega / 2\pi$$

$$\mu = iA = (Q\omega / 2\pi)(\pi R^2) = Q\omega R^2 / 2$$

$$L = I\omega = \frac{1}{2}MR^2\omega$$

$$g = \frac{2M\mu}{QL} = \frac{2MQ\omega R^2/2}{QMR^2\omega/2} = 2$$

(b) The entire charge is on the equatorial ring, which rotates with frequency $f = \omega / 2\pi$.

$$i = Qf = Q\omega / 2\pi$$

$$\mu = iA = (Q\omega / 2\pi)(\pi R^2) = Q\omega R^2 / 2$$

$$g = \frac{2M\mu}{QL} = \frac{2MQ\omega R^2/2}{QMR^2\omega/5} = 5/2 = 2.5$$

7-31. Angular momentum $S = I\omega = (2/5)mr^2(v/r)$ or

$$v = (5/2)S(1/mr) = 5S/2mr = 5(3/4)^{1/2}\hbar/2mr$$

$$= \frac{5(3/4)^{1/2}(1.055\times10^{-34}J\bullet s)}{2(9.11\times10^{-31}kg)(10^{-15}m)} = 2.51\times10^{11}m/s = 837c$$

7-32. (a) The K ground state is $\ell = 0$, so two lines due to spin of the single s electron would be seen.

(b) The Ca ground state is $\ell = 0$ with two s electrons whose spins are opposite resulting in $S = 0$, so there will be one line.

(c) The electron spins in the O ground state are coupled to zero, the orbital angular momentum is 2, so five lines would be expected.

(d) The total angular momentum of the Sn ground state is $j = 0$, so there will be one line.

7-33. $|F_z| = m_s g_L \mu_B (dB/dz) = m_{Ag} a_z$ (From Equation 7-51)

and $a_z = m_S g_L \mu_B (dB/dz)/m_{Ag}$

Each atom passes through the magnet's 1m length in t = (1/250)s and cover the additional 1m to the collector in the same time. Within the magnet they deflect in the z direction an amount z_1 given by: $z_1 = (1/2)a_z t^2 = (1/2)\left[m_s g_L \mu_B (dB/dz)/m_{Ag}\right](1/250)^2$ and leave the magnet with a z-component of velocity given by $v_z = a_z t$. The additional z deflection in the field-free region is $z_2 = v_z t = a_z t^2$.

The total deflection is then $z_1 + z_2 = 0.5mm = 5.0\times10^{-4}m$.

$$5.0\times10^{-4}m = z_1 + z_2 = (3/2)a_z t^2 = (3/2)\left[m_s g_L \mu_B (dB/dz)/m_{Ag}\right]\left[1/250\right]^2 \text{ or,}$$

$$\frac{dB}{dz} = \frac{(5.0\times10^{-4}m)(250)^2(m_{Ag})(2)}{m_s g_L \mu_B}$$

$$= \frac{(5.0\times10^{-4}m)(250s^{-1})^2(1.79\times10^{-25}kg)(2)}{3(1/2)(1)(9.27\times10^{-24}J/T)} = 0.805T/m$$

7-34. (a) There should be four lines corresponding to the four m_J values −3/2, −1/2, +1/2, +3/2.

(b) There should be three lines corresponding to the three m_ℓ values −1, 0, +1.

7-35. (a) For the hydrogen atom the n = 4 levels in order of increasing energy are:

$4\,^2S_{1/2}, 4\,^2P_{1/2}, 4\,^2P_{3/2}, 4\,^2D_{3/2}, 4\,^2D_{5/2}, 4\,^2F_{5/2}, 4\,^2F_{7/2}$

(b) $2j + 1$

7-36. For $\ell = 2,\quad L = \sqrt{\ell(\ell+1)}\hbar = \sqrt{6}\hbar = 2.45\hbar,\quad j = \ell \pm 1/2 = 3/2, 5/2$ and $J = \sqrt{j(j+1)}\hbar$

For $j = 3/2,\quad J = \sqrt{(3/2)(3/2+1)}\hbar = \sqrt{15/4}\hbar = 1.94\hbar$

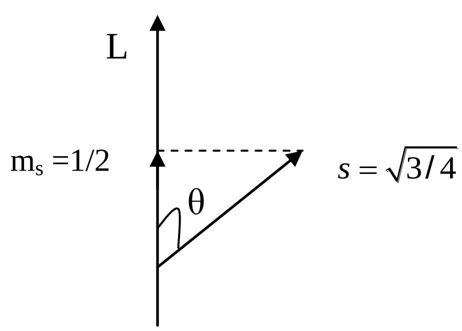
For $j = 5/2,\quad J = \sqrt{(5/2)(5/2+1)}\hbar = \sqrt{35/4}\hbar = 2.96\hbar$

7-37. (a) $j = \ell \pm 1/2 = 2 \pm 1/2 = 5/2$ or $3/2$

(b) $J = \sqrt{j(j+1)}\hbar = \sqrt{\frac{5}{2}(5/2+1)}\hbar = 2.96\hbar$ or $\sqrt{\frac{3}{2}(3/2+1)}\hbar = 1.94\hbar$

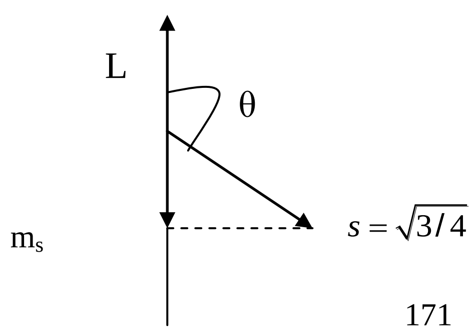
(c) $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}\quad J_Z = L_Z + S_Z = m_\ell\hbar + m_s\hbar = m_j\hbar$ where $m_j = -j, -j+1, \ldots, j-1, j$. For j = 5/2 the z-components are $-5/2,\ -3/2,\ -1/2,\ +1/2,\ +3/2,\ +5/2$. For j = 3/2 the z-components are $-3/2,\ -1/2,\ +1/2,\ +3/2$.

7-38. (a) $d_{5/2}$



$$\theta = \cos^{-1}\frac{1/2}{\sqrt{3/4}} = 54.7°$$

(b) $d_{3/2}$



$$\theta = \cos^{-1}\frac{(-1/2)}{\sqrt{3/4}} = 125.3°$$

7-39.

n	ℓ	m_ℓ	m_s
4	3 2 1 0	−3, −2, −1, 0, 1, 2, 3 −2, −1, 0, 1, 2 −1, 0, 1 0	$\pm 1/2$ for each m_ℓ state
3	2 1 0	−2, −1, 0, 1, 2 −1, 0, 1 0	$\pm 1/2$ for each m_ℓ state
2	1 0	−1, 0, 1 0	$\pm 1/2$ for each m_ℓ state

7-40. (a) $\boldsymbol{L} = \boldsymbol{L}_1 + \boldsymbol{L}_2$

$$\ell = (\ell_1 + \ell_2),\ (\ell_1 + \ell_2 - 1), \ldots, |\ell_1 - \ell_2| = (1+1),\ (1+1-1),\ (1-1) = 2, 1, 0$$

(b) $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$

$$s = (s_1 + s_2),\ (s_1 + s_2 - 1), \ldots, |s_1 - s_2| = (1/2 + 1/2),\ (1/2 - 1/2) = 1, 0$$

(c) $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$

$$j = (\ell + s),\ (\ell + s - 1), \ldots, |\ell - s|$$

For $\ell = 2$ and $s = 1$, $j = 3,\ 2,\ 1$
$\ell = 2$ and $s = 0$, $j = 2$

For $\ell = 1$ and $s = 1$, $j = 2,\ 1,\ 0$
$\ell = 1$ and $s = 0$, $j = 1$

For $\ell = 0$ and $s = 1$, $j = 1$
$\ell = 0$ and $s = 0$, $j = 0$

(d) $\boldsymbol{J}_1 = \boldsymbol{L}_1 + \boldsymbol{S}_1$ $\quad j_1 = \ell_1 \pm 1/2 = 3/2,\ 1/2$

$\boldsymbol{J}_2 = \boldsymbol{L}_2 + \boldsymbol{S}_2$ $\quad j2 = \ell_2 \pm 1/2 = 3/2,\ 1/2$

(e) $\boldsymbol{J} = \boldsymbol{J}_1 + \boldsymbol{J}_2$ $\quad j = (j_1 + j_2),\ (j_1 + j_2 - 1), \ldots, |j_1 - j_2|$

For $j_1 = 3/2$ and $j_2 = 3/2$, $j = 3,\ 2,\ 1,\ 0$
$j_1 = 3/2$ and $j_2 = 1/2$, $j = 2,\ 1$

For $j_1 = 1/2$ and $j_2 = 3/2$, $j = 2,\ 1$
$j_1 = 1/2$ and $j_2 = 1/2$, $j = 1,\ 0$

These are the same values as found in (c).

7-41. (*a*)

$$E = hf \Rightarrow f = E/h = (4.372\times10^{-6}\text{ eV})(1.602\times10^{-19}\text{ J/eV})/6.63\times10^{-34}\text{ J}\cdot\text{s} = 1.056\times10^{9}\text{ Hz}$$

(*b*) $c = f\lambda \Rightarrow \lambda = c/f = (3.00\times10^{8}\text{ m/s})/(1.056\times10^{9}\text{ Hz}) = 0.284\,\text{m} = 28.4\,\text{cm}$

(*c*) short wave radio region of the EM spectrum

7-42. (a) $E_{3/2} = \dfrac{hc}{\lambda}$ Using values from Figure 7-22,

$$E_{3/2} = \frac{1239.852eV\bullet nm}{588.99nm} = 2.10505eV \qquad E_{1/2} = \frac{1239.852eV\bullet nm}{589.59nm} = 2.10291eV$$

(b) $\Delta E = E_{3/2} - E_{1/2} = 2.10505eV - 2.10291eV = 2.14\times10^{-3}eV$

(c) $\Delta E = 2\mu_B B \rightarrow B = \dfrac{\Delta E}{2\mu_B} = \dfrac{2.14\times10^{-3}eV}{2\left(5.79\times10^{-4}eV/T\right)} = 18.5T$

7-43. $\psi_{12} = \psi\left(x_1, x_2\right) = C\sin\dfrac{\pi x_1}{L}\sin\dfrac{2\pi x_2}{L}$ Substituting into Equation 7-57 with $V = 0$,

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi_{12}}{\partial x_1^2} + \frac{\partial^2\psi_{12}}{\partial x_2^2}\right) = \left(\frac{\hbar^2}{2m}\right)(1+4)\left(\frac{\pi^2}{L^2}\right)\psi_{12} = E\psi_{12}$$

Obviously, ψ_{12} is a solution if $E = \dfrac{5\hbar^2\pi^2}{2mL^2}$

7-44. $E_n = \dfrac{n^2\hbar^2\pi^2}{2mL^2}$ Neutrons have antisymmetric wave functions, but if spin is ignored then one is in the state $n = 1$ state, but the second is in the $n = 2$ state, so the minimum energy is: $E = E_1 + E_2 = \left(1^2 + 2^2\right)E_1 = 5E_1$ where

$$E_1 = \frac{(hc)^2\pi^2}{2mc^2L^2} = \frac{(197.3)^2\pi^2}{2(939.6)(2.0)^2} = 51.1MeV \qquad E = 5E_1 = 255MeV$$

7-45. (a) For electrons: Including spin, two are in the n = 1 state, two are in the n = 2 state, and one is in the n = 3 state. The total energy is then:

$$E = 2E_1 + 2E_2 + E_3 \quad \text{where } E_n = \frac{n^2\hbar^2\pi^2}{2mL^2} \quad E = 2E_1 + 2\left(2^2 E_1\right) + \left(3^2 E_1\right) = 19E_1$$

$$\text{where } E_1 = \frac{(hc)^2 \pi^2}{2m_e c^2 L^2} = \frac{(197.3)^2 \pi^2}{2\left(0.511\times10^6\right)(1.0)^2} = 0.376eV \qquad E = 19E_1 = 7.14eV$$

(b) Pions are bosons and all five can be in the n = 1 state, so the total energy is:

$$E = 5E_1 \quad \text{where } E_1 = \frac{0.376eV}{264} = 0.00142eV \qquad E = 5E_1 = 0.00712eV$$

7-46. (a) Carbon: $Z = 6$; $1s^2 2s^2 2p^2$

(b) Oxygen: $Z = 8$; $1s^2 2s^2 2p^4$

(c) Argon: $Z = 18$; $1s^2 2s^2 2p^6 3s^2 3p^6$

7-47. Using Figure 7-34:

Sn (Z = 50)

$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6\ 4d^{10}\ 5s^2\ 5p^2$

Nd (Z = 60)

$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6\ 4d^{10}\ 5s^2\ 5p^6\ 4f^4\ 6s^2$

Yb (Z = 70)

$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6\ 4d^{10}\ 4f^{14}\ 5s^2\ 5p^6\ 6s^2$

Comparison with Appendix C.

Sn: agrees

Nd: $5p^6$ and $4f^4$ are in reverse order

Yb: agrees

7-48. Both *Ga* and *In* have electron configurations $(ns)^2(np)$ outside of closed shells $(n-1,s)^2(n-1,p)^6(n-1,d)^{10}$. The last p electron is loosely bound and is more easily removed than one of the s electrons of the immediately preceding elements *Zn* and *Cd*.

7-49. The outermost electron outside of the closed shell in *Li, Na, K, Ag,* and *Cu* has $\ell = 0$. The ground state of these atoms is therefore not split. In *B, Al,* and *Ga* the only electron not in a closed shell or subshell has $\ell = 1$, so the ground state of these atoms will be split by the spin-orbit interaction.

7-50. $E_n = -\frac{Z_{eff}^2 E_1}{n^2}$ (Equation 7-25)

$$Z_{eff} = n\sqrt{\frac{-E_n}{E_1}} = 3\sqrt{\frac{5.14eV}{13.6eV}} = 1.84$$

7-51. (a) Fourteen electrons, so $Z = 14$. Element is silicon.

(b) Twenty electrons, so $Z = 20$. Element is calcium.

7-52. (a) For a *d* electron, $\ell = 2$, so $L_z = -2\hbar, -1\hbar, 0, 1\hbar, 2\hbar$

(b) For an *f* electron, $\ell = 3$, so $L_z = -3\hbar, -2\hbar, -1\hbar, 0, 1\hbar, 2\hbar, 3\hbar$

7-53. Like *Na*, the following atoms have a single *s* electron as the outermost shell and their energy level diagrams will be similar to sodium's: *Li, Rb, Ag, Cs, Fr.*

The following have two *s* electrons as the outermost shell and will have energy level diagrams similar to mercury: *He, Ca, Ti, Cd, Mg, Ba, Ra.*

7-54. Group with 2 outer shell electrons: beryllium, magnesium, calcium, nickel, and barium.

Group with 1 outer shell electron: lithium, sodium, potassium, chromium, and cesium.

7-55. Similar to H: Li, Rb, Ag, and Fr. Similar to He: Ca, Ti, Cd, Ba, Hg, and Ra.

7-56.

n	ℓ	j
4	0	1/2
4	1	1/2
4	1	3/2
5	0	1/2
3	2	3/2
3	2	5/2
5	1	1/2
5	1	3/2
4	2	3/2
4	2	5/2
6	0	1/2
4	3	5/2
4	3	7/2

Energy is increasing downward in the table.

7-57. Selection rules: $\Delta\ell = \pm 1 \quad \Delta j = \pm 1, 0$

Transition	$\Delta\ell$	Δj	Comment
$4S_{1/2} \rightarrow 3S_{1/2}$	0	0	ℓ - forbidden
$4S_{1/2} \rightarrow 3P_{3/2}$	+1	+1	allowed
$4P_{3/2} \rightarrow 3S_{1/2}$	−1	−1	allowed
$4D_{5/2} \rightarrow 3P_{1/2}$	−1	−2	j – forbidden
$4D_{3/2} \rightarrow 3P_{1/2}$	−1	−1	allowed
$4D_{3/2} \rightarrow 3S_{1/2}$	−2	−1	ℓ - forbidden
$5D_{3/2} \rightarrow 4S_{1/2}$	−2	−1	ℓ - forbidden
$5P_{1/2} \rightarrow 3S_{1/2}$	−1	0	allowed

7-58. (a) $E_1 = -13.6eV\left(Z-1\right)^2 = -13.6eV\left(74-1\right)^2 = -7.25\times10^4 eV = -72.5keV$

(b) $E_1\left(\text{exp}\right) = -69.5keV = -13.6eV\left(Z-\sigma\right)^2 = -13.6eV\left(74-1\right)^2$

$$74-\sigma = \left(69.5\times10^3 eV/13.6eV\right)^{1/2} = 71.49$$

$$\sigma = 74-71.49 = 2.51$$

7-59. $\Delta j = \pm 1, 0 \quad (\text{no } j = 0 \rightarrow j = 0)$ (Equation 7-66)

The four states are $^2P_{3/2}$, $^2P_{1/2}$, $^2D_{5/2}$, $^2D_{3/2}$

Transition	$\Delta\ell$	Δj	Comment
$D_{5/2} \rightarrow P_{3/2}$	−1	−1	allowed
$D_{5/2} \rightarrow P_{1/2}$	−1	−2	*j* - forbidden
$D_{3/2} \rightarrow P_{3/2}$	−1	0	allowed
$D_{3/2} \rightarrow P_{1/2}$	−1	−1	allowed

7-60. (a) $\Delta E = hc/\lambda$

$$E(3P_{1/2}) - E(3S_{1/2}) = \frac{1240eV \bullet nm}{589.59nm} = 2.10eV$$

$$E(3P_{1/2}) = E(3S_{1/2}) + 2.10eV = -5.14eV + 2.10eV = -3.04eV$$

$$E(3D) - E(3P_{1/2}) = \frac{1240eV \bullet nm}{818.33nm} = 1.52eV$$

$$E(3D) = E(3P_{1/2}) + 1.52eV = -3.04eV + 1.52eV = -1.52eV$$

(b) For $3P$: $\quad Z_{eff} = 3\sqrt{\dfrac{3.04eV}{13.6eV}} = 1.42$

For $3D$: $\quad Z_{eff} = 3\sqrt{\dfrac{1.52eV}{13.6eV}} = 1.003$

(c) The Bohr formula gives the energy of the 3*D* level quite well, but not the 3*P* level.

7-61. (a) $\Delta E = g m_j \mu_B B$ (Equation 7-73) where $s = 1/2$, $\ell = 0$ gives $j = 1/2$ and (from Equation 7-73) $g = 2$. $\quad m_j = \pm 1/2$.

$$\Delta E = (2)(\pm 1/2)(5.79 \times 10^{-5} eV/T)(0.55T) = \pm 3.18 \times 10^{-5} eV$$

The total splitting between the $m_j = \pm 1/2$ states is $6.37 \times 10^{-5} eV$.

(b) The $m_j = 1/2$ (spin up) state has the higher energy.

(c) $\Delta E = hf \rightarrow f = \Delta E/h = 6.37 \times 10^{-5} eV / 4.14 \times 10^{-15} eV \bullet s = 1.54 \times 10^{10} Hz$

This is in the microwave region of the spectrum.

7-62. $E = \frac{hc}{\lambda} \rightarrow \Delta E \approx \frac{dE}{d\lambda}\Delta\lambda = -\frac{hc}{\lambda^2} \rightarrow \Delta\lambda \approx -\frac{\lambda^2}{hc}\Delta E$

7-63. (a) $\Delta E = \frac{e\hbar}{2m}B = \left(5.79\times10^{-5}eV/T\right)\left(0.05T\right) = 2.90\times10^{-6}eV$

(b) $\left|\Delta\lambda\right| = \frac{\lambda^2}{hc}\Delta E = \frac{\left(579.07nm\right)^2\left(2.90\times10^{-6}eV\right)}{1240eV\bullet nm} = 7.83\times10^{-4}nm$

(c) The smallest measurable wavelength change is larger than this by the ratio $0.01nm/7.83\times10^{-4}nm = 12.8$. The magnetic field would need to be increased by this same factor because $B \propto \Delta E \propto \Delta\lambda$. The necessary field would be 0.638T.

7-64. $E_n = -13.6eV\left(Z_{eff}^2/n^2\right)$

$E_2 = -13.6eV\left(Z_{eff}^2/2^2\right) = -5.39eV$

$Z_{eff} = 2\left(5.39/13.6\right)^{1/2} = 1.26$

7-65. $\psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$ (Equations 7-30 and 7-31)

$P\left(r\right) = 4\pi r^2\psi_{100}^*\psi_{100}$ (Equation 7-32)

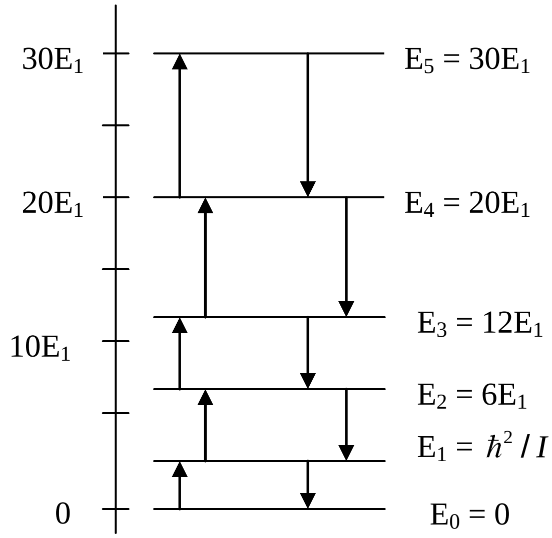
$$= 4\pi r^2\frac{Z^3}{\pi a_0^3}e^{-Zr/a_0} = \frac{4Z^3}{a_0^3}r^2e^{-2Zr/a_0}$$

$$\left\langle r\right\rangle = \int_0^\infty rP\left(r\right)dr = \int_0^\infty \frac{4Z^3}{a_0^3}r^3e^{-2Zr/a_0}dr$$

$$= \frac{a_0}{4Z}\int_0^\infty\left(\frac{2Zr}{a_0}\right)^3 e^{-2Zr/a_0}d\left(2Zr/a_0\right) = \frac{a_0}{4Z}\times3! = \frac{3a_0}{2Z}$$

7-66. (a) $E_\ell = \dfrac{\ell(\ell+1)\hbar^2}{2I}$

ℓ:	0	1	2	3	4	5	…
$\ell+1$:	1	2	3	4	5	6	…
E_ℓ:	0	$1E_1$	$6E_1$	$12E_1$	$20E_1$	$30E_1$	…



(b) $E_{\ell+1} - E_\ell = \dfrac{\hbar^2}{2I}\left[(\ell+1)(\ell+2) - \ell(\ell+1)\right]$

$$= \frac{\hbar^2}{2I}\left[(\ell+1)(\ell+2-\ell)\right] = \frac{\hbar^2}{I}(\ell+1) = (\ell+1)E_1$$

The values of $\ell = 0, 1, 2, \ldots$ yield all the positive integer multiples of E_1.

(c) $I = \dfrac{1}{2}m_p r^2 \rightarrow E_1 = \dfrac{\hbar^2}{I} = \dfrac{2\hbar^2}{m_p r^2} = \dfrac{2(\hbar c)^2}{m_p c^2 r^2}$

$$= \frac{2(197.3eV\bullet nm)^2}{(938.28\times10^6 eV)(0.074nm)^2} = 1.52\times10^{-2}eV$$

(d) $\lambda = \dfrac{hc}{E_1} = \dfrac{1.24\times10^{-6}eV\bullet nm}{1.52\times10^{-2}eV} = 8.18\times10^{-5}m = 81.8\mu m$

7-67. (a) $|F_z| = m_s g_L \mu_B (dB/dz)$ (From Equation 7-51)

From Newton's 2nd law, $|F_z| = m_H a_z = m_s g_L \mu_B (dB/dz)$

$a_z = m_s g_L (dB/dz)/m_H = (1/2)(1)(9.27\times10^{-24}J/T)(600T/m)/(1.67\times10^{-27}kg)$

$= 1.67\times10^6 m/s^2$

(Problem 7-67 continued)

(b) At $14.5km/s = v = 1.45\times 10^4 m/s$, the atom takes $t_1 = 0.75m/\left(1.45\times 10^4 m/s\right)$

$= 5.2\times 10^{-5}s$ to traverse the magnet. In that time, its z deflection will be:

$$z_1 = (1/2)(a_z)t_1^2 = (1/2)\left(1.67\times 10^6 m/s^2\right)\left(5.2\times 10^{-5}s\right)^2 = 2.26\times 10^{-3}m = 2.26mm$$

Its v_z velocity component as it leaves the magnet is $v_z = a_z t_1$ and its additional z deflection before reaching the detector 1.25m away will be:

$$z_2 = v_z t_2 = (a_z t_1)\left(1.25m/\left[1.45\times 10^4 m/s\right]\right)$$

$$= \left(1.67\times 10^6 m/s^2\right)\left(5.2\times 10^{-5}s\right)(1.25)/\left(1.45\times 10^4 m/s\right)$$

$$= 7.49\times 10^{-3}m = 7.49mm$$

Each line will be deflected $z_1 + z_2 = 9.75mm$ from the central position and, thus, separated by a total of 19.5mm = 1.95cm.

7-68. $\theta_{\text{min}} = \cos^{-1}\left[m_\ell \hbar/\sqrt{\ell(\ell+1)}\,\hbar\right]$ with $m_\ell = \ell$.

$\cos\theta_{\text{min}} = \ell\sqrt{\ell(\ell+1)}$. Thus, $\cos^2\theta_{\text{min}} = \ell^2/\left[\ell(\ell+1)\right] = 1 - \sin^2\theta_{\text{min}}$

or, $\sin^2\theta_{\text{min}} = 1 - \dfrac{\ell^2}{\ell(\ell+1)} = \dfrac{\ell(\ell+1)-\ell^2}{\ell(\ell+1)} = \dfrac{\ell^2+\ell-\ell^2}{\ell(\ell+1)}$

And, $\sin\theta_{\text{min}} = \left(\dfrac{1}{\ell+1}\right)^{1/2}$ For large ℓ, θ_{min} is small.

Then $\sin\theta_{\text{min}} \approx \theta_{\text{min}} = \left(\dfrac{1}{\ell+1}\right)^{1/2} \approx \dfrac{1}{(\ell)^{1/2}}$

7-69. (a) $E_1 = hf = hc/\lambda_1 = 1240eV\bullet nm/766.41nm = 1.6179eV$

$E_2 = hf = hc/\lambda_2 = 1240eV\bullet nm/769.90nm = 1.6106eV$

(b) $\Delta E = E_1 - E_2 = 1.6179eV - 1.6106eV = 0.0073eV$

(c) $\Delta E/2 = gm_j\mu_B B \rightarrow B = \dfrac{\Delta E}{2gm_j\mu_B} = \dfrac{0.0073eV}{2(2)(1/2)\left(5.79\times 10^{-5}eV/T\right)} = 63T$

7-70. $P(r) = \frac{4Z^3}{a_0^3} r^2 e^{-2Zr/a_0}$ (See Problem 7-65)

For hydrogen, $Z = 1$ and at the edge of the proton $r = R_0 = 10^{-15} m$. At that point, the exponential factor in P(r) has decreased to:

$$e^{-2R_0/a_0} = e^{-2(10^{-15})/(0.529\times10^{-10}m)} = e^{-(3.78\times10^{-5})} \approx 1 - 3.78\times10^{-5} \approx 1$$

Thus, the probability of the electron in the hydrogen ground state being inside the nucleus, to better than four figures, is:

$$P(r) = \frac{4r^2}{a_0^3} \qquad P = \int_0^{r_0} P(r)dr = \int_0^{R_0} \frac{4r^2}{a_0^3} = \frac{4}{a_0^3}\int_0^{R_0} r^2 dr = \frac{4}{a_0^3}\frac{r^3}{3}\bigg|_0^{R_0}$$

$$= \frac{4}{a_0^3}\left(\frac{R_0^3}{3}\right) = \frac{4\left(10^{-15}m\right)^3}{3\left(0.529\times10^{-10}m\right)^3} = 9.0\times10^{-15}$$

7-71. (a) $g = 1 + \frac{j(j+1) + s(s+1) - \ell(\ell+1)}{2j(j+1)}$ (Equation 7-73)

For ${}^2P_{1/2}$: $j = 1/2$, $\ell = 1$, and $s = 1/2$

$$g = 1 + \frac{1/2(1/2+1) + 1/2(1/2+1) - 1(1+1)}{2\times1/2(1/2+1)} = 1 + \frac{3/4 + 3/4 - 2}{3/2} = 2/3$$

For ${}^2S_{1/2}$: $j = 1/2$, $\ell = 0$, and $s = 1/2$

$$g = 1 + \frac{1/2(1/2+1) + 1/2(1/2+1) - 0}{2\times1/2(1/2+1)} = 1 + \frac{3/4 + 3/4}{3/2} = 2$$

The ${}^2P_{1/2}$ levels shift by:

$$\Delta E = gm_j\mu_B B = \frac{2}{3}\left(\pm\frac{1}{2}\right)\mu_B B = \pm\frac{1}{3}\mu_B B \qquad \text{(Equation 7-72)}$$

The ${}^2S_{1/2}$ levels shift by:

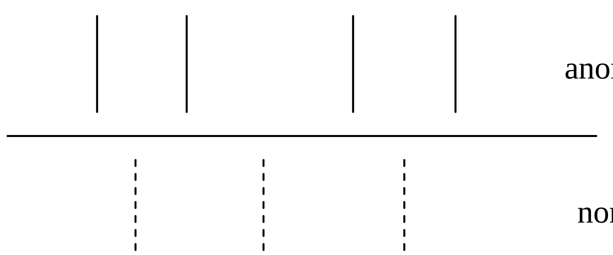
$$\Delta E = gm_j\mu_B B = 2\left(\pm\frac{1}{2}\right)\mu_B B = \pm\mu_B B$$

(Problem 7-71 continued)

To find the transition energies, tabulate the several possible transitions and the corresponding energy values (let E_p and E_s be the $B = 0$ unsplit energies of the two states.):

Transition	Energy
$P_{1/2,1/2} \to S_{1/2,1/2}$	$\left(E_p + \frac{1}{3}\mu_B B\right) - \left(E_s + \mu_B B\right) = \left(E_p - E_s\right) - \frac{2}{3}\mu_B B$
$P_{1/2,-1/2} \to S_{1/2,1/2}$	$\left(E_p - \frac{1}{3}\mu_B B\right) - \left(E_s + \mu_B B\right) = \left(E_p - E_s\right) - \frac{4}{3}\mu_B B$
$P_{1/2,1/2} \to S_{1/2,-1/2}$	$\left(E_p + \frac{1}{3}\mu_B B\right) - \left(E_s - \mu_B B\right) = \left(E_p - E_s\right) + \frac{4}{3}\mu_B B$
$P_{1/2,-1/2} \to S_{1/2,-1/2}$	$\left(E_p - \frac{1}{3}\mu_B B\right) - \left(E_s - \mu_B B\right) = \left(E_p - E_s\right) + \frac{2}{3}\mu_B B$

Thus, there are four different photon energies emitted. The energy or frequency spectrum would appear as below (normal Zeeman spectrum shown for comparison).



(b) For $^2P_{3/2}$: $j = 3/2$, $\ell = 1$, and $s = 1/2$

$$g = 1 + \frac{3/2(3/2+1) + 1/2(1/2+1) - 1(1+1)}{2 \times 3/2(3/2+1)} = 1 + \frac{15/4 + 3/4 - 2}{30/4} = 4/3$$

These levels shift by:

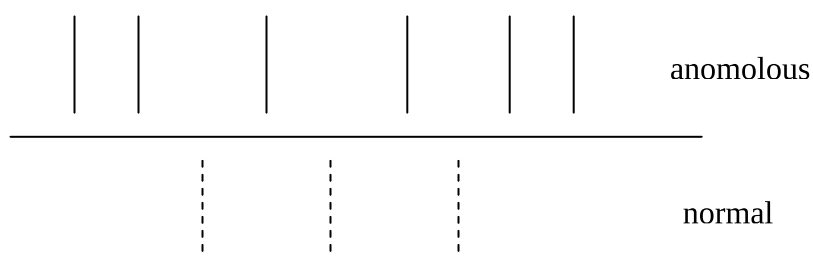
$$\Delta E = g m_j \mu_B B = \frac{4}{3}\left(\pm\frac{1}{2}\right)\mu_B B = \pm\frac{2}{3}\mu_B B \qquad \Delta E = \frac{4}{3}\left(\pm\frac{3}{2}\right)\mu_B B = \pm 2\mu_B B$$

(Problem 7-71 continued)

Tabulating the transitions as before:

Transition	Energy
$P_{3/2,3/2} \to S_{1/2,1/2}$	$\left(E_p + 2\mu_B B\right) - \left(E_s + \mu_B B\right) = \left(E_p - E_s\right) + \mu_B B$
$P_{3/2,3/2} \to S_{1/2,-1/2}$	forbidden, $\Delta m_j = 2$
$P_{3/2,1/2} \to S_{1/2,1/2}$	$\left(E_p - \frac{2}{3}\mu_B B\right) - \left(E_s + \mu_B B\right) = \left(E_p - E_s\right) - \frac{1}{3}\mu_B B$
$P_{3/2,1/2} \to S_{1/2,-1/2}$	$\left(E_p + \frac{2}{3}\mu_B B\right) - \left(E_s - \mu_B B\right) = \left(E_p - E_s\right) + \frac{5}{3}\mu_B B$
$P_{3/2,-1/2} \to S_{1/2,1/2}$	$\left(E_p - \frac{2}{3}\mu_B B\right) - \left(E_s + \mu_B B\right) = \left(E_p - E_s\right) - \frac{5}{3}\mu_B B$
$P_{3/2,-1/2} \to S_{1/2,-1/2}$	$\left(E_p - \frac{2}{3}\mu_B B\right) - \left(E_s - \mu_B B\right) = \left(E_p - E_s\right) + \frac{1}{3}\mu_B B$
$P_{3/2,-3/2} \to S_{1/2,1/2}$	forbidden, $\Delta m_j = 2$
$P_{3/2,-3/2} \to S_{1/2,-1/2}$	$\left(E_p - 2\mu_B B\right) - \left(E_s - \mu_B B\right) = \left(E_p - E_s\right) - \mu_B B$

There are six different photon energies emitted (two transitions are forbidden); their spectrum looks as below:



7-72. (a) Substituting $\psi\left(r,\theta\right)$ into Equation 7-9 and carrying out the indicated operations yields (eventually)

$$-\frac{\hbar^2}{2\mu}\psi\left(r,\theta\right)\left[2/r^2 - 1/4a_0^2\right] - \frac{\hbar^2}{2\mu}\psi\left(r,\theta\right)\left(-2/r^2\right) + V\psi\left(r,\theta\right) = E\psi\left(r,\theta\right)$$

(Problem 7-72 continued)

Canceling $\psi(r,\theta)$ and recalling that $r^2 = 4a_0^2$ (because ψ given is for *n* = 2) we have $-\frac{\hbar^2}{2\mu}\left(-1/4a_0^2\right) + v = E$

The circumference of the *n* = 2 orbit is: $C = 2\pi(4a_0) = 2\lambda \rightarrow a_0 = \lambda/4\pi = 1/2k$.

Thus, $-\frac{\hbar^2}{2\mu}\left(-\frac{1}{4/4k^2}\right) + V = E \;\rightarrow\; \frac{\hbar^2 k^2}{2\mu} + V = E$

(b) or $\frac{p^2}{2m} + v = E$ and Equation 7-9 is satisfied.

$$\int_0^\infty \psi^2 dx = \int A^2 \left(\frac{r}{a_0}\right)^2 e^{-r/a_0} \cos^2\theta r^2 \sin\theta dr\, d\theta d\phi = 1$$

$$A^2 \int_0^\infty \left(\frac{r}{a_0}\right)^2 e^{-r/a_0} r^2 dr \int_0^\pi \cos^2\theta \sin\theta d\theta \int_0^{2\pi} d\phi = 1$$

Integrating (see Problem 7-23),

$$A^2\left(6a_0^3\right)(2/3)(2\pi) = 1$$

$$A^2 = 1/8a_0^3\pi \rightarrow A = \sqrt{1/8a_0^3\pi}$$

7-73. $\mu = -g_L \mu_B L/\hbar$ (Equation 7-43)

(a) The 1*s* state has $\ell = 0$, so it is unaffected by the external *B*.

The 2*p* state has $\ell = 1$, so it is split into three levels by the external *B*.

(b) The $2p \rightarrow 1s$ spectral line will be split into three lines by the external *B*.

(c) In Equation 7-43 we replace μ_B with $\mu_k = e\hbar/2m_k$, so

$$\mu_{kz} = -(1)(1)(e\hbar/2m_k) = -\mu_B(m_e/m_k) \quad \text{(From Equation 7-45)}$$

Then $\Delta E = \mu_B(m_e/m_k)B$

$$= \left(5.79\times10^{-5} eV/T\right)\left[\left(0.511\times10^6 MeV/c^2\right)/\left(497.7 MeV/c^2\right)\right](1.0T)$$

$$= 5.94\times10^{-8} eV$$

(Problem 7-73 continued)

$\frac{\Delta\lambda}{\lambda} = -\frac{\lambda}{hc}\Delta E$ (From Problem 7-62) where λ for the (unsplit) 2*p* → 1*s* transition is given by

$\lambda = hc/\Delta E_k$ and $\Delta E_k = E_2 - E_1 = -13.6eV\left(m_k / m_e\right)\left(1-1/4\right) = 9.93\times10^3 eV$

and $\lambda = 1240eV\bullet nm/9.93\times10^3 eV = 0.125nm$

and $\frac{\Delta\lambda}{\lambda} = \frac{0.125nm\left(5.94\times10^{-8}eV\right)}{1240eV\bullet nm} = 5.98\times10^{-12}$

7-74. $\Delta E = -\boldsymbol{\mu}\bullet\boldsymbol{B} = \frac{ke^2}{r^3 m\left(mc^2\right)}\boldsymbol{S}\bullet\boldsymbol{L}$ where, for $n = 3$, $r = a_0 n^2 = 9a_0$

For 3*P* states $\boldsymbol{S}\bullet\boldsymbol{L} \approx \hbar^2$

$$\Delta E \approx \frac{1.440eV\bullet nm\left(3.00\times10^8 m/s\times10^9 nm/m\right)^2\left(6.58\times10^{-16}eV\bullet s\right)^2}{9\left(0.053nm\right)^3\left(0.511\times10^6 eV\right)^2} = 1.60\times10^{-4}eV$$

For 3*D* states $\boldsymbol{S}\bullet\boldsymbol{L} \approx \hbar^2/3$

$\Delta E \approx 1.60\times10^{-4}eV/3 \approx 0.53\times10^{-4}eV$

7-75. (a) $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ $\quad\boldsymbol{\mu} = -\mu_B\left(\boldsymbol{L} + 2\boldsymbol{S}\right)/\hbar$ (Equation 7-71)

$$\boldsymbol{\mu}_J = \frac{\boldsymbol{\mu}\bullet\boldsymbol{J}}{J} = \frac{\left[-\mu_B\left(\boldsymbol{L}+2\boldsymbol{S}\right)/\hbar\right]\times\left[\boldsymbol{L}+\boldsymbol{S}\right]}{J} \equiv -\frac{\mu_B}{\hbar J}\left(\boldsymbol{L}\bullet\boldsymbol{L} + 2\boldsymbol{S}\bullet\boldsymbol{S} + 3\boldsymbol{S}\bullet\boldsymbol{L}\right)$$

$$= -\frac{\mu_B}{\hbar J}\left(L^2 + 2S^2 + 3\boldsymbol{S}\bullet\boldsymbol{L}\right)$$

(b) $J^2 = \boldsymbol{J}\bullet\boldsymbol{J} = \left(\boldsymbol{L}+\boldsymbol{S}\right)\bullet\left(\boldsymbol{L}+\boldsymbol{S}\right) = \boldsymbol{L}\bullet\boldsymbol{L} + \boldsymbol{S}\bullet\boldsymbol{S} + 2\boldsymbol{S}\bullet\boldsymbol{L} \quad \therefore \boldsymbol{S}\bullet\boldsymbol{L} = \frac{1}{2}\left(J^2 - L^2 - S^2\right)$

(c) $\mu_J = -\frac{\mu_B}{\hbar J}\left[L^2 + 2S^2 + \frac{3}{2}\left(J^2 - L^2 - S^2\right)\right] = -\frac{\mu_B}{2\hbar J}\left(3J^2 + S^2 - L^2\right)$

(d) $\mu_Z = \mu_J\frac{J_Z}{J} = -\frac{\mu_B}{2\hbar J}\left(3J^2 + S^2 - L^2\right)\frac{J_Z}{J} = -\frac{\mu_B}{2\hbar J}\left(3J^2 + S^2 - L^2\right)$

(Problem 7-75 continued)

$$= -\mu_B \left(1 + \frac{J^2 + S^2 - L^2}{2J^2}\right)\frac{J_Z}{\hbar}$$

(e) $\Delta E = -\mu_Z B$ (Equation 7-69)

$$= +\mu_B B\left[1 + \frac{j(j+1) + s(s+1) - \ell(\ell+1)}{2j(j+1)}\right]m_j$$

$= g m_j \mu_B B$ (Equation 7-72)

where $g = \left[1 + \frac{j(j+1) + s(s+1) - \ell(\ell+1)}{2j(j+1)}\right]$ (Equation 7-73)

7-76. The number of steps of size unity between two integers (or half-integers) a and b is $b - a$. Including both values of a and b, the number of distinct values in this sequence is $b - a + 1$. For $\boldsymbol{F} = \boldsymbol{I} + \boldsymbol{J}$, the largest value of f is $I + J = b$. If $I < J$, the smallest value of f is $J - I = a$. The number of different values of f is therefore $(I + J) - (J - I) + 1 = 2I + 1$. For $I > J$, the smallest value of f is $I - J = a$. In that case, the number of different values of f is $(I + J) - (I - J) + 1 = 2J + 1$. The two expressions are equal if $I = J$.

7-77. (a) $\mu_N = \dfrac{e\hbar}{2m_p} = 5.05\times10^{-27}\,J/T$

$$B = \frac{2k_m\mu}{r^3} = \frac{2k_m(2.8\mu_N)}{r^3} = \frac{2k_m(2.8\mu_N)}{a_0^3}$$

$$= \frac{2(10^{-7}\,H/m)(2.8)(5.05\times10^{-27}\,J/T)}{(0.529\times10^{-10}\,m)^3} = 0.0191T$$

(b) $\Delta E \approx 2\mu_B B = 2(5.79\times10^{-4}\,eV/T)(0.0191T) = 2.21\times10^{-6}\,eV$

(c) $\lambda = \dfrac{hc}{\Delta E} = \dfrac{1.24\times10^{-6}\,eV\bullet m}{2.21\times10^{-6}\,eV} = 0.561m = 56.1cm$

Chapter 8 – Statistical Physics

8-1. (a) $v_{rms} = \sqrt{\dfrac{3RT}{M}} = \left[\dfrac{3(8.31 J/mole\bullet K)(300K)}{2(.0079\times10^{-3} kg/mole)}\right]^{1/2} = 1930 m/s$

(b) $T = \dfrac{Mv_{rms}^2}{3R} = \dfrac{2(1.0079\times10^{-3} kg/mole)(11.2\times10^3 m/s)^2}{3(8.31 J/mole\bullet K)} = 1.01\times10^4 K$

8-2. (a) $\overline{E_k} = \dfrac{3}{2}kT \quad \therefore T = \dfrac{2\overline{E_k}}{3k} = \dfrac{2(13.6 eV)}{3(8.617\times10^{-5} eV/K)} = 1.05\times10^5 K$

(b) $\overline{E_k} = \dfrac{3}{2}kT = \dfrac{3}{2}(8.67\times10^{-5} eV/K)(10^7 K) = 1.29 keV$

8-3. $v_{rms} = \sqrt{\dfrac{3RT}{M}}$

(a) For O_2: $v_{rms} = \sqrt{\dfrac{3(8.31 J/K\bullet mol)(273K)}{32\times10^{-3} kg/mol}} = 461 m/s$

(b) For H_2: $v_{rms} = \sqrt{\dfrac{3(8.31 J/K\bullet mol)(273K)}{2\times10^{-3} kg/mol}} = 1840 m/s$

8-4. $\left[\dfrac{3RT}{M}\right]^{1/2} = \left[\dfrac{(J/mole\bullet K)(K)}{kg/mole}\right]^{1/2} = \left[\dfrac{kg\bullet m^2/s^2}{kg}\right]^{1/2} = m/s$

8-5. (a) $E_K = n\times\dfrac{3}{2}RT = (1\,mole)\dfrac{3}{2}(8.31 J/mole\bullet K)(273) = 3400 J$

(b) One mole of any gas has the same translational energy at the same temperature.

8-6. $\langle v^2\rangle = \frac{1}{N}\int_0^\infty v^2 n(v)\,dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^4 e^{-\lambda v^2}dv$ where $\lambda = m/2kT$

$\langle v^2\rangle = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} I_4$ where I_4 is given in Table B1-1.

$$I_4 = \frac{3}{8}\pi^{1/2}\lambda^{-5/2} = \frac{3}{8}\pi^{1/2}(m/2kT)^{-5/2}$$

$$\langle v^2\rangle = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\frac{3}{8}\right)\pi^{1/2}\left(\frac{2kT}{m}\right)^{5/2} = \frac{3kT}{m} = \frac{3RT}{mN_A} = \frac{3RT}{M}$$

$$v_{rms} = \sqrt{\langle v^2\rangle} = \sqrt{\frac{3RT}{M}}$$

8-7. $\langle v\rangle = \sqrt{\frac{8kT}{\pi m}} = \left[\frac{8(1.381\times10^{-23}J/K)(300K)}{\pi(1.009u)(1.66\times10^{-27}kg/u)}\right]^{1/2} = 2510m/s$

$$v_m = \sqrt{\frac{2kT}{m}} = \left[\frac{2(1.381\times10^{-23}J/K)(300K)}{\pi(1.009u)(1.66\times10^{-27}kg/u)}\right]^{1/2} = 2220m/s$$

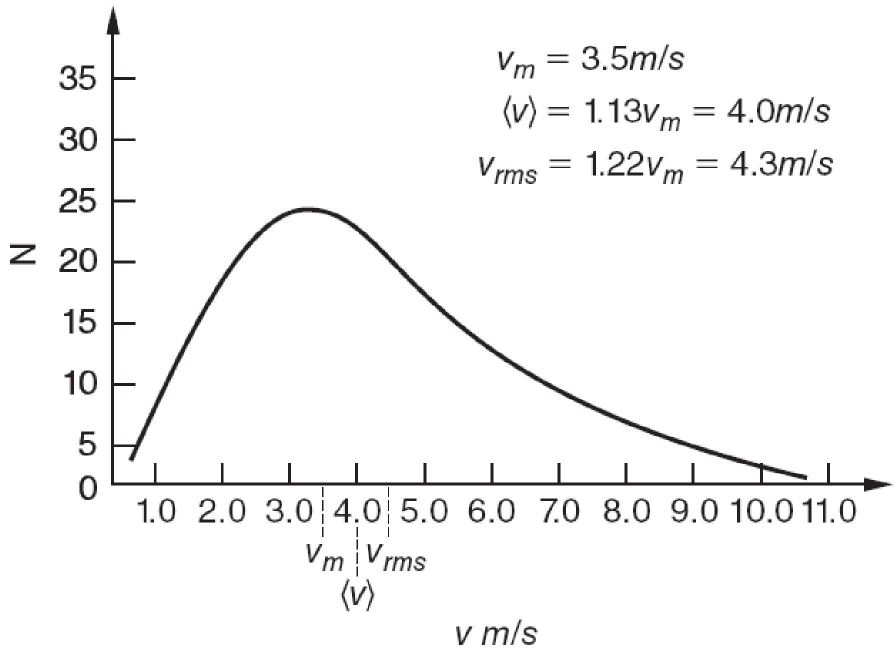
$n(v) = 4\pi N(m/2\pi kT)^{3/2} v^2 e^{-mv^2/kT}$ (Equation 8-28)

At the maximum: $\frac{dn}{dv} = 0 = 4\pi N(m/2\pi kT)^{3/2}\left(2v + v^2(-mv/kT)\right)e^{-mv^2/2kT}$

$$0 = ve^{-mv^2/2kT}\left(2 - mv^2/kT\right)$$

The maximum corresponds to the vanishing of the last factor. (The other two factors give minima at v = 0 and $v = \infty$.) So $2 - mv^2/kT = 0$ and $v_m = (2kT/m)^{1/2}$.

8-8.



8-9. $n\ \ v\ \ dv = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT} dv$ (Equation 8-8)

$\frac{dn}{dv} = A\left[v^2\left(-\frac{2vm}{2kT}\right) + 2v\right]e^{-mv^2/2kT}$ The v for which $dn/dv = 0$ is v_m.

$A\left[-\frac{2mv^3}{2kT} + 2v\right]e^{-mv^2/2kT} = 0$

Because A = constant and the exponential term is only zero for $v \to \infty$, only the quantity in [] can be zero, so $-\frac{2mv^3}{2kT} + 2v = 0$

or $v^2 = \frac{2kT}{m} \to v_m = \sqrt{\frac{2kT}{m}}$ (Equation 8-9)

8-10. The number of molecules N in 1 liter at 1 atm, 20°C is:

$N = 1\ell\ \ 1g{\bullet}mol/22.4\ell\ \ \ N_A\ molecules/g{\bullet}mol$

Each molecule has, on the average, $3kT/2$ kinetic energy, so the total translational kinetic

(Problem 8-10 continued)

energy in one liter is: $KE = \dfrac{6.02\times 10^{23}}{22.4}\left[\dfrac{3\ 1.381\times 10^{-23}J/K\quad 293K}{2}\right] = 163J$

8-11. $\dfrac{n_2}{n_1} = \dfrac{g_2 e^{-E_2/kT}}{g_1 e^{-E_1/kT}} = \dfrac{g_2}{g_1} e^{-\ E_2-E_1\ /kT}$

$$e^{\ E_2-E_1\ /kT} = \frac{g_2}{g_1}\times\frac{n_1}{n_2} = \ E_2 - E_1\ /kT = \ln\left(\frac{g_2}{g_1}\times\frac{n_1}{n_2}\right)$$

$$T = \frac{E_2 - E_1}{k\ln\left[\ g_2/g_1\quad n_1/n_2\ \right]} = \frac{10.2eV}{8.617\times 10^{-5}eV/K\ \ \ln\ 4\times 10^6} = 7790K$$

8-12. $\dfrac{n_2}{n_1} = \dfrac{g_2}{g_1} e^{-\ E_2-E_1\ /kT} = \dfrac{3}{1} e^{-\left[\frac{4\times 10^{-3}eV}{8.617\times 10^{-5}eV/K\quad 300K}\right]} = 2.57$

8-13. There are two degrees of freedom, therefore,

$C_v = 2\ \ R/2\ = R,\quad C_p = R + R = 2R,$ and $\gamma = 2R/R = 2.$

8-14. $c_v = 3R/M$

(a) Al: $c_v = \dfrac{3\ 1.99cal/mole\bullet K}{27.0g/mole} = 0.221cal/g\bullet K \qquad 0.215cal/g\bullet K$

(b) Cu: $c_v = \dfrac{3\ 1.99cal/mole\bullet K}{62.5g/mole} = 0.0955cal/g\bullet K \qquad 0.0920cal/g\bullet K$

(c) Pb: $c_v = \dfrac{3\ 1.99cal/mole\bullet K}{207g/mole} = 0.0288cal/g\bullet K \qquad 0.0305cal/g\bullet K$

The values for each element shown in brackets are taken from the *Handbook of Chemistry and Physics* and apply at 25° C.

8-15. $n(E) = \frac{2\pi N}{\left(\pi kT\right)^{3/2}} E^{1/2} e^{-E/kT}$ (Equation 8-13)

At the maximum: $\frac{dn}{dE} = 0 = \frac{2\pi N}{\left(\pi kT\right)^{3/2}} \left\{ \frac{1}{2} E^{-1/2} + E^{1/2} \left(-\frac{1}{kT} \right) \right\} e^{-E/kT}$

$$= E^{-1/2} e^{-E/kT} \left(\frac{1}{2} - E/kT \right)$$

The maximum corresponds to the vanishing of the last factor. (The vanishing of the other two factors corresponds to minima at $E = 0$ and $E = \infty$.)

$1/2 - E/kT = 0 \rightarrow E = 1/2kT.$

8-16. (a) $n(v) = 4\pi N \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-mv^2/2kT}$ (Equation 8-8)

$$= \frac{4\pi N}{\pi^{3/2}} \left(\frac{m}{2kT} \right)^{3/2} v^2 e^{-v^2 (m/2kT)}$$

$$= \frac{4N}{\sqrt{\pi}} \frac{v^2}{v_m^3} e^{-v^2/v_m^2} \quad \text{where } v_m = \sqrt{2kT/m}$$

$$= \frac{4N}{\sqrt{\pi}} \frac{1}{v_m} \left(\frac{v}{v_m} \right)^2 e^{-(v/v_m)^2}$$

$$\Delta N = n(v)\,\Delta v = \frac{4N_A}{\sqrt{\pi}} \frac{1}{v_m} \left(\frac{v}{v_m} \right)^2 e^{-(v/v_m)^2} (0.01 v_m)$$

$$= 1.36 \times 10^{22} (v/v_m)^2 e^{-(v/v_m)^2}$$

(b) $\Delta N = 1.36 \times 10^{22} (0)^2 e^{-0} = 0$

(c) $\Delta N = 1.36 \times 10^{22} (1)^2 e^{-1} = 5.00 \times 10^{21}$

(d) $\Delta N = 1.36 \times 10^{22} (2)^2 e^{-(2)^2} = 9.96 \times 10^{20}$

(e) $\Delta N = 1.36 \times 10^{22} (8)^2 e^{-(8)^2} = 1.369 \times 10^{-4}$ (or no molecules most of the time)

8-17. For hydrogen: $E_n = -\frac{mk^2e^4}{2\hbar^2}\frac{1}{n^2} = -\frac{13.605687}{n^2}eV$ using values of the constants accurate to six decimal places.

$E_1 = -13.605687eV$

$E_2 = -3.401422eV \qquad E_2 - E_1 = 10.204265eV$

$E_3 = -1.511743eV \qquad E_3 - E_1 = 12.093944eV$

(a) $\frac{n_2}{n_1} = \frac{g_2}{g_1}e^{-(E_2-E_1)/kT} = \frac{8}{2}e^{-10.20427/0.02586} = 4e^{-395} = 4\times10^{-172} \approx 0$

$\frac{n_3}{n_1} = \frac{g_3}{g_1}e^{-(E_3-E_1)/kT} = \frac{18}{2}e^{-12.09394/0.02586} = 9e^{-468} = 9\times10^{-203} \approx 0$

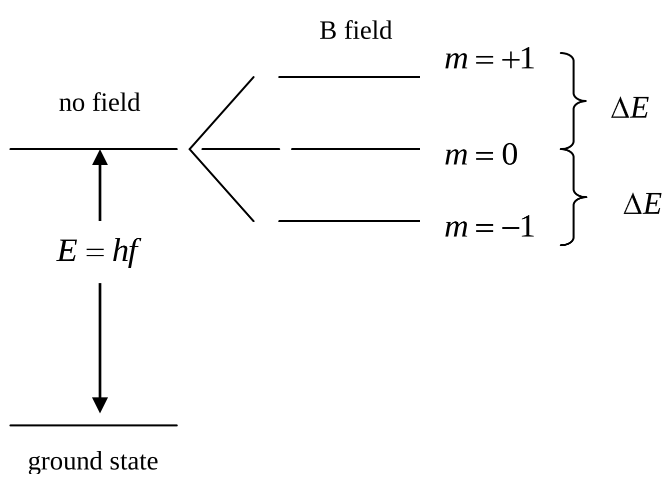
(b) $\frac{n_2}{n_1} = 0.01 = 4e^{-10.20427/kT} \quad \rightarrow \quad e^{-10.20427/kT} = 0.0025$

$-10.20427/kT = \ln(0.0025) = -5.99146$

$$T = \frac{10.20427eV}{(5.99146)(8.61734\times10^{-5}eV\bullet K)} = 19,760K$$

(c) $\frac{n_3}{n_1} = 9e^{-12.09394/(8.61734\times10^{-5})(19,760)} = 0.00742 = 0.7\%$

8-18.



Neglecting the spin, the 3*p* state is doubly degenerate: $\ell = 0,1$ hence, there are two *m* = 0 levels equally populated.

$E = hf = hc/\lambda = 1.8509eV \qquad \lambda = 670.79nm$

(Problem 8-18 continued)

$$\Delta E = \frac{e\hbar B}{2m_e} = 2.315\times 10^{-4}\,eV$$

(a) The fraction of atoms in each *m*-state relative to the ground state is: (Example 8-2)

$$\frac{n_{+1}}{n} = e^{-1.8511/0.02586} = e^{-71.58} = 10^{-31.09} = 8.18\times 10^{-32}$$

$$\frac{n_0}{n} = 2\times e^{-1.8509/0.02586} = 2e^{-71.57} = 2\times 10^{-31.08} = 1.64\times 10^{-31}$$

$$\frac{n_0}{n_{-1}} = e^{-1.8507/0.02586} = e^{-71.56} = 10^{-31.08} = 8.30\times 10^{-32}$$

(b) The brightest line with the B-field "on" will be the transition from the m = 0 level, the center line of the Zeeman spectrum. With that as the "standard", the relative intensities will be: $8.30/16.4/8.18 \quad \rightarrow \quad 0.51/1.00/0.50$

8-19. (a) $e^{-\alpha} = \dfrac{N}{V}\dfrac{h^3}{2\ \ 2\pi m_e kT^{\ \ 3/2}}$ (Equation 8-44)

$$\frac{N}{V} = e^{-\alpha}\,\frac{2\ \ 2\pi m_e kT^{\ \ 3/2}}{h^3} = e^{-\alpha}\,\frac{2\ \ 2\pi m_e c^2 kT^{\ \ 3/2}}{hc^{\ \ 3}}$$

$$= 1\times 2\,\frac{\left[2\pi\ \ 5.11\times 10^5 eV\ \ \ 2.585\times 10^{-2} eV\ \ \right]^{1/2}}{1240 eV\bullet nm^{\ \ 3}}\left(\frac{10^7\,nm}{1cm}\right)^3 = 2.51\times 10^{19}/cm^3$$

8-20. (a) $e^{-\alpha}\ \ O_2\ \ = \dfrac{N}{V}\dfrac{h^3}{2\pi MkT^{\ \ 3/2}}$ (Equation 8-44)

$$= \frac{N_A}{V_M}\frac{hc^{\ \ 3}}{2\pi Mc^2 kT^{\ \ 3/2}}$$

$$= \frac{6.022\times 10^{23}/mole}{22.4\times 10^3 cm^3/mole}\,\frac{1.24\times 10^{-4} eV\bullet cm^{\ \ 3}}{\left[2\pi\ \ 32uc^2\ \ \ 931.5\times 10^6 eV/u\ \ \ 8.617\times 10^{-5} eV/K\ \ \ 273K\ \ \right]}$$

$$= 1.75\times 10^{-7}$$

(Problem 8-20 continued)

(b) At temperature T, $e^{-\alpha}\left(O_2\right)=1.75\times10^{-7}\left(273\right)^{3/2}/T^{3/2}$

$$1=\left(1.75\times10^{-7}\right)\left(273K\right)^{3/2}/T^{3/2}\rightarrow T^{3/2}=\left(1.75\times10^{-7}\right)\left(273K\right)^{3/2}$$

$$\therefore T=\left(1.75\times10^{-7}\right)^{2/3}\left(273K\right)=8.5mK$$

8-21. Assuming the gasses are ideal gases, the pressure is given by: $P=\frac{2}{3}\frac{N\langle E\rangle}{V}$ for classical, FD, and BE particles. P_{FD} will be highest due to the exclusion principle, which, in effect, limits the volume available to each particle so that each strikes the walls more frequently than the classical particles. On the other hand, P_{BE} will be lowest, because the particles tend to be in the same state, which in effect, is like classical particles with a mutual attraction, so they strike the walls less frequently.

8-22. (a) $f_{BE}=\frac{1}{e^{\alpha}e^{E/kT}-1}$ For $\alpha=0$ and $f_{BE}=1$, at $T=5800K$

$$\frac{1}{e^{E/5800k}-1}=1\rightarrow e^{E/5800k}=2$$

$$E\Big/\left(5800K\right)\left(8.62\times10^{-5}eV/K\right)=\ln 2$$

$$E=0.347eV$$

(b) For $E=0.35V$, $\alpha=0$ and $f_{BE}=0.5$

$$\frac{1}{e^{0.35/kT}-1}=0.5\rightarrow e^{0.35/kT}=3$$

$$0.35eV\Big/\left(8.62\times10^{-5}eV/K\right)T=\ln 3$$

$$T=\frac{0.35eV}{\ln 3\left(8.62\times10^{-5}eV/K\right)}=3700K$$

8-23. $\lambda=\frac{h}{p}=\frac{h}{\sqrt{2m\langle E\rangle}}=\frac{h}{\sqrt{2m\left(3kT/2\right)}}=\frac{h}{\left(3mkT\right)^{1/2}}$

The distance between molecules in an ideal gas $\left(V/N\right)^{1/3}$ is found from

(Problem 8-23 continued)

$$PV = nRT = nRT\left(N_A/N_A\right) = NkT \rightarrow (V/N)^{1/3} = (kT/P)^{1/3}$$

and equating this to λ above, $(kT/P)^{1/3} = \dfrac{h}{(3mkT)^{1/2}}$

$\dfrac{kT}{P} = \dfrac{h^3}{(3mkT)^{3/2}}$ and solving for T, yields: $T^{5/2} = \dfrac{P}{k}\dfrac{h^3}{(3mk)^{3/2}}$

$$T = \left[\frac{Ph^3}{k(3mk)^{3/2}}\right]^{2/5} = \left[\frac{(101kPa)(6.63\times10^{-34}J\bullet s)^3}{3(2\times1.67\times10^{-27}kg)(1.38\times10^{-23}J/K)^{5/2}}\right]^{2/5} = 4.4K$$

8-24. $\dfrac{N_0}{N} \approx 1 - \left(\dfrac{T}{T_C}\right)^{3/2}$ (Equation 8-52)

(a) For $T = 3T_C/4 \rightarrow \dfrac{N_0}{N} \approx 1 - \left(\dfrac{3T}{4T_C}\right)^{3/2} = 0.351$

(b) For $T = T_C/2 \rightarrow \dfrac{N_0}{N} \approx 1 - \left(\dfrac{T}{2T_C}\right)^{3/2} = 0.646$

(c) For $T = T_C/4 \rightarrow \dfrac{N_0}{N} \approx 1 - \left(\dfrac{T}{4T_C}\right)^{3/2} = 0.875$

(d) For $T = T_C/8 \rightarrow \dfrac{N_0}{N} \approx 1 - \left(\dfrac{T}{8T_C}\right)^{3/2} = 0.956$

8-25. For small values of α, $e^{\alpha} = 1 + \alpha + (\alpha^2/2!) + \cdots$ and $N_0 = \dfrac{1}{e^{\alpha}-1} \rightarrow N_0(e^{\alpha}-1) = 1$

which for small α values becomes: $N_0(1+\alpha+\cdots-1) = N_0\alpha = 1$ or $N_0 = \dfrac{1}{\alpha}$

8-26. $T_C = \dfrac{h^2}{2mk}\left[\dfrac{N}{2\pi(2.315)V}\right]^{2/3}$ (Equation 8-48)

The density of liquid *Ne* is 1.207 g/cm^3, so

(Problem 8-26 continued)

$$\frac{N}{V} = \frac{(1.207\,g/cm^3)(6.022\times10^{23}\,molecules/mol)(10^6\,cm^2/m^3)}{20.18\,g/mol} = 3.601\times10^{28}/m^3$$

$$T = \frac{(6.626\times10^{-34}\,J\bullet s)^2}{2(20u\times1.66\times10^{-27}\,kg/u)(.381\times10^{-23}\,J/K)}\left[\frac{3.601\times10^{28}\,m^3}{2\pi(2.315)}\right]^{2/3} = 0.895K$$

Thus, T_C at which ^{20}Ne would become a superfluid is much lower than its freezing temperature of 24.5*K*.

8-27. Power per unit area *R* arriving at Earth is given by the Stefan-Boltzmann law: $R = \sigma T^4$ where σ is Stefan's constant. For a 5% decrease in the Sun's temperature,

$$\frac{R(T) - R(0.95T)}{R(T)} = \frac{\sigma T^4 - \sigma(0.95T)^4}{\sigma T^4} = 1 - (0.95)^4 = 0.186$$, or a decrease of 18.6%.

8-28. $\langle E\rangle = \dfrac{hf}{e^{hf/kT} - 1}$ (Equation 8-60)

(a) For $T = 10hf/k$; $hf = kT/10 \rightarrow \langle E\rangle = \dfrac{hf}{e^{1/10} - 1} = \dfrac{kT/10}{0.1051} = 0.951kT$

(b) For $T = hf/k$; $hf = kT \rightarrow \langle E\rangle = \dfrac{hf}{e^1 - 1} = \dfrac{kT}{1.718} = 0.582kT$

(c) For $T = 0.1hf/k$; $hf = 10kT \rightarrow \langle E\rangle = \dfrac{hf}{e^{10} - 1} = \dfrac{10kT}{2.20\times10^4} = 4.54\times10^{-4}kT$

According to equipartition $\langle E\rangle = kT$ in each case.

8-29. $C_V = 3N_A k\left(\dfrac{hf}{kT}\right)^2 \dfrac{e^{hf/kT}}{(e^{hf/kT} - 1)^2}$ As $T \rightarrow \infty$, hf/kT gets small and

$e^{hf/kT} \approx 1 + hf/kT + \cdots$

$$C_V = 3N_A k\left(\frac{hf}{kT}\right)^2 \frac{1 + hf/kT + \cdots}{(hf/kT)^2} \approx 3N_A k = 3N_A(R/N_A) = 3R$$

The rule of Dulong and Petit.

8-30. $C_V = 3R\left(\frac{hf}{kT}\right)^2 \frac{e^{hf/kT}}{\left(e^{hf/kT}-1\right)^2}$ (Equation 8-62)

Writing $hf/kT = Af$ where $A = h/kT = 2.40\times10^{-13}$ when $T = 200K$,

$$C_V = 3R\,(Af)^2 \frac{e^{Af}}{\left(e^{Af}\right)^2 - 2e^{Af} + 1} = eR\,(Af)^2 \frac{1}{e^{Af} - 2 + \left(1/e^{Af}\right)}$$

Because Af is "large", $1/e^{Af} \approx 0$ and e^{Af} dominates $(Af)^2$, so

$C_V \approx 3R/e^{Af} \rightarrow e^{Af} \approx 3R/C_V \rightarrow f \approx \ln\left(3R/C_V\right)(1/A)$

For Al, $C_V(200K) = 20.1J/K{\bullet}mol$ (From Figure 8-13)

$$f = \ln\left(\frac{3(8.31)}{20.1}\right)\left(1/2.40\times10^{-13}\right) = 8.97\times10^{11}\,Hz$$

For Si, $C_V(200K) = 13.8J/K{\bullet}mol$ (From Figure 8-13)

$$f = \ln\left(\frac{3(8.31)}{13.8}\right)\left(1/2.40\times10^{-13}\right) = 2.46\times10^{12}\,Hz$$

8-31. $C_V = 3R\left(\frac{hf}{kT}\right)^2 \frac{e^{hf/kT}}{\left(e^{hf/kT}-1\right)^2}$ (Equation 8-62)

At the Einstein temperature $T_E = hf/k$,

$$C_V = 3R(1)^2 \frac{e^1}{\left(e^1-1\right)^2} = 3R(0.9207) = 3(8.31J/K{\bullet}mol)(0.9207)$$

$$= 22.95K/K{\bullet}mol = 5.48cal/K{\bullet}mol$$

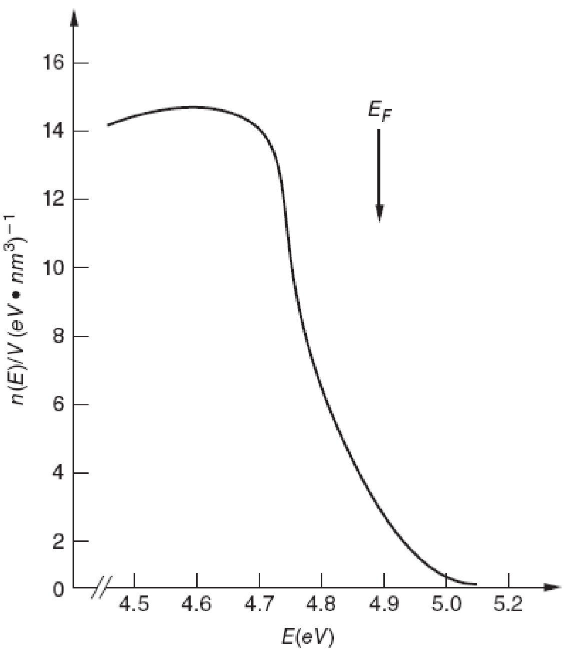
8-32. Rewriting Equation 8-69 as $\frac{n(E)}{V} = \frac{\pi}{2}\left[\frac{8mc^2}{(hc)^2}\right]^{3/2} \frac{E^{1/2}}{e^{(E-E_F)/kT}+1}$

Set up the equation on a spreadsheet whose initial and final columns are $E(eV)$ and $n(E)/V\ (eV{\bullet}nm^3)^{-1}$, respectively.

(Problem 8-32 continued)

$E\ \ eV$	$n\ \ E\ \ /V\ \ \ eV \bullet nm^3\ \ ^{-1}$
4.5	14.4
4.6	14.6
4.7	14.5
4.8 (=E_F)	7.46
4.9	0.306
5	0.0065
5.1	0.00014

The graph of these values is below.



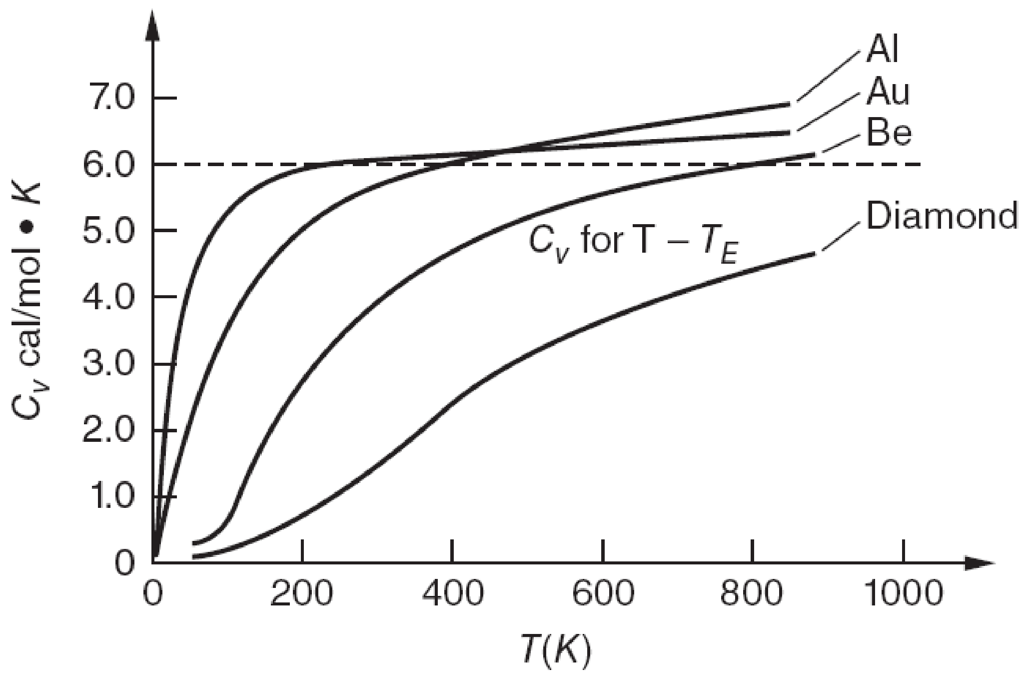
From the graph, about 0.37 $electrons/nm^3$ or $3.7 \times 10^{26} electrons / m^3$ within $0.1 eV$ below E_F have been excited to levels above E_F.

8-33. The photon gas has the most states available, since any number of photons may be in the ground state. In contrast, at $T = 1K$ the electron gas's available states are limited to those within about $2kT = 2\left(8.62\times10^{-5}eV\bullet K\right)\left(1K\right) = 1.72\times10^{-4}eV$ of the Fermi level. All other states are either filled, hence unavailable, or higher than kT above the Fermi level, hence not accessible.

8-34. From the graph.

$$T_E\left(Au\right) = 136K \quad T_E\left(Al\right) = 243K$$

$$T_E\left(Be\right) = 575K \quad T_E\left(Diamond\right) = \text{ off the graph (well over 1000K)}$$



8-35. Approximating the nuclear potential with an infinite square well and ignoring the Coulomb repulsion of the protons, the energy levels for both protons and neutrons are given by $E_n = \left(n^2h^2\right)\big/\left(8mL^2\right)$ and six levels will be occupied in ^{22}Ne, five levels with 10 protons and six levels with 12 neutrons.

$$E_F\left(\text{protons}\right) = \frac{\left(5\right)^2\left(1240MeV\bullet fm\right)^2}{8\left(1.0078u\times931.5MeV/u\right)\left(3.15fm\right)^2} = 516MeV$$

(Problem 8-35 continued)

$$E_F(\text{neutrons}) = \frac{(6)^2(1240MeV\bullet fm)^2}{8(1.0087u\times 931.5MeV/u)(3.15fm)^2} = 742MeV$$

$$\langle E\rangle(\text{protons}) = (3/5)E_F = 310MeV$$

$$\langle E\rangle(\text{neutrons}) = (3/5)E_F = 445MeV$$

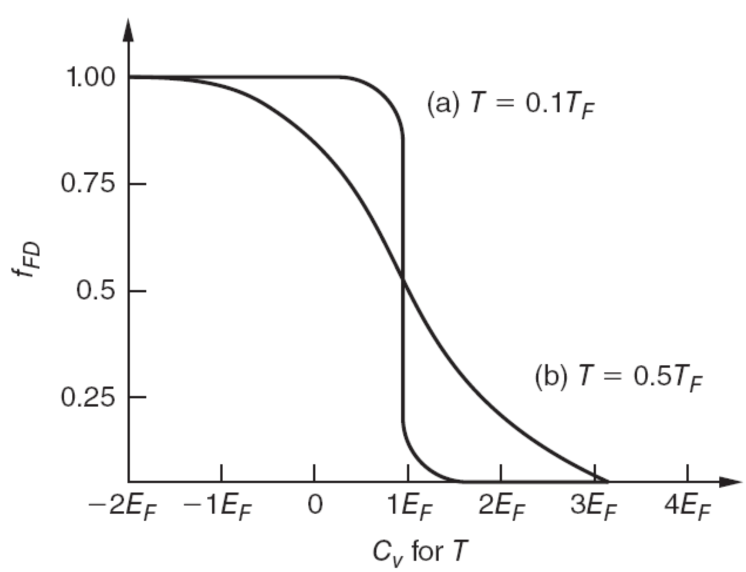
As we will discover in Chapter 11, these estimates are nearly an order of magnitude too large. The number of particles is not a large sample.

8-36. $E_1 = h^2/8mL^2$. All 10 bosons can be in this level, so $E_1(\text{total}) = 10h^2/8mL^2$.

8-37. (a) $f_{FD}(E) = \frac{1}{e^{(E-E_F)/kT}+1}$ (Equation 8-68)

$$= \frac{1}{e^{(E-E_F)/0.1E_F}+1} = \frac{1}{e^{10(E-E_F)/E_F}+1}$$

(b) $f_{FD}(E) = \frac{1}{e^{(E-E_F)/0.5E_F}+1} = \frac{1}{e^{2(E-E_F)/E_F}+1}$



8-38. $\frac{N_O}{N} \approx 1 - \left(\frac{T}{T_c}\right)^{3/2}$ (Equation 8-52)

(a) $\frac{N_O}{N} \approx 1 - \left(\frac{T_c/2}{T_c}\right)^{3/2} = 1 - \left(\frac{1}{2}\right)^{3/2} = 0.646$

(b) $\frac{N_O}{N} \approx 1 - \left(\frac{T_c/4}{T_c}\right)^{3/2} = 1 - \left(\frac{1}{4}\right)^{3/2} = 0.875$

8-39. For a one-dimensional well approximation, $E_n = n^2h^2 \big/ 8mL^2$. At the Fermi level E_F, $n = N/2$, where N = number of electrons.

$$E_F = \frac{(N/2)^2 h^2}{8mL^2} = \frac{h^2}{32m}\left(\frac{N}{L}\right)^2$$ where N/L = number of electrons/unit length,

i.e., the density of electrons. Assuming 1 free electron/*Au* atom,

$$\frac{N}{L} = \left[\frac{N_A\rho}{M}\right]^{1/3} = \left[\frac{(6.02\times10^{23}\,electrons/mol)(19.32\,g/cm^3)(10^2\,cm/m)^3}{197\,g/mol}\right]^{1/3} = 3.81\times10^9\,m^{-1}$$

$$E_F = \frac{(6.63\times10^{-34}\,J\bullet s)^2(3.81\times10^9\,m^{-1})^2}{32(9.11\times10^{-31}\,kg)(1.602\times10^{-19}\,J/eV)} = 1.37eV$$

This is the energy of an electron in the Fermi level above the bottom of the well. Adding the work function to such an electron just removes it from the metal, so the well is $1.37eV + 4.8eV = 6.2eV$ deep.

8-40. (a) At $T = 850K$ $v_m = \left(\frac{2kT}{m}\right)^{1/2} = \left[\frac{2(1.3807\times10^{-23}\,J/K)(850K)}{6.94\times10^{-25}\,kg}\right]^{1/2} = 183.9m/s$

$$\langle v\rangle = \left(\frac{8kT}{\pi m}\right)^{1/2} = \left(\frac{4}{\pi}\right)^{1/2} v_m = 207.5m/s$$

$$v_{rms} = \left(\frac{3kT}{m}\right)^{1/2} = \left(\frac{3}{2}\right)^{1/2} v_m = 225.2m/s$$

(Problem 8-40 continued)

The times for molecules with each of these speeds to travel across the 10*cm* diameter of the rotating drum is:

$$t\left(v_m\right) = \frac{0.10m}{183.9m/s} = 5.44\times10^{-4}s$$

$$t\left(\langle v\rangle\right) = \frac{0.10m}{207.5m/s} = 4.82\times10^{-4}s$$

$$t\left(v_{rms}\right) = \frac{0.10m}{225.2m/s} = 4.44\times10^{-4}s$$

The drum is rotating at 6250*rev/min* = 104.2*rev/s* or $9.600\times10^{-3}s/rev$. The fraction of a revolution made by the drum while molecules with each of these three speeds are crossing the diameter is:

$$\text{for } v_m: \quad \frac{5.44\times10^{-4}s}{9.600\times10^{-3}s/rev} = 0.05667rev$$

$$\text{for } \langle \text{v}\rangle: \quad \frac{4.82\times10^{-4}s}{9.600\times10^{-3}s/rev} = 0.05021rev$$

$$\text{for } v_{rms}: \quad \frac{4.44\times10^{-4}s}{9.600\times10^{-3}s/rev} = 0.04625rev$$

Assuming that point *A* is directly opposite the slit s_2 when the first (and fastest) molecules enter the drum, molecules with each of the three speeds will strike the plate at the following distances from *A*: (The circumference of the drum $C = 0.10\pi m$.)

$$v_m: \quad 0.05667rev\times0.314159m/rev = 0.01780m = 1.780cm$$

$$\langle v\rangle: \quad 0.05021rev\times0.314159m/rev = 0.01577m = 1.577cm$$

$$v_{rms}: \quad 0.04625rev\times0.314159m/rev = 0.01453m = 1.453cm$$

(b) Correction is necessary because faster molecules in the oven will approach the oven's exit slit more often than slower molecules, so the speed distribution in the exit beam is slightly skewed toward higher speeds.

(c) No. The mean speed of N_2 molecules at 850*K* is 710.5*m/s*, since they have a smaller mass than Bi_2 molecules. Repeating for them the calculations in part (a), N_2 molecules moving at v_m would strike the plate only 0.4*cm* from *A*. Molecules moving at $\langle v\rangle$ and v_{rms} would be even closer to *A*.

8-41. $\left\langle E_{K(\text{escape})}\right\rangle = \dfrac{1}{2}m\left\langle v^2_{\text{escape}}\right\rangle = \int_0^\infty \left(\dfrac{1}{2}mv^2\right)F\;\;v\;\;dv$

$$= \frac{\int_0^\infty \left(\frac{1}{2}mv^2\right)v^3 e^{-mv^2/2kT}dv}{\int_0^\infty v^3 e^{-mv^2/2kT}dv} = \frac{1}{2}m\frac{I_5}{I_3}$$

$$= \frac{1}{2}m\frac{\lambda^{-3}}{\lambda^{-2}/2} = \frac{m}{\lambda} = m\frac{2kT}{m} = 2kT \qquad \text{where } \lambda = \frac{m}{2kT}$$

8-42. (a) $f\;\;u\;\;du = Ce^{-E/kT}du = Ce^{-Au^2/kT}du$ (from Equation 8-5)

$$1 = \int_{-\infty}^{+\infty} f\;\;u\;\;du = \int_{-\infty}^{+\infty} Ce^{-Au^2/kT}du = 2C\int_{-\infty}^{+\infty} e^{-Au^2/kT}du$$

$$= 2CI_0 = 2C\sqrt{\pi}\;\lambda^{-1/2}/2 \quad \text{where } \lambda = A/kT$$

$$= C\sqrt{\pi}\sqrt{kT/A} \rightarrow C = \sqrt{A/\pi kT}$$

(b) $\langle E\rangle = \left\langle Au^2\right\rangle = \int_{-\infty}^{+\infty} Au^2 f\;\;u\;\;du = \int_{-\infty}^{+\infty} Au^2\sqrt{A/\pi kT}\,e^{-Au^2/kT}du$

$$= A\sqrt{A/\pi kT}\;\;2I_2\;\; = A\sqrt{A/\pi kT}\,2\times\;\sqrt{\pi}\;/4\;\;\lambda^{-3/2} \quad \text{where } \lambda = A/kT$$

$$= \frac{1}{2}A\sqrt{A/kT}\;\;kT/A\;\;^{3/2} = \frac{1}{2}kT$$

8-43. $f\;\;v_x\;\; = \;\;m/2\pi kT\;\;^{1/2}\,e^{-mv_x^2/2kT}$ (from Equation 8-6)

$$\left\langle |v_x|\right\rangle = \int_{-\infty}^{+\infty}|v_x|\,f\;\;v_x\;\;dv_x$$

$$= \int_{-\infty}^{0} -v_x\;\;\;m/2\pi kT\;\;^{1/2}\,e^{-mv_x^2/2kT}dv_x + \int_0^{+\infty} v_x\;\;m/2\pi kT\;\;^{1/2}\,e^{-mv_x^2/2kT}dv_x$$

$$= 2\int_0^\infty \;m/2\pi kT\;\;^{1/2}\,v_x e^{-mv_x^2/2kT}dv_x$$

$$= 2\;\;m/2\pi kT\;\;^{1/2}\,I_1 \quad \text{with } \lambda = m/2kT$$

(Problem 8-43 continued)

$$= 2\left(m/2\pi kT\right)^{1/2} \times 1/2 \times \frac{2kT}{m} = \sqrt{\frac{2kT}{\pi m}}$$

8-44. $f_{FD} = \dfrac{1}{e^{\alpha}e^{E/kT}+1} = \dfrac{1}{e^{\left(E-E_F\right)/kT}+1}$ where $\alpha = -\dfrac{E_F}{kT}$

For $E \gg E_F$, $e^{\left(E-E_F\right)/kT} \gg 1$ and

$$f_{FD} \approx \frac{1}{e^{\left(E-E_F\right)/kT}} = \frac{1}{e^{\left(-E_F\right)/kT} \times e^{E/kT}} = \frac{1}{e^{\alpha}e^{E/kT}} = f_B$$

8-45. $N = e^{-\alpha}\dfrac{4\pi\left(2m_e\right)^{3/2}V}{h^3}\displaystyle\int_0^{\infty} E^{1/2}e^{-E/kT}dE$ (Equation 8-43)

Considering the integral, we change the variable: $E/kT = u^2$, then

$E = kTu^2$, $E^{1/2} = \left(kT\right)^{1/2}u$, and $dE = kT\left(2u\right)du$. So,

$$\int_0^{\infty} E^{1/2}e^{-E/kT}dE = 2\left(kT\right)^{3/2}\int_0^{\infty} u^2 e^{-u^2}du$$

The value of the integral (from tables) is $\sqrt{\pi}/4$, so

$N = e^{-\alpha}\dfrac{4\pi\left(2m_e\right)^{3/2}V}{h^3}\dfrac{2\left(kT\right)^{3/2}\sqrt{\pi}}{4}$ or $e^{\alpha} = \dfrac{2\left(2m_e\pi kT\right)^{3/2}V}{Nh^3}$, which is Equation 8-44.

8-46. (a) $N = \sum_i n_i = f_0\left(E_0\right) + f_1\left(E_1\right)$ (with $g_0 = g_1 = 1$)

$$= Ce^{0} + Ce^{-\varepsilon/kT} = C\left(1+e^{-\varepsilon/kT}\right)$$

So, $C = \dfrac{N}{1+e^{-\varepsilon/kT}}$

(b) $\langle E\rangle = \dfrac{0\bullet n_0 + \varepsilon n_1}{N} = \dfrac{\varepsilon C e^{-\varepsilon/kT}}{N} = \dfrac{N\varepsilon e^{-\varepsilon/kT}}{\left(1+e^{-\varepsilon/kT}\right)N} = \dfrac{\varepsilon e^{-\varepsilon/kT}}{1+e^{-\varepsilon/kT}}$

(Problem 8-46 continued)

As $T \to 0$, $\quad e^{-\varepsilon/kT} = 1/e^{-\varepsilon/kT} \to 0$, so $\langle E\rangle \to 0$

As $T \to \infty$, $\quad e^{-\varepsilon/kT} = 1/e^{-\varepsilon/kT} \to 0$, so $\langle E\rangle \to \varepsilon/2$

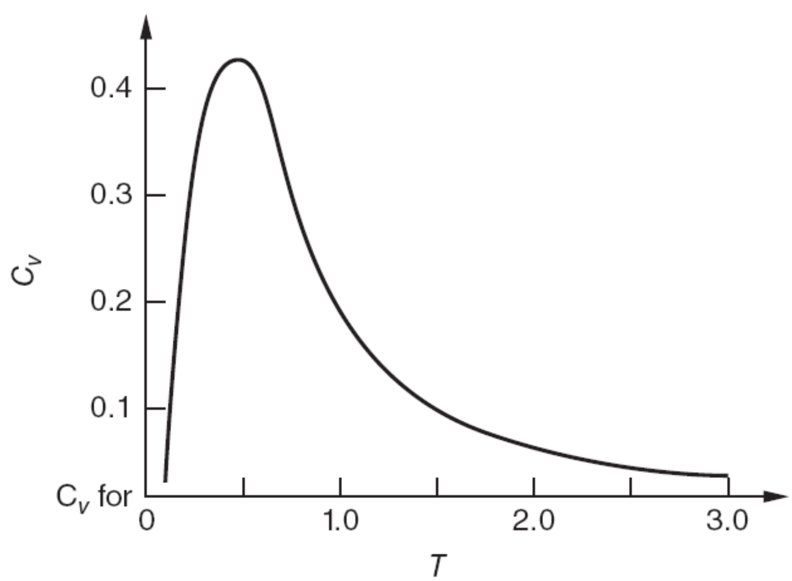
(c) $$C_V = \frac{dE}{dT} = \frac{d\left(N\langle E\rangle\right)}{dT} = \frac{d}{dT}\left(\frac{N\varepsilon e^{-\varepsilon/kT}}{1+e^{-\varepsilon/kT}}\right)$$

$$= \frac{N\varepsilon^2}{kT^2}\left[\frac{-\left(e^{-\varepsilon/kT}\right)^2}{\left(1+e^{-\varepsilon/kT}\right)^2} + \frac{e^{-\varepsilon/kT}}{1+e^{-\varepsilon/kT}}\right]$$

$$= Nk\left(\frac{\varepsilon}{kT}\right)^2 \frac{e^{-\varepsilon/kT}}{\left(1+e^{-\varepsilon/kT}\right)^2}$$

(d)

$T\ (\times \varepsilon/k)^{-1}$	0.1	0.25	0.5	1.0	2.0	3.0
$C_V\ (\times Nk)$	0.005	0.28	0.42	0.20	0.06	0.03



8-47. (a) $$p = \hbar\left[k_x^2 + k_y^2 + k_z^2\right]^{1/2} = \hbar\left[\left(\frac{n_1\pi}{L}\right)^2 + \left(\frac{n_2\pi}{L}\right)^2 + \left(\frac{n_3\pi}{L}\right)^2\right]^{1/2}$$

$$= \frac{\hbar\pi}{L}\left[n_1^2 + n_2^2 + n_3^2\right]^{1/2} = \frac{\hbar\pi N}{L}$$

(Problem 8-47 continued)

$$E = pc = \frac{\hbar c \pi N}{L}$$

(b) Considering the space whose axes are $n_1, n_2,$ and n_3. The points in space correspond to all possible integer values of $n_1, n_2,$ and n_3, all of which are located in the all positive octant. Each state has unit volume associated with it. Those states between N and $N + dN$ lie in a spherical shell of the octant whose radius is N and whose thickness is dN. Its volume is $(1/8)(4\pi N^2 dN)$. Because photons can have two polarizations (spin directions), the number of possible state is $2\times(1/8)(4\pi N^2 dN) = \pi N^2 dN.$

(c) This number of photon states has energy between E and $E+dE$, where $N = EL/\pi\hbar c.$ The density of states $g(E)$ is thus:

$$g(E)\,dE = \text{number of photon states at } E \in dE$$
$$= \text{number of photon states at } N \in dN$$
$$= \pi N^2 dN$$
$$= \pi\left(EL/\pi\hbar c\right)^2\left(L/\pi\hbar c\right)dE$$
$$= \frac{8\pi L^3}{(2\pi\hbar c)^3}E^2 dE = \frac{8\pi L^3}{(hc)^3}E^2 dE$$

The probability that a photon exists in a state is given by:

$$f_{BE}(E) = \frac{1}{e^{\alpha}e^{E/kT}-1} = \frac{1}{e^{E/kT}-1} \qquad \text{(Equation 8-24)}$$

The number of photons with energy between E and $E+dE$ is then:

$$n(E)\,dE = f_{BE}(E)\,g(E)\,dE = \frac{8\pi(L/hc)^3E^2dE}{e^{E/kT}-1}$$

(d) The number of photons per unit volume within this energy range is $n(E)\,dE/L^3.$ Because each photon has energy E, the energy density for photons is:

$$u(E)\,dE = E\bullet n(E)\,dE/L^3 = \frac{8\pi E^3 dE}{(hc)^3\left(e^{E/kT}-1\right)}$$

which is also the density of photons with wavelength between λ and $\lambda+d\lambda$, where

(Problem 8-47 continued)

$\lambda = hc/E \rightarrow E = hc/\lambda$. So,

$$d\lambda = \left|\frac{d\lambda}{dE}\right| dE = \frac{hc}{E^2} dE = \frac{\lambda^2}{hc} dE \rightarrow dE = \frac{hc}{\lambda^2} d\lambda$$

$$u(\lambda)\,d\lambda = u(E)\,dE = \frac{8\pi\,(hc/\lambda)^3\,(hc/\lambda^2)\,d\lambda}{(hc)^3\left(e^{hc/\lambda kT}-1\right)} = \frac{8\pi hc\lambda^{-3}d\lambda}{e^{hc/\lambda kT}-1}$$

Chapter 9 – Molecular Structure and Spectra

9-1. (a) $$1\frac{eV}{molecule}=\left(1\frac{eV}{molecule}\right)\left(\frac{1.609\times10^{-19}J}{eV}\right)\left(\frac{6.022\times10^{23}molecules}{mole}\right)$$

$$=\left(96472\frac{J}{mole}\right)\left(\frac{1cal}{4.184J}\right)=23057\frac{cal}{mole}=23.06\frac{kcal}{mole}$$

(b) $$E_d=\left(4.27\frac{eV}{molecule}\right)\left(\frac{23.06kcal/mole}{1eV/molecule}\right)=98.5kcal/mole$$

(c) $$E_d=\left(106\frac{eV}{molecule}\right)\left(\frac{1eV/molecule}{96.47kJ/mole}\right)=1.08eV/molecule$$

9-2. Dissociation energy of $NaCl$ is $4.27eV$, which is the energy released when the $NaCl$ molecule is formed from neutral Na and Cl atoms. Because this is more than enough energy to dissociate a Cl_2 molecule, the reaction is exothermic. The net energy release is $4.27eV - 2.48eV = 1.79eV$.

9-3. From Cs to F: $3.89eV - 3.40eV = 0.49eV$

From Li to I: $5.39eV - 3.06eV = 2.33eV$

From Rb to Br: $4.18eV - 3.36eV = 0.82eV$

9-4. $$E_d=\left|U_C\right|=-\frac{ke^2}{r_0}+E_{ion}$$

$$CsI:\quad -\frac{ke^2}{r_0}+E_{ion}=-\frac{1.440eV\bullet nm}{0.337nm}+\left(3.89eV-3.06eV\right)\rightarrow E_d=3.44eV$$

$$NaF:\quad -\frac{ke^2}{r_0}+E_{ion}=-\frac{1.440eV\bullet nm}{0.193nm}+\left(5.14eV-3.40eV\right)\rightarrow E_d=5.72eV$$

$$LiI:\quad -\frac{ke^2}{r_0}+E_{ion}=-\frac{1.440eV\bullet nm}{0.238nm}+\left(5.39eV-3.06eV\right)\rightarrow E_d=3.72eV$$

While E_d for CsI is very close to the experimental value, the other two are both high. Exclusion principle repulsion was ignored.

9-5. (a) Total potential energy: $U(r) = -\dfrac{ke^2}{r} + E_{ex} + E_{ion}$ (Equation 9-1)

attractive part of $U(r_0) = -\dfrac{ke^2}{r_0} = -\dfrac{1.44eV{\bullet}nm}{0.267nm} = -5.39eV$

(b) The net ionization energy is:

$$E_{ion} = (\text{ionization energy of } Rb) - (\text{electron affinity of } Cl)$$
$$= 4.18eV - 3.62eV = 0.56eV$$

Neglecting the exclusion principle repulsion energy E_{ex},

dissociation energy $= -U(r_0) = 5.39eV - 0.56eV = 4.83eV$

(c) Including exclusion principle repulsion,

dissociation energy $= 4.37eV - U(r_0) = 5.39eV - 0.56eV - E_{ex}$

$$E_{ex} = 5.39eV - 4.37eV - 0.56eV = 0.46eV$$

9-6. $U_c = -\dfrac{ke^2}{r_0} + E_{ion} = -\dfrac{1.440eV{\bullet}nm}{0.282nm} + (4.34eV - 3.36eV) = -4.13eV$

The dissociation energy is 3.94*eV*.

$$E_d = |U_c + E_{ex}| = 3.94eV = |-4.13eV + E_{ex}|$$

$E_{ex} = 0.19eV$ at $r_0 = 0.282nm$

9-7. $E_{ex} = \dfrac{A}{r^n}$ (Equation 9-2) $0.19eV = \dfrac{A}{(0.282nm)^n}$

At r_0 the net force on each ion is zero, so we have (from Example 9-2)

$$\frac{U_c(r_0)}{r_0} = \frac{ke^2}{r_0^2} = 18.11eV/nm = \frac{nA}{r_0^{n+1}} = \frac{n}{r_0} \times \frac{A}{r_0^n} = \frac{n}{r_0}(0.19eV)$$

$$n = \frac{(18.11eV/nm)(0.282nm)}{0.19eV} = 26.9 \approx 27$$

$$A = E_{ex} r_0^n = (0.19eV)(0.282nm)^{27} = 2.73 \times 10^{-16} eV{\bullet}nm^{27}$$

9-8. $E_d = 3.81eV$ per molecule of $NaBr$ (from Table 9-2)

$$1eV/molecule = (1eV/molecule)(1.609\times10^{-19}\,J/eV)\times$$

$$(6.02\times10^{23}\,molecules/mol)/(1cal/4.186J) = 23.0kcal/mol$$

$$E_d(NaBr) = (3.81eV/molecule)(23.0kcal/mol)/(1eV/molecule) = 87.6kcal/mol$$

9-9. For KBr: $U_C = \dfrac{1.440eV\bullet nm}{0.282nm} + (4.34eV - 3.36eV) = -4.13eV$

$$E_d = 3.94eV = |U_C + E_{ex}| = |-4.13eV + E_{ex}|$$

$$E_{ex} = 0.19eV$$

For $RbCl$: $U_C = \dfrac{1.440eV\bullet nm}{0.279nm} + (4.18eV - 3.62eV) = -4.60eV$

$$E_d = 4.37eV = |U_C + E_{ex}| = |-4.60eV + E_{ex}|$$

$$E_{ex} = 0.23eV$$

9-10. H_2S, H_2Te, H_3P, H_3Sb

9-11. (a) KCl should exhibit ionic bonding.

(b) O_2 should exhibit covalent bonding.

(c) CH_4 should exhibit covalent bonding.

9-12. Dipole moment $p_{ionic} = er_0$ (Equation 9-3)

$$= (1.609\times10^{-19}C)(0.0917nm)$$

$$= 1.47\times10^{-20}C\bullet nm\times10^{-9}m/nm$$

$$= 1.47\times10^{-29}C\bullet m$$

if the HF molecule were a pure ionic bond. The measured value is $6.64\times10^{-29}C\bullet m$, so the HF bond is $(6.40\times10^{-30}C\bullet m)/(1.47\times10^{-29}C\bullet m) = 0.44$ or 44% ionic.

9-13. $p_{ionic} = er_0 = (1.609\times10^{-19}C)(0.2345\times10^{-9}m)$ (Equation 9-3)

$= 3.757\times10^{-29}C\bullet m,$ if purely ionic.

The measured value should be:

$$p_{ionic}(measured) = 0.70p_{ionic} = 0.70(3.757\times10^{-29}C\bullet m) = 2.630\times10^{-29}C\bullet m$$

9-14. $p_{ionic} = er_0 = (1.609\times10^{-19}C)(0.193\times10^{-9}m)$ (Equation 9-3)

$= 3.09\times10^{-29}C\bullet m$

The measured values is $2.67\times10^{-29}C\bullet m$, so the BaO bond is

$$(2.67\times10^{-29}C\bullet m)/(3.09\times10^{-29}C\bullet m) = 0.86 \text{ or } 86\% \text{ ionic.}$$

9-15. Silicon, germanium, tin, and lead have the same outer shell configuration as carbon. Silicon and germanium have the same hybrid bonding as carbon (their crystal structure is diamond, like carbon); however, tin and lead are metallic bonded. (See Chapter 10.)

9-16. $\mathbf{p} = \mathbf{p_1} + \mathbf{p_2}$ and $p = 6.46\times10^{-30}C\bullet m$ and $p = p_1\cos 52.25° + p_2\cos 52.25°$

If bonding were ionic, $p_{ionic} = er_0 = (1.609\times10^{-19}C)(0.0956\times10^{-9}m) = 1.532\times10^{-29}C\bullet m$

$$p_1(actual) = p/2(\cos 52.25°) = 6.46\times10^{-30}C\bullet m/2(\cos 52.25°) = 5.276\times10^{-30}C\bullet m$$

Ionic fraction = fraction of charge transferred = $\dfrac{5.276\times10^{-30}C\bullet m}{1.532\times10^{-29}C\bullet m} = 0.34$ or 34%

9-17. $U = \alpha k^2 p_1^2 / r^2$ (Equation 9-10)

(a) Kinetic energy of $N_2 = 0.026eV$, so when $|U| = 0.026eV$ the bond will be broken.

$$0.026eV = \frac{(1.1\times10^{-37}m\bullet C^2/N)(9\times10^9 N\bullet m^2/C^2)^2(6.46\times10^{-30}C\bullet m)^2}{r^6}$$

$$r^6 = \frac{(1.1\times10^{-37}m\bullet C^2/N)(9\times10^9 N\bullet m^2/C^2)^2(6.46\times10^{-30}C\bullet m)^2}{0.026eV(1.60\times10^{-19}J/eV)} = 8.94\times10^{-56}m^6$$

$$r = 6.7\times10^{-10}m = 0.67nm$$

(Problem 9-17 continued)

(b) $U \approx \frac{ke^2}{r} \rightarrow |U| = 0.026eV = \frac{1.440eV \bullet nm}{r} \rightarrow r \approx 55nm$

(c) *H_2O-Ne* bonds in the atmosphere would be very unlikely. The individual molecules will, on average, be about 4*nm* apart, but if a *H_2O-Ne* molecule should form, its $U \approx 0.003eV$ at $r = 0.95nm$, a typical (large) separation. Thus, a N_2 molecule with the average kinetic energy could easily dissociate the *H_2O-Ne* bond.

9-18. (a) $\Delta E = 0.3eV = hc/\lambda = 1240eV \bullet nm/\lambda \rightarrow \lambda = 1240eV \bullet nm/0.3eV = 4.13 \times 10^3 nm$

(b) Infrared

(c) The infrared is absorbed causing increased molecular vibrations (heat) long before it gets to the DNA.

9-19. (a) *NaCl* is polar. The Na^+ ion is the positive charge center, the Cl^- ion is the negative charge center.

(b) O_2 is nonpolar. The covalent bond involves no separation of charges, hence no polarization of the molecule.

9-20. For N_2 $\quad E_0 r = 2.48 \times 10^{-4} eV = \hbar^2 / 2I$ where $I = \frac{1}{2} m r_0^2$ and $m = 14.0067u$

$$2.48 \times 10^{-4} eV (2I) = \hbar^2$$

$$r_0^2 = \frac{\hbar^2}{(2.48 \times 10^{-4} eV)(14.0067u)}$$

$$r_0 = \left[\frac{(1.055 \times 10^{-34} J \bullet s)^2}{(2.48 \times 10^{-4} eV)(1.60 \times 10^{-19} J/eV)(14.0067u)(1.66 \times 10^{-27} kg/u)} \right]^{1/2}$$

$$= 1.61 \times 10^{-10} m = 0.161nm$$

9-21. $E_{0r} = \dfrac{\hbar^2}{2I}$ (Equation 9-14) where $I = \dfrac{1}{2}mr_0^2$ for a symmetric molecule.

$$E_{0r} = \frac{\hbar^2}{mr_0^2} = \frac{(\hbar c)^2}{mc^2 r_0^2} = \frac{(197.3eV \bullet nm)^2}{(16uc)^2 (931.5\times10^6 eV/uc^2)(0.121nm)^2} = 1.78\times10^{-4}eV$$

9-22. For Co: $f = 6.42\times10^{13}Hz$ (See Example 9-6)

$E_V = (v + 1/2)hf$ (Equation 9-20)

(a) $E_1 - E_0 = 3hf/2 - hf/2 = hf$

$$= (4.14\times10^{-15}eV\bullet s)(6.42\times10^{13}Hz)$$

$$= 0.27eV$$

(b) $\dfrac{n_1}{n_0} = e^{-(E_1-E_0)/kT}$ (from Equation 8-2)

$$0.01 = e^{-(0.27)/(8.62\times10^{-5})T}$$

$$\ln(0.01) = -(0.27eV)/(8.62\times10^{-5}eV/K)T$$

$$T = \frac{-(0.27eV)}{\ln(0.01)(8.62\times10^{-5}eV/K)}$$

$$T = 680K$$

9-23. For LiH: $f = 4.22\times10^{13}Hz$ (from Table 9-7)

(a) $E_V = (v+1/2)hf = E_0 = hf/2 = (4.14\times10^{-15}eV\bullet s)(4.22\times10^{13}Hz)/2$

$$E_0 = 0.087eV$$

(b) $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ (Equation 9-17)

$$\mu = \frac{(7.0160u)(1.0078u)}{(7.0160u)+(1.0078u)} = 0.8812u$$

(c) $f = \dfrac{1}{2\pi}\sqrt{\dfrac{K}{\mu}}$ (Equation 9-21)

(Problem 9-23 continued)

$$K = (2\pi f)^2 \mu = (2\pi)^2 (4.22\times10^{13}Hz)(0.8812)(1.66\times10^{-27}kg/u)$$

$$K = 117N/m$$

(d) $E_n = n^2h^2/8mr_0^2 \rightarrow r_0^2 = n^2h^2/8mE_n$

$$r_0 \approx h/(8mE_0)^{1/2}$$

$$r_0 \approx \frac{6.63\times10^{-34}J\bullet s}{\left[8(0.8812u)(1.66\times10^{-27}kg/u)(0.087eV)(1.60\times10^{-19}J/eV)\right]^{1/2}}$$

$$r_0 \approx 5.19\times10^{-11}m = 0.052nm$$

9-24. (a) For H_2: $\mu = \frac{m_1m_2}{m_1+m_2} = \frac{(1.0078u)^2}{2(1.0078u)} = 0.504u$

(b) For N_2: $\mu = \frac{(14.0067u)^2}{2(14.0067u)} = 7.0034u$

(c) For CO: $\mu = \frac{(12.0111u)(15.9994u)}{12.0111u+15.9994u} = 6.8607u$

(d) For HCl: $\mu = \frac{(1.0078u)(35.453u)}{1.0078u+35.453u} = 0.980u$

9-25. (a) $\mu = \frac{m_1m_2}{m_1+m_2} = \frac{(39.1u)(35.45u)}{39.1u+35.45u} = 18.6u$

(b) $E_{0r} = \frac{\hbar^2}{2I}$ (Equation 9-14) $I = \mu r_0^2$

$$E_{0r} = \frac{\hbar^2}{2\mu r_0^2} = \frac{(\hbar c)^2}{2\mu c^2 r_0^2} \rightarrow r_0^2 = \frac{(\hbar c)^2}{2\mu c^2 E_V}$$

$$\therefore r_0 = \frac{\hbar c}{(2\mu c^2 E_V)^{1/2}} = \frac{197.3eV\bullet nm}{\left[2(10.6uc^2)(931.5\times10^6 eV/uc^2)(1.43\times10^{-5}eV)\right]^{1/2}}$$

$$r_0 = 0.280nm$$

9-26. $f = \frac{1}{2\pi}\sqrt{\frac{K}{\mu}}$ (Equation 9-21)

(a) For $H^{35}Cl$: μ = 0.980u and $f = 8.97\times10^{13}\,Hz$.

$$K = (2\pi f)^2\mu = (2\pi)^2(8.97\times10^{13}\,Hz)^2(0.980u)(1.66\times10^{-27}\,kg/u) = 517\,N/m.$$

(b) For $K^{79}Br$: $\mu = \frac{m_1 m_2}{m_1+m_2} = \frac{(39.102u)(78.918u)}{39.102u+78.918u} = 26.147u$ and $f = 6.93\times10^{12}\,Hz$

$$K = (2\pi f)^2\mu = (2\pi)^2(6.93\times10^{12}\,Hz)^2(26.147u)(1.66\times10^{-27}\,kg/u) = 82.3\,N/m$$

9-27. 1. $f = \frac{1}{2\pi}\sqrt{\frac{K}{\mu}}$ (Equation 9-21) Solving for the force constant, $K = (2\pi f)^2\mu$

2. The reduced mass μ of the NO molecule is

$$\mu = \frac{m_{\mathrm{N}}\, m_{\mathrm{O}}}{m_{\mathrm{N}} + m_{\mathrm{O}}} = \frac{(14.01\,\mathrm{u})(16.00\,\mathrm{u})}{14.01\,\mathrm{u}+16.00\,\mathrm{u}} = 7.47\,\mathrm{u}$$

3. $K = (2\pi\times5.63\times10^{13}\ \mathrm{Hz})^2\times7.47\,\mathrm{u}\times1.66\times10^{-27}\ \mathrm{kg/u} = 1.55\times10^3\ \mathrm{N/m}$

(Note: This is equivalent to about 8.8 lbs/ft, the force constant of a moderately strong spring.)

9-28. $E_{0r} = \hbar^2/2I$ Treating the *Br* atom as fixed,

$$I = m_H r_0^2 = (1.0078u)(1.66\times10^{-27}\,kg/u)(0.141nm)^2$$

$$E_{0r} = \frac{(1.055\times10^{-34}\,J\bullet s)^2}{2(1.0078u)(1.66\times10^{-27}\,kg/u)(0.141nm)^2(10^{-9}\,m/nm)^2}$$

$$= 1.67\times10^{-22}\,J = 1.04\times10^{-3}\,eV$$

$E_\ell = \ell(\ell+1)E_{0r}$ for $\ell = 0, 1, 2, \cdots$ (Equation 9-13)

(Problem 9-28 continued)

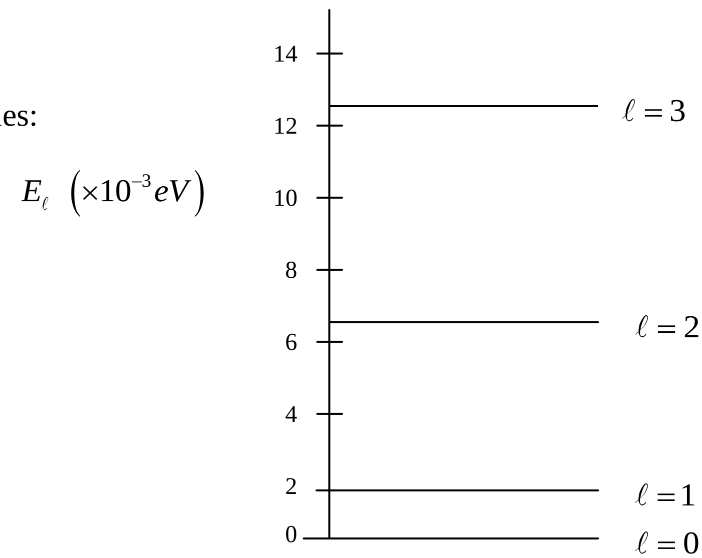
The four lowest states have energies:

$E_0 = 0$

$E_1 = 2E_{0r} = 2.08\times10^{-3}eV$

$E_2 = 6E_{0r} = 6.27\times10^{-3}eV$

$E_3 = 12E_{0r} = 12.5\times10^{-3}eV$



9-29. $\Delta E = hf$ where $f = 1.05\times10^{13}\,Hz$ for *Li*. Approximating the potential (near the bottom) with a square well, $\Delta E(2\rightarrow1) = (2^2-1)\left(\frac{\pi^2}{2}\right)\frac{\hbar^2}{mr_0^2} = hf$

For Li_2: $r_0^2 = \frac{3\pi^2}{2}\frac{\hbar}{2\pi}\frac{1}{f\mu} = \frac{3\pi}{4}\frac{\hbar}{f\mu}$

$$r = \left[\left(\frac{3\pi}{4}\right)\frac{1.055\times10^{-34}\,J\bullet s}{(1.05\times10^{13}\,Hz)(6.939u)(1.66\times10^{-27}\,kg/u)}\right]^{1/2}$$

$$= 4.53\times10^{-11}m = 0.045nm$$

9-30. $E_{0r} = \frac{\hbar^2}{2I}$ where $I = \mu r_0^2$ (Equation 9-14)

For $K^{35}Cl$: $\mu = \frac{(39.102u)(34.969u)}{39.102u + 34.969u} = 18.46u$

For $K^{37}Cl$: $\mu = \frac{(39.102u)(34.966u)}{39.102u + 34.966u} = 19.00u$

$r_0 = 0.267nm$ for *KCl*.

(Problem 9-30 continued)

$$E_{0r}\left(K^{35}Cl\right)=\frac{\left(1.055\times10^{-34}\,J\bullet s\right)^2}{2\left(18.46u\right)\left(1.66\times10^{-27}\,kg/u\right)\left(0.267\times10^{-9}\,m\right)^2}$$

$$=2.55\times10^{-24}\,J=1.59\times10^{-5}\,eV$$

$$E_{0r}\left(K^{37}Cl\right)=\frac{\left(1.055\times10^{-34}\,J\bullet s\right)^2}{2\left(19.00u\right)\left(1.66\times10^{-27}\,kg/u\right)\left(0.267\times10^{-9}\,m\right)^2}$$

$$=2.48\times10^{-24}\,J=1.55\times10^{-5}\,eV$$

$$\Delta E_{0r}=0.04\times10^{-5}\,eV$$

9-31. (a) NaF – ionic (b) KBr – ionic

(c) N_2 – covalent (d) Ne – dipole-dipole

9-32. $$\Delta E_{0,1}=\frac{(\ell+1)\hbar^2}{I}=\frac{\hbar^2}{\mu r_0^2}$$

(a) For $NaCl$: $r_0=0.251nm$ (from Table 9-7)

$$\mu=\frac{m(Na)m\left({}^{35}Cl\right)}{m(Na)+m\left({}^{35}Cl\right)}=\frac{(22.9898)(34.9689)}{(22.9898)+(34.9689)}=13.8707u$$

$$\Delta E_{0,1}=\frac{\left(1.055\times10^{-34}\,J\bullet s\right)^2}{\left(13.8707u\times1.66\times10^{-27}\,kg/u\right)\left(0.251\times10^{-9}\,m\right)^2}$$

$$\Delta E_{0,1}=7.67\times10^{-24}\,J=4.80\times10^{-5}\,eV$$

(b) $\Delta E_{0,1}=hf\rightarrow f=\Delta E_{0,1}/h=\dfrac{7.67\times10^{-24}\,J}{6.63\times10^{-34}\,J\bullet s}$

$$f=1.16\times10^{10}\,Hz$$

$$\lambda=c/f=\left(3.00\times10^{8}\,m/s\right)\left(1.16\times10^{10}\,Hz\right)=0.0259nm=2.59cm$$

9-33. (a) $\lambda = 2400nm \rightarrow E = hc/2400nm = \dfrac{1240eV\bullet nm}{2400nm} = 0.517eV$

$E_2 - E_1 = 3.80eV \quad E_3 - E_2 = 0.500eV \quad E_4 - E_3 = 2.9eV \quad E_5 - E_4 = 0.30eV$

The $E_3 - E_2$ and $E_5 - E_4$ transitions can occur.

(b) None of these can occur, as a minimum of 3.80*eV* is needed to excite higher states.

(c) $\lambda = 250nm \rightarrow E = 1240eV\bullet nm/250nm = 4.96eV$. All transitions noted in (a) can occur. If the temperature is low so only E_1 is occupied, states up to E_3 can be reached, so the $E_2 - E_1$ and $E_3 - E_2$ transitions will occur, as well as $E_3 - E_1$.

(d) $E_4 - E_3 = 2.9eV = hc/\lambda$ or $\lambda = 1240eV\bullet nm/2.9eV = 428nm$

$E_4 - E_2 = 3.4eV = hc/\lambda$ or $\lambda = 1240eV\bullet nm/3.4eV = 365nm$

$E_4 - E_1 = 7.2eV = hc/\lambda$ or $\lambda = 1240eV\bullet nm/7.2eV = 172nm$

9-34. $\dfrac{A_{21}}{B_{21}u(f)} = e^{hf/kT} - 1$ (Equation 9-42)

For the Hα line λ=656.1*nm*

At T = 300*K*, $\dfrac{hf}{kT} = \dfrac{hc}{\lambda kT} = \dfrac{1240eV\bullet nm}{(656.1nm)(8.62\times10^{-5}eV/K)(300K)} = 73.1$

$e^{hf/kT} - 1 = e^{73.1} - 1 \approx 5.5\times10^{31}$

Spontaneous emission is more probable by a very large factor!

9-35. $\dfrac{n(E_1)}{n(E_0)} = \dfrac{e^{-E_1/kT}}{e^{-E_0/kT}}$ i.e., the ratio of the Boltzmann factors.

For O_2: $f = 4.74\times10^{13}Hz$ and

$$E_0 = hf/2 = (4.14\times10^{-15}eV\bullet s)(4.74\times10^{13}Hz)/2 = 0.0981eV$$

$$E_1 = 3hf/2 = 0.294eV$$

At 273*K*, $kT = (8.62\times10^{-5}eV/K)(273K) = 0.0235eV$

(Problem 9-35 continued)

$$\frac{n(E_1)}{n(E_0)} = \frac{e^{-0.0294/0.0235}}{e^{-0.0981/0.0235}} = \frac{e^{-12.5}}{e^{-4.17}} = 2.4\times10^{-4}$$

Thus, about 2 of every 10,000 molecules are in the E_1 state.

Similarly, at 77K, $\frac{n(E_1)}{n(E_0)} = 1.4\times10^{-13}$

9-36. $E = \ell(\ell+1)E_{0r}$ for $\ell = 0, 1, 2, \cdots$ (Equation 9-13)

Where $E_{0r} = \frac{\hbar^2}{2I}$ and $I = \mu r_0^2$ with $\mu = m/2$

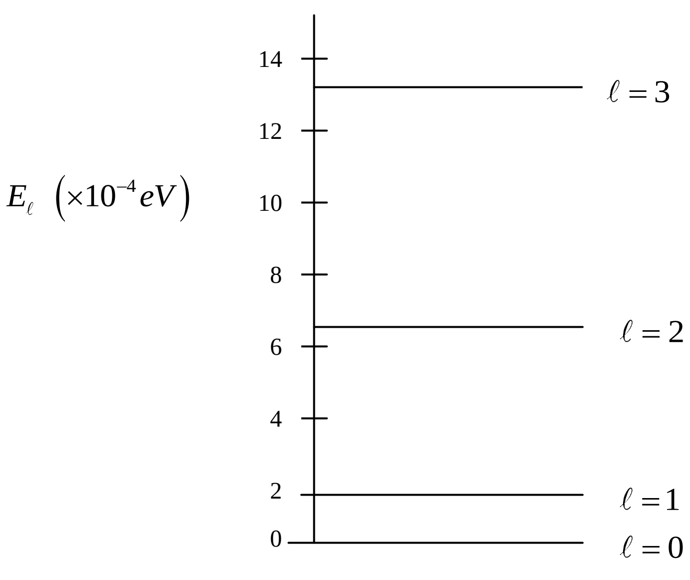
$$E_{0r} = \frac{(1.055\times10^{-34}\,J\bullet s)^2}{2(18.99u)(1.66\times10^{-27}\,kg/u)(0.14\times10^{-9}\,m)^2} = 1.80\times10^{-23}\,J = 1.12\times10^{-4}\,eV$$

(a) $E_0 = 0$

$E_1 = 2E_{0r} = 2.24\times10^{-4}\,eV \qquad E_1 - E_0 = 2.24\times10^{-4}\,eV$

$E_2 = 6E_{0r} = 6.72\times10^{-4}\,eV \qquad E_2 - E_1 = 4.48\times10^{-4}\,eV$

$E_3 = 12E_{0r} = 13.4\times10^{-4}\,eV \qquad E_3 - E_2 = 6.72\times10^{-4}\,eV$



(Problem 9-36 continued)

(b) $\Delta\ell \pm 1 \qquad \Delta E = hc/\lambda \rightarrow \lambda = hc/\Delta E$

For $E_1 - E_0 : \lambda = \dfrac{1240eV \bullet nm}{2.24\times10^4 eV} = 5.54\times10^6 nm = 5.54nm$

For $E_2 - E_1 : \lambda = \dfrac{1240eV \bullet nm}{4.48\times10^4 eV} = 2.77\times10^6 nm = 2.77nm$

For $E_3 - E_2 : \lambda = \dfrac{1240eV \bullet nm}{6.72\times10^4 eV} = 1.85\times10^6 nm = 1.85nm$

9-37. (a) $10MW = 10^7 J/s \rightarrow E = \left(10^7 J/s\right)\left(1.5\times10^{-9} s\right) = 1.5\times10^{-2} J$

(b) For ruby laser: $\lambda = 694.3nm$, so the energy/photon is:

$E = hc/\lambda = 1240eV \bullet nm/694.3nm = 1.786eV$

Number of photons = $\dfrac{\left(1.5\times10^{-2} J\right)}{\left(1.786eV\right)\left(1.60\times10^{-19} J/eV\right)} = 5.23\times10^6$

9-38. $4mW = 4\times10^{-3} J/s$

$E = hc/\lambda = \dfrac{1240eV \bullet nm}{632.8nm} = 1.960eV \;\; per \;\; photon$

Number of photons = $\dfrac{4\times10^{-3} J/s}{\left(1.960eV\right)\left(1.60\times10^{-19} J/eV\right)} = 1.28\times10^{16}/s$

9-39. (a) $\sin\theta = 1.22\lambda/D = 1.22\left(600\times10^{-9} m\right)/\left(10\times10^{-2} m\right) = 7.32\times10^{-6}$

$\approx \theta \approx 7.32\times10^{-6} radians$

$\theta = S/R$ where $S =$ diameter of the beam on the moon and $R =$ distance to moon.

$S = R\theta = \left(3.84\times10^8 m\right)\left(7.32\times10^{-6} radians\right) = 2.81\times10^3 m = 2.81km$

(b) $\sin\theta = 1.22\left(600\times10^{-9} m\right)/\left(1m\right) = 7.32\times10^{-7} radians$

$S = R\theta = \left(3.84\times10^8 m\right)\left(7.32\times10^{-7} radians\right) = 281m$

9-40. (a) $\dfrac{n(E_2)}{n(E_1)} = \dfrac{e^{-E_2/kT}}{e^{-E_1/kT}} = e^{(E_2-E_1)/kT}$ $\quad E_2 - E_1 = hc/\lambda = 1240eV \bullet nm/420nm = 2.95eV$

At $T = 297K, \quad kT = (8.61\times10^{-5}eV/K)(297K) = 0.0256eV$

$n(E_2) = n(E_1)e^{-2.95/0.0256} = 2.5\times10^{21}e^{-115} = 2\times10^{-29} \approx 0$

(b) Energy emitted = $(1.8\times10^{21})(2.95eV/photon) = 5.31\times10^{21}eV = 850J$

9-41. (a) Total potential energy: $U(r) = -\dfrac{ke^2}{r} + E_{ex} + E_{ion}$

the electrostatic part of $U(r)$ at r_0 is $-\dfrac{ke^2}{r_0} = -\dfrac{1.44eV\bullet nm}{0.24nm} = -6.00eV$

(b) The net ionization energy is:

$E_{ion} = (ionization\ energy\ of\ \ Na) - (electron\ affinity\ of\ \ Cl)$

$= 5.14eV - 3.62eV = 1.52eV$

dissociation energy of $NaCl$ =$4.27eV$ (from Table 9-2)

$4.27eV = -U(r_0) = 6.00eV - 1.52eV = 4.67eV - E_{ex}$

$E_{ex} = 6.00eV - 4.27eV - 1.52eV = 0.21eV$

(c) $E_{ex} = \dfrac{A}{r^n}$ (Equation 9-2)

At $r_0 = 0.24nm, E_{ex} = 0.21eV$

At $r_0 = 0.14nm, U(r) = 0$ and $E_{ex} = \dfrac{ke^2}{r} - E_{ion} = 8.77eV$

At $r_0 : E_{ex} = 0.21eV = \dfrac{A}{(0.24nm)^n} \to A = (0.21eV)(0.24nm)^n$

At $r = 0.14nm : E_{ex} = 8.77eV = \dfrac{A}{(0.14nm)^n} \to A = (8.77eV)(0.14nm)^n$

Setting the two equations for A equal to each other:

$\dfrac{(0.24nm)^2}{(0.14nm)^n} = \left(\dfrac{0.24}{0.14}\right)^2 = \left(\dfrac{8.77eV}{0.21eV}\right) \to (1.71)^n = 41.76$

(Problem 9-41 continued)

$$n\log 1.71 = \log 41.76$$

$$n = (\log 41.76)/(\log 1.71) = 6.96$$

$$A = 0.21eV\,(0.24nm)^n = 0.21eV\,(0.24nm)^{6.96} = 1.02\times10^{-5}\,eV\bullet nm^{6.96}$$

9-42. (a) $\sin\theta = 1.22\lambda / D = 1.22\left(694.3\times10^{-9}m\right)/\left(0.01m\right) = 8.47\times10^{-5}$

$\therefore\ \theta = 8.47\times10^{-5}\,radians$

(b) $E_{photon} = hc/\lambda = 1240eV\bullet nm / 694.3nm = 1.786eV / photon$

For $10^{18}\ photons/s$:

$$E_{total} = (1.786eV / photon)\left(1.602\times10^{-19}J/eV\right)\left(10^{18}\ photons/s\right)$$

$$= 0.286J/s = 0.286W$$

Area of spot A is: $A = \pi d^2/4 = \pi(8.47cm)^2/4$

and $E = E_{total}/A = 0.286W/\pi\left[(8.47cm)^2/4\right] = 5.08\times10^{-3}W/cm^2$

9-43. (a) $U_{att} = -\dfrac{ke^2}{r} = \dfrac{1.440eV\bullet nm}{0.267nm} = -5.39eV$

(b) To form K^+ and Cl^- requires $E_{ion} = 4.34eV - 3.61eV = 0.73eV$

$$E_d = -U_C = -\left(-\frac{ke^2}{r} + E_{ion}\right) = 5.39eV - 0.73eV = 4.66eV$$

(c) $E_{ex} = 4.66eV - 4.43eV = 0.23eV$ at r_0

9-44. $E_{0r} = \dfrac{\hbar^2}{2I}$ where $I = \mu r_0^2$ with $r_0 = 0.267nm$ and

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{(39.102u)(35.453u)}{39.102u + 35.453u} = 18.594u$$

$$E_{0r} = \frac{\left(1.055\times10^{-34}J\bullet s\right)^2}{2(18.594u)\left(1.66\times10^{-27}kg/u\right)\left(0.267\times10^{-9}m\right)^2} = 2.53\times10^{-24}J = 1.58\times10^{-5}eV$$

9-45. (a) $E_d = \dfrac{kp_1}{x^3}$ where $p_1 = qa,$ being the separation of the charges $+q$ and $-q$ of the dipole

(b) $U = -\boldsymbol{p}\cdot\boldsymbol{E}$ and $\boldsymbol{p} \propto \boldsymbol{E} \rightarrow \boldsymbol{p} = \alpha\boldsymbol{E}$

So the individual dipole moment of a nonpolar molecule in the field produced by p_1 is

$p_2 = \alpha E_d = \alpha k p_1 / x^3$ and $U = -\boldsymbol{p_2}\cdot\boldsymbol{E_d} = \alpha\left(kp_1\right)^2 / x^6$

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left[\alpha\left(k^2 p_1^2\right)/x^6\right] = 6\alpha k^2 p_1^2 / x^7$$

9-46. 1. The energies E_ν of the vibrational levels are given by Equation 9-20:

$$E_\nu = (\nu + \frac{1}{2})hf \quad \text{for} \quad \nu = 0, 1, 2, 3, \cdots$$

The frequencies are found from Equation 9-21 and requires first that the reduced mass for each of the molecules be found using Equation 9-17.

$$\mu_{H_2} = \frac{m_H m_H}{m_H + m_H} = \frac{(1.01\text{u})(1.01\text{u})}{1.01\text{u} + 1.01\text{u}} = 0.51\text{u}$$

Similarly,

$$\mu_{HD} = 0.67\,\text{u}$$
$$\mu_{D_2} = 1.01\text{u}$$

2. The vibrational frequencies for the molecules are then:

$$f_{H_2} = \frac{1}{2\pi}\sqrt{\frac{K}{\mu_{H_2}}} = \frac{1}{2\pi}\sqrt{\frac{580\,\text{N/m}}{(0.51\text{u})(1.66\times10^{-27}\ \text{kg/u})}} = 1.32\times10^{14}\ \text{Hz}$$

Similarly,

$$f_{HD} = 1.15\times10^{14}\ \text{Hz}$$
$$f_{D_2} = 9.36\times10^{13}\ \text{Hz}$$

3. The energies of the four lowest vibrational levels are then:

For H_2:

$$E_0 = \frac{1}{2}hf_{H_2} = \frac{1}{2}(6.63\times10^{-34}\ \text{J}\cdot\text{s})(1.32\times10^{14}\ \text{Hz}) = 4.38\times10^{-20}\ \text{J}$$

$$E_0 = \frac{4.38\times10^{-20}\ \text{J}}{1.60\times10^{-19}\ \text{J/eV}} = 0.27\,\text{eV}$$

(Problem 9-46 continued)

$$E_1 = 0.82\,\text{eV}$$

Similarly, $$E_2 = 1.37\,\text{eV}$$

$$E_3 = 1.91\,\text{eV}$$

For HD:

$$E_0 = \frac{1}{2}hf_{\text{HD}} = \frac{1}{2}(6.63\times10^{-34}\,\text{J}\cdot\text{s})(1.15\times10^{14}\,\text{Hz}) = 3.81\times10^{-20}\,\text{J}$$

$$E_0 = \frac{3.81\times10^{-20}\,\text{J}}{1.60\times10^{-19}\,\text{J/eV}} = 0.24\,\text{eV}$$

And again similarly,

$$E_1 = 0.72\,\text{eV}$$
$$E_2 = 1.19\,\text{eV}$$
$$E_3 = 1.67\,\text{eV}$$

For D_2:

$$E_0 = \frac{1}{2}hf_{\text{D}_2} = \frac{1}{2}(6.63\times10^{-34}\,\text{J}\cdot\text{s})(9.36\times10^{13}\,\text{Hz}) = 3.10\times10^{-20}\,\text{J}$$

$$E_0 = \frac{3.10\times10^{-20}\,\text{J}}{1.60\times10^{-19}\,\text{J/eV}} = 0.19\,\text{eV}$$

And once again similarly,

$$E_1 = 0.58\,\text{eV}$$
$$E_2 = 0.97\,\text{eV}$$
$$E_3 = 1.36\,\text{eV}$$

4. There are three transitions for each molecule:

$$\nu = 3 \rightarrow \nu = 2; \quad \nu = 2 \rightarrow \nu = 1; \quad \nu = 1 \rightarrow \nu = 0$$

For H_2: $\Delta E = hf = hc/\lambda \Rightarrow \lambda = hc/\Delta E = 1240\,\text{eV}\cdot\text{nm}/\Delta E\,\text{eV}$

$$\Delta E = E_{\nu=3} - E_{\nu=2} = (1.91 - 1.37)\text{eV} = 0.54\,\text{eV}$$
$$\therefore \lambda_{3\rightarrow2} = 1240\,\text{eV}\cdot\text{nm}/0.54\,\text{eV} = 2.30\times10^{3}\,\text{nm}$$

Similarly, $$\lambda_{2\rightarrow1} = 2.25\times10^{3}\,\text{nm}$$

$$\lambda_{1\rightarrow0} = 2.25\times10^{3}\,\text{nm}$$

For HD:

$$\Delta E = E_{\nu=3} - E_{\nu=2} = (1.67 - 1.19)\text{eV} = 0.48\,\text{eV}$$
$$\therefore \lambda_{3\rightarrow2} = 1240\,\text{eV}\cdot\text{nm}/0.48\,\text{eV} = 2.58\times10^{3}\,\text{nm}$$

(Problem 9-46 continued)

And again similarly, $\lambda_{2\to1} = 2.64\times10^3\ \text{nm}$

$\lambda_{1\to0} = 2.58\times10^3\ \text{nm}$

For D_2 :

$$\Delta E = E_{v=3} - E_{v=2} = (1.36-0.97)\text{eV} = 0.39\,\text{eV}$$
$$\therefore \lambda_{3\to2} = 1240\,\text{eV}\cdot\text{nm}/0.39\,\text{eV} = 3.18\times10^3\ \text{nm}$$

And once again similarly,

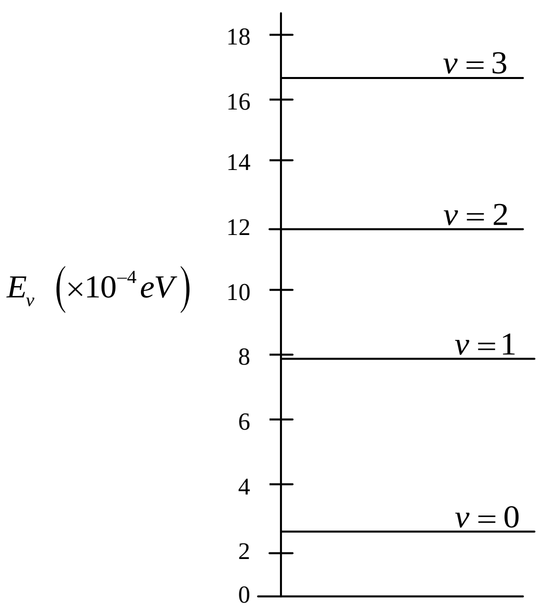
$$\lambda_{2\to1} = 3.18\times10^3\ \text{nm}$$
$$\lambda_{1\to0} = 3.18\times10^3\ \text{nm}$$

9-47. (a) $E_3 = hc/\lambda = 1240eV\bullet nm/(0.86mm)(10^6 nm/mm) = 1.44\times10^{-3} eV$

$E_2 = 1240eV\bullet nm/(1.29mm)(10^6 nm/mm) = 9.61\times10^{-4} eV$

$E_1 = 1240eV\bullet nm/(2.59mm)(10^6 nm/mm) = 4.79\times10^{-4} eV$

These are vibrational states, because they are equally spaced. Note the $v = 0$ state at the ½ level spacing.



(Problem 9-47 continued)

(b) Approximating the potential with a square well (at the bottom),

$$E_1 = 4.79\times10^{-4}eV = n^2\frac{\pi^2}{2}\frac{\hbar^2}{mr_0^2}$$

$$r_0 = \left[\frac{\left(2^2-1^2\right)\pi^2\left(1.055\times10^{-34}J\bullet s\right)^2}{2\left(28.01u\right)\left(1.66\times10^{-27}kg/u\right)\left(4.79\times10^{-4}eV\right)\left(1.60\times10^{-19}J/eV\right)}\right]^{1/2}$$

$$= 2.15\times10^{-10}m = 0.215nm$$

9-48. Using the *NaCl* potential energy versus separation graph in Figure 9-23(b) as an example (or you can plot one using Equation 9-1):

The vibrational frequency for *NaCl* is $1.14\times10^{13}Hz$ (from Table 9-7) and two vibrational levels, for example v = 0 and v = 10 yield (from Equation 9-20)

$$E_0 = 1/2hf = 0.0236eV \qquad E_{10} = 11/2hf = 0.496eV$$

above the bottom of the well. Clearly, the average separation for $v_{10} > v_0$.

9-49. (a) $E_{0r} = \dfrac{\hbar^2}{2\mu r_0^2}$ where $r_0 = 0.128nm$ for *HCl* and

$$\mu = \frac{m_1m_2}{m_1+m_2} = \frac{\left(1.0079u\right)\left(35.453u\right)}{1.0079u+35.453u} = 0.980u$$

$$E_{0r} = \frac{\left(1.055\times10^{-34}J\bullet s\right)^2}{2\left(0.980u\right)\left(1.66\times10^{-27}kg/u\right)\left(0.128\times10^{-9}m\right)^2} = 2.089\times10^{-22}J = 1.303\times10^{-3}eV$$

$$E_\ell = \ell\left(\ell+1\right)E_{0r}$$

$$E_0 = 0 \qquad E_1 = 2E_{0r} = 2.606\times10^{-3}eV \qquad E_2 = 6E_{0r} = 7.82\times10^{-3}eV$$

$$\Delta E_{01} = E_1 - E_0 = 2.606\times10^{-3}eV \qquad \Delta E_{12} = E_2 - E_1 = 5.214\times10^{-3}eV$$

$$\Delta f_{01} = \Delta E_{01}/h = \frac{2.606\times10^{-3}eV}{4.136\times10^{-15}eV\bullet s} = 0.630\times10^{12}Hz$$

(Problem 9-49 continued)

$$\Delta f_{12} = \Delta E_{12}/h = \frac{5.214\times10^{-3}eV}{4.136\times10^{-15}eV\bullet s} = 1.26\times10^{12}Hz$$

$$f'_{01} = f \pm \Delta f_{01} = 6.884\times10^{14}Hz \pm 0.63\times10^{12} = 6.890\times10^{14}Hz;\ \ 6.878\times10^{14}Hz$$

$$\lambda'_{01} = c/f'_{01} = 435.5nm;\ \ 436.2nm$$

$$f'_{02} = f \pm \Delta f_{02} = 6.884\times10^{14}Hz \pm 1.26\times10^{12} = 6.897\times10^{14}Hz;\ \ 6.871\times10^{14}Hz$$

$$\lambda'_{02} = c/f'_{02} = 435.0nm;\ \ 436.6nm$$

(b) From Figure 9-29: $\Delta f_{01} = 0.6\times10^{12}Hz$ and $\Delta f_{12} = 1.2\times10^{12}Hz$

The agreement is very good!

9-50. (a) Li_2 : $E_v = (v+1/2)hf$

$$E_1 = (3/2)(4.14\times10^{-15}eV\bullet s)(1.05\times10^{13}Hz) = 0.0652eV = 6.52\times10^{-2}eV$$

$$E_\ell = \ell(\ell+1)E_{0r}$$

$$E_1 = 2(8.39\times10^{-5}eV) = 1.68\times10^{-4}eV$$

(b) $K^{79}Br$: $E_v = (v+1/2)hf$

$$E_1 = (3/2)(4.14\times10^{-15}eV\bullet s)(6.93\times10^{12}Hz) = 4.30\times10^{-2}eV$$

$$E_\ell = \ell(\ell+1)E_{0r}$$

$$E_1 = 2(9.1\times10^{-6}eV) = 1.8\times10^{-5}eV$$

9-51. $\mu(HCl) = 0.980u$ (See solution to Problem 9-49)

From Figure 9-29, the center of the gap is the characteristic oscillation frequency f :

$f = 8.65\times10^{13}Hz \rightarrow E = 0.36eV$ Thus, $f = \frac{1}{2\pi}\sqrt{\frac{K}{\mu}}$ or $K = (2\pi f)^2\mu$

$$K = (2\pi)^2(8.65\times10^{13}Hz)^2(0.980u)(1.66\times10^{-27}kg/u) = 480N/m$$

9-52. $n\ E_n\ = g\ E_n\ e^{-E_n/kT}$

$$\lambda_{21} = 694.3nm \rightarrow E_2 - E_1 = hc/\lambda_{21} = \frac{1240eV\bullet nm}{694.3nm} = 1.7860eV$$

$$E_2' - E_1 = 1.7860eV + 0.0036eV = 1.7896eV$$

Where E_2 is the lower energy level of the doublet and E_2' is the upper.

Let T = 300K, so kT = 0.0259eV.

(a) $\frac{n\ E_2'}{n\ E_1'} = \frac{g\ E_2}{g\ E_1} e^{-\ E_2-E_1\ /kT} = \frac{2}{4} e^{-1.7896/0.0259} = \frac{1}{2} e^{-69} = 4.91\times 10^{-31}$

$$\frac{n\ E_2}{n\ E_1} = \frac{1}{2} e^{-1.7896/0.0259} = 5.64\times 10^{-31}$$

(b) If only $E_2 \rightarrow E_1$ transitions produce lasing, but E_2 and E_2' are essentially equally populated, in order for population inversion between levels E_2 and E_1, at least 2/3 rather than 1/2) of the atoms must be pumped. The required power density (see Example 9-8) is:

$$p \approx \frac{2N}{3}\left(\frac{hf}{t_s}\right) \approx \frac{2\ \ 2\times 10^{19} cm^3\ \ \ 6.63\times 10^{-34} J\bullet s\ \ \ 4.32\times 10^{14} Hz}{3\ \ 3\times 10^{-3} s} = 1273W/cm^3$$

9-53. (a) $E_v = \ v + 1/2\ \ hf$ (Equation 9-20)

For v = 0, $E_0 = hf/2 = \ 6.63\times 10^{-34} J\bullet s\ \ \ 8.66\times 10^{13} Hz\ \ /2 = 0.179eV$

(b) For $\Delta\ell = \pm 1, \quad \Delta E_\ell = \ell^2\hbar/I = \ell h\Delta f$

so $I = h/\ 4\pi^2\Delta f\ \ = \frac{6.63\times 10^{-34} J\bullet s}{4\pi^2\ \ 6\times 10^{11} Hz} = 2.8\times 10^{-47} kg\bullet m^2$

(c) $I = \mu r^2$, where μ is given by Equation 9-17.

$$\mu = \frac{m_H m_{Cl}}{m_H + m_{Cl}} = 0.973u \rightarrow r = 0.132nm$$

9-54. (a) $\dfrac{dU}{dr} = U_0\left[-12a^{12}r^{-13} - 2\ \ -6a^6\ \ r^{-7}\right]$

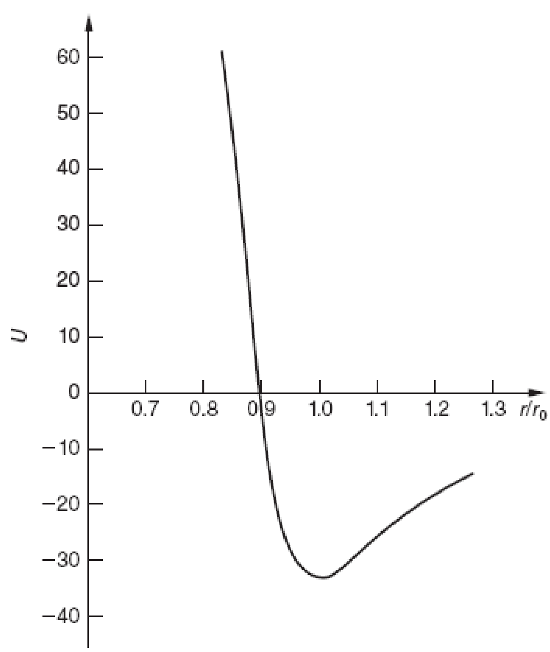
For $U_{\min}$, $dU/dr = 0$, so $-12a^{12}r^{-6} + 12a^6 = 0 \rightarrow r^{-6} = a^{-6} \rightarrow r = a$

(b) For $U = U_{\min}$, $r = a$ then $U_{\min} = U_0\left[\left(\dfrac{a}{a}\right)^{12} - 2\left(\dfrac{a}{a}\right)^6\right] = \ 1-2\ U_0 = -U_0$

(c) From Figure 9-8(b): $r_0 = 0.074nm\ \ (= a)$ $\quad U_0 = 32.8eV$

(d)

r/r_0	$r_0/r\ ^{12}$	$-2\ \ r_0/r\ ^{6}$	U
0.85	7.03	−5.30	+56.7
0.90	3.5	−3.8	−9.8
0.95	1.85	−2.72	−28.5
1.00	1	−2.0	−32.8
1.05	0.56	−1.5	−30.8
1.10	0.32	−1.12	−26.2
1.15	0.19	−0.86	−22.0
1.20	0.11	−0.66	−18.0



9-55. (a) $U(r) = -\frac{ke^2}{r} + E_{ex} + E_{ion}$ (Equation 9-1)

For $NaCl$, $E_d = 4.27eV$ and $r_0 = 0.236nm$ (Table 9-1).

$E_{ion} = E_{ion}(Na) + E_{aff}(Cl) = 5.14 - 3.62 = 1.52eV$ and $U(r_0) = -E_d = -4.27eV$

$$E_{ex} = -4.27 + \frac{ke^2}{0.236} - 1.52 = 0.31eV$$

(b) $E_{ex} = Ar^{-n} = 0.31eV$ (Equation 9-2)

Following Example 9-2, $\frac{ke^2}{r_0^2} = 25.85eV/nm = \frac{n}{r_0}\frac{A}{r_0^n} = \frac{n}{r_0}(0.31eV)$

Solving for n: $n = (25.85eV/nm)(0.236nm)/0.31eV = 19.7 \approx 20$

$$A = (0.31eV)(0.236nm)^{20} = 8.9\times10^{-14}\,eV\bullet nm^{20}$$

9-56. For $H^+ - H^-$ system, $U(r) = -\frac{ke^2}{r} + E_{ion}$

There is no E_{ex} term, the two electrons of H^- are in the n = 1 shell with opposite spins.

E_{ion} = ionization energy for H^+ − electron affinity for $H^- = 13.6eV - 0.75eV = 12.85eV$.

$$U(r) = -\frac{1.440eV\bullet nm}{r} + 12.85eV \qquad \frac{dU(r)}{dr} = \frac{1.440}{r^2}$$

For $U(r)$ to have a minimum and the ionic $H^+ - H^-$ molecule to be bound, $dU/dr = 0$. As we see from the derivative, there is no non-zero or finite value of r for which this occurs.

9-57. (a) $u(35) = \frac{(1.007825u)(34.968851u)}{1.007825u + 34.968851u} = 0.979593u$

$$u(37) = \frac{(1.007825u)(36.965898u)}{1.007825u + 36.965898u} = 0.981077u$$

$$\frac{\Delta\mu}{\mu(35)} = 1.52\times10^{-3}$$

(b) The energy of a transition from one rotational state to another is:

(Problem 9-57 continued)

$$\Delta E_{\ell,\,\ell+1} = (\ell+1)\hbar^2/I = hf \quad \text{(Equation 9-15)}$$

$$f = \frac{(\ell+1)\hbar^2}{hI} = \frac{(\ell+1)h^2}{4\pi^2 h\mu r_0^2} = \frac{(\ell+1)h}{4\pi^2\mu r_0^2}$$

$$\Delta f \approx \frac{df}{d\mu}\Delta\mu = \left[\frac{(\ell+1)h}{4\pi^2\mu r_0^2}\right]\left(-\frac{1}{\mu^2}\right)\Delta\mu$$

$$\frac{\Delta f}{f} = \left[\frac{(\ell+1)h}{4\pi^2\mu r_0^2}\right]\left(-\frac{\Delta\mu}{\mu}\right)\left[\frac{(\ell+1)h}{4\pi^2\mu r_0^2}\right]^{-1} = -\frac{\Delta\mu}{\mu}$$

(c) $\frac{\Delta f}{f} = -\frac{\Delta\mu}{\mu} = -1.53\times10^{-3}$ from part (b). In Figure 9-29 the Δf between the ^{35}Cl lines (the taller ones) and the ^{37}Cl lines is of the order of $0.01\times10^{13}\,Hz$, so $\Delta f / f \approx 0.0012$, about 20% smaller than $\Delta\mu/\mu$.

9-58. (a) For CO: $r_0 = 0.113$ $\qquad \mu = \frac{m_1 m_2}{m_1+m_2} = \frac{(12.0112u)(15.9994u)}{12.0112u + 15.9994u} = 6.861u$

$$I = \mu r_0^2 = (6.861u)(1.66\times10^{-27}kg/u)(0.113\times10^{-9})^2 = 1.454\times10^{-46}kg\bullet m^2$$

$$E_{0r} = \frac{\hbar^2}{2I} = \frac{(1.055\times10^{-34}J\bullet s)^2}{2(1.454\times10^{-46}kg\bullet m^2)} = 3.827\times10^{-23}J = 2.39\times10^{-4}eV$$

(b) $E_\ell = \ell(\ell+1)E_{0r}$

$E_0 = 0$

$E_1 = 2E_{0r} = 4.78\times10^{-4}eV$

$E_2 = 6E_{0r} = 1.43\times10^{-3}eV$

$E_3 = 12E_{0r} = 2.87\times10^{-3}eV$

$E_4 = 20E_{0r} = 4.78\times10^{-3}eV$

$E_5 = 30E_{0r} = 7.17\times10^{-3}eV$

(Problem 9-58 continued)

(c) (See diagram)

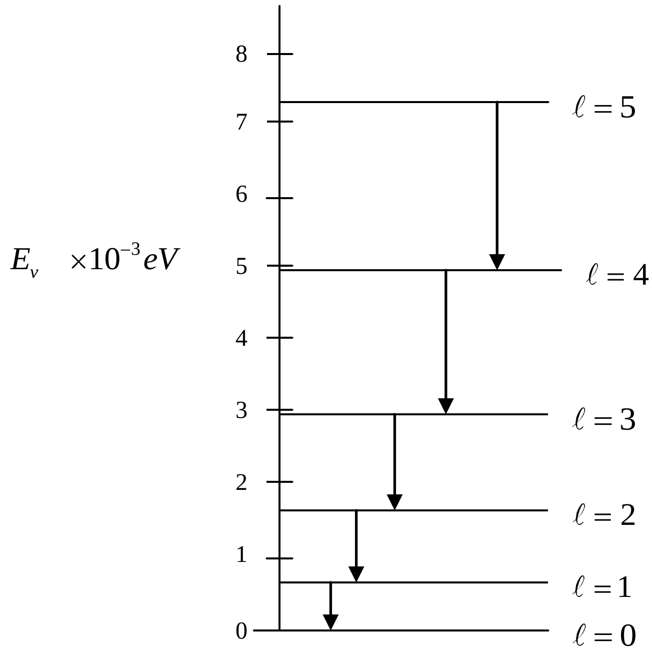
$$E_{54} = \ 7.17 - 4.78 \ \times 10^{-3} eV = 2.39 \times 10^{-3} eV$$

$$E_{43} = \ 4.78 - 2.87 \ \times 10^{-3} eV = 1.91 \times 10^{-3} eV$$

$$E_{32} = \ 2.87 - 1.43 \ \times 10^{-3} eV = 1.44 \times 10^{-3} eV$$

$$E_{21} = \ 1.43 - 0.48 \ \times 10^{-3} eV = 0.95 \times 10^{-3} eV$$

$$E_{10} = 4.78 \times 10^{-4} eV$$



(d) $\lambda = hc / E$

$$\lambda_{54} = \frac{1240 eV \bullet nm}{2.39 \times 10^{-3} eV} = 5.19 \times 10^{5} nm = 0.519 mm$$

$$\lambda_{43} = \frac{1240 eV \bullet nm}{1.91 \times 10^{-3} eV} = 6.49 \times 10^{5} nm = 0.649 mm$$

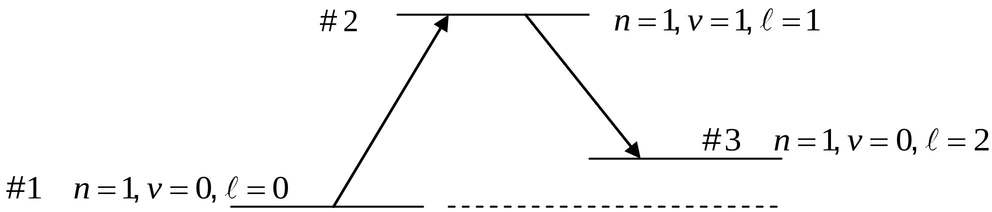
$$\lambda_{32} = \frac{1240 eV \bullet nm}{1.44 \times 10^{-3} eV} = 8.61 \times 10^{5} nm = 0.861 mm$$

$$\lambda_{21} = \frac{1240 eV \bullet nm}{0.95 \times 10^{-3} eV} = 13.05 \times 10^{5} nm = 1.31 mm$$

$$\lambda_{10} = \frac{1240 eV \bullet nm}{4.78 \times 10^{-4} eV} = 25.9 \times 10^{5} nm = 2.59 mm$$

All of these are in the microwave region of the electromagnetic spectrum.

9-59.



(a) $E(1) = \frac{1}{2}hf_{H_2} + \ell(\ell+1)E_{0r} = \frac{1}{2}hf_{H_2}$ since $\ell = 0$

$E(2) = \frac{3}{2}hf_{H_2} + 2E_{0r}$ since $\ell = 1$

$E(3) = \frac{1}{2}hf_{H_2} + 6E_{0r}$ since $\ell = 2$

A $\quad E(2) - E(1) = \left(\frac{3}{2}hf_{H_2} + 2E_{0r}\right) - \frac{1}{2}hf_{H_2} = h(1.356\times10^{14}\,Hz)$

B $\quad E(2) - E(3) = \left(\frac{3}{2}hf_{H_2} + 2E_{0r}\right) - \left(\frac{1}{2}hf_{H_2} + 6E_{0r}\right) = h(1.246\times10^{14}\,Hz)$

Re-writing [A] and [B] with $E_{0r} = \hbar^2/2I$:

A1 $\quad hf_{H_2} + \hbar^2/I = h(1.356\times10^{14}\,Hz)$

B1 $\quad hf_{H_2} - 2\hbar^2/I = h(1.246\times10^{14}\,Hz)$

Subtracting [B1] from [A1] and cancelling an *h* from each term gives:

$3h/4\pi^2 I = 0.110\times10^{14}\,Hz$

$$I = \frac{3h}{4\pi^2(0.110\times10^{14}\,Hz)} = 4.58\times10^{-48}\,kg\bullet m^2$$

(b) $I = \mu r_0^2$ $\qquad$ For H_2: $\quad \mu = \frac{(1.007825)^2}{2(1.007825)} = 0.503912u$

$r_0 = (I/\mu)^{1/2} = \left[(4.58\times10^{-48}\,kg\bullet m^2)/(0.5039u\times1.66\times10^{-27}\,kg/u)\right]^{1/2}$

$r_0 = 7.40\times10^{-11}\,m = 0.0740nm$ in agreement with Table 9-7.

Canceling an *h* from [B1] and substituting the value of *I* from (a) gives:

$f_{H_2} = 2h/4\pi^2 I + 1.246\times10^{14}\,Hz$

$f_{H_2} = 1.32\times10^{14}\,Hz$ also in agreement with Table 9-7.

Chapter 10 – Solid State Physics

10-1. $U\left(r_0\right) = -\alpha \dfrac{ke^2}{r_0}\left(1 - \dfrac{1}{n}\right)$ (Equation 10-6)

$$E = -U\left(r_0\right) = \alpha \frac{ke^2}{r_0}\left(1 - \frac{1}{n}\right)$$

$$1 - \frac{1}{n} = \frac{E_d r_0}{\alpha ke^2} = \frac{\left(741kJ/mol\right)\left(0.257nm\right)}{1.7476\left(1.44eV\bullet nm\right)} \times \frac{1eV/ion\ pair}{96.47kJ/mol} = 0.7844$$

$$n = \frac{1}{1 - 0.7844} = 4.64$$

10-2. The molar volume is $\dfrac{M}{\rho} = 2N_A r_0^3$

$$r_0 = \left[\frac{M}{2N_A\rho}\right] = \left[\frac{74.55g/mole}{2\left(6.022\times10^{23}/mole\right)\left(1.984g/cm^3\right)}\right]^{1/3} = 3.15\times10^{-8}cm = 0.315nm$$

10-3. The molar volume is $\dfrac{M}{\rho} = 2N_A r_0^3$

$$\rho = \frac{M}{2N_A r_0^3} = \frac{42.4g/mole}{2\left(6.022\times10^{23}/mole\right)\left(0.257\times10^{-7}cm\right)^3} = 2.07g/cm^3$$

10-4. (a) $U_{att} = -\alpha \dfrac{ke^2}{r_0}$ (Equation 10-1)

(b) $E_d = -U\left(r_0\right) = \alpha \dfrac{ke^2}{r_0}\left(1 - \dfrac{1}{n}\right)$ (Equation 10-6)

$$= \left(8.01eV\right)\left(1 - \frac{1}{9}\right) = 7.12eV/ion\ pair$$

$$= \left(7.12eV/ion\ pair\right)\left(\frac{96.47kJ/mol}{1eV/ion\ pair}\right)\left(\frac{1cal}{4/186J}\right) = 164kcal/mole$$

(Problem 10-4 continued)

(c) $1-\frac{1}{n}=\frac{E_d r_0}{\alpha ke^2}=\frac{(165.5kcal/mol)(0.314nm)}{1.7476(1.44eV\bullet nm)}\times\frac{4.186J}{1cal}\left(\frac{1eV/ion\ pair}{96.47kJ/mol}\right)=0.8960$

Therefore, $n=\frac{1}{1-0.8960}=9.62$

10-5. Cohesive energy (LiBr)

$$= 788\times10^3 J/mol\left(\frac{1mol}{6.02\times10^{23}ion\ pairs}\right)\left(\frac{1eV}{1.60\times10^{-19}}\right)=8.182ev/ion\ pair$$

$$=4.09eV/atom$$

This is about 32% larger than the value in Table 10-1.

10-6. Molecular weight Na = 22.990

Molecular weight Cl = 35.453

$\therefore$ the $NaCl$ molecule is by weight 0.3934 Na and 0.6066 Cl.

Since the density of $NaCl$ = 2.16 g/cm^3, then

mol of $Na/cm^3=(0.3934)(2.16g/cm^3)/(2.990g/mol)=0.03696\,mol/cm^3$

mol of $Cl/cm^3=0.03696\,mol/cm^3$, also

since there is one ion of each element per molecule.

Number of Na ions$/cm^3=0.03696N_A$

Number of Cl ions$/cm^3=0.03696N_A$

Total number of ions$/cm^3=(0.07392)(6.022\times10^{23})=4.45\times10^{22}$

Nearest neighbor distance = equilibrium separation r_0.

$$r_0=\left[\frac{1}{(4.45\times10^{22}ions/cm^3)(10^2cm/m)^3}\right]^{1/3}=2.88\times10^{-10}m=0.288nm$$

10-7. $r_0\left(KCl\right)=0.315nm=3.15\times10^{-10}m$

$N\left(ions/m^3\right)=1/r_0^3=3.20\times10^{28}/m^3=3.20\times10^{22}/cm^3$

Half of the ions in $1cm^3$ are K and half are Cl, so there are $1.60\times10^{22}/cm^3$ of each element.

This number of ions equals:

$$\frac{1.60\times10^{22}ions}{N_A}=\frac{1.60\times10^{22}ions}{6.022\times10^{23}/mol}=0.02657\,mol$$

This is the moles of each ion in 1 cm^3.

Molecular weight of K = 39.102 *g/mol*.

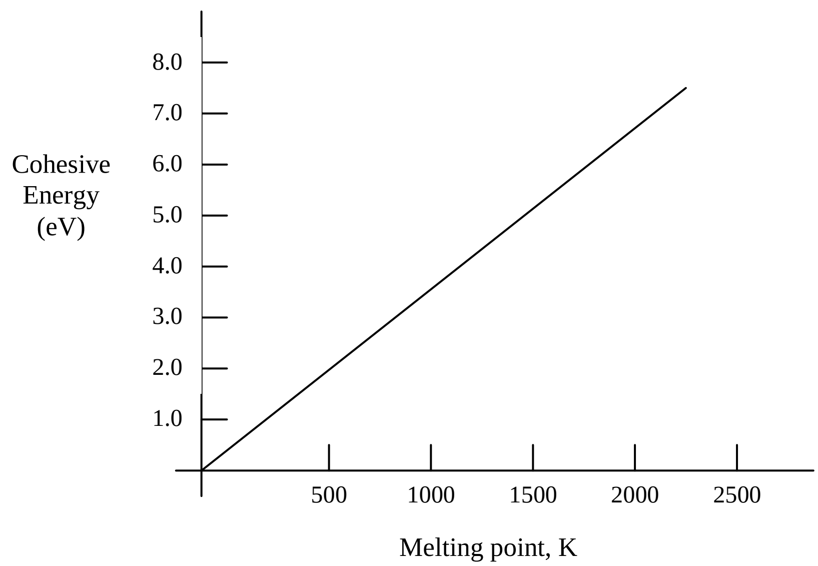
Molecular weight of Cl = 35.453 *g/mol*.

Weight of K/cm^3 = $\left(39.102g/mol\right)\left(0.02657mol/cm^3\right)=1.039\,g/cm^3$

Weight of Cl/cm^3 = $\left(35.453g/mol\right)\left(0.02657mol/cm^3\right)=0.942\,g/cm^3$

$\therefore$ density of $KCl=\left(1.039+0.942\right)g/cm^3=1.98\,g/cm^3$

10-8. (a)



(b) Noting that the melting points are in kelvins on the graph,

Co melting point = 1768 *K*, cohesive energy = 5.15 *eV*

Ag melting point = 1235 *K*, cohesive energy = 3.65 *eV*

Na melting point = 371 *K*, cohesive energy = 1.25 *eV*

10-9. $U_{att} = -ke^2\left(\frac{2}{a} + \frac{2}{2a} - \frac{2}{3a} + \frac{2}{4a} + \frac{2}{5a} - \frac{2}{6a} + \cdots\right)$

$$U_{att} = -ke^2\left(2 + 1 - \frac{2}{3} + \frac{1}{3} + \frac{2}{5} - \frac{1}{3} + \cdots\right)$$

The quantity in parentheses is the Madelung constant α. The 35th term of the series (2/35) is approximately 1% of the total, where α = 4.18.

10-10. (a) $\rho = \frac{m_e\langle v\rangle}{ne^2\lambda}$ (Equation 10-13)

$$= \frac{\left(9.11\times10^{-31}kg\right)\left(1.17\times10^5 m/s\right)}{\left(8.47\times10^{28} electrons/m^3\right)\left(1.60\times10^{-19}C\right)^2\left(0.4\times10^{-9}m\right)} = 1.23\times10^{-7}\,\Omega\bullet m$$

(b) $\langle v\rangle \propto \left(kT/m_e\right)^{1/2}$ (from Equation 10-9)

$$\langle v\rangle_{100} = \left(\frac{100K}{300K}\right)^{1/2} = \frac{1}{\sqrt{3}}$$

$$\rho_{100} = \rho_{300}\big/\sqrt{3} = 7.00\times10^{-8}\,\Omega\bullet m$$

10-11. (a) $j = \frac{I}{A} = \frac{I}{\pi d^2/4} = \frac{4\left(10^{-3}A\right)}{\pi\left(1.63\times10^{-3}m\right)^2} = 479A/m^3$ (from Equation 10-10)

(b) $v_d = \frac{I}{Ane} = \frac{d}{ne} = \frac{479A/m^2}{\left(8.47\times10^{28}/m^3\right)\left(1.602\times10^{-19}C\right)} = 3.53\times10^{-8}m/s$

$$= 3.53\times10^{-6}cm/s$$

10-12. (a) There are n_a conduction electrons per unit volume, each occupying a sphere of volume $4\pi r_s^3/3$: $n_a \times \left(4\pi r_s^3/3\right) = 1$

$$r_s^3 = \frac{3}{4\pi n_a} \quad\rightarrow\quad r_s = \left(3/4\pi n_a\right)^{1/3}$$

(Problem 10-12 continued)

(b) $$r_s = \left[\frac{3}{4\pi\left(8.47\times10^{28}/m^3\right)}\right]^{1/3} = 1.41\times10^{-10}m = 0.141nm$$

10-13. (a) $n = \rho N_A / M$ for 1 electron/atom

$$n = \frac{\left(10.5g/cm^3\right)\left(6.022\times10^{23}/mole\right)}{107.9g/mole} = 5.86\times10^{22}/cm^3$$

(b) $$n = \frac{\left(19.3g/cm^3\right)\left(6.022\times10^{23}/mole\right)}{196.97g/mole} = 5.90\times10^{22}/cm^3$$

Both agree with the values given in Table 10-3.

10-14. (a) $n = 2\rho N_A / M$ for two free electrons/atom

$$n = \frac{2\left(1.74g/cm^3\right)\left(6.022\times10^{23}/mole\right)}{24.31g/mole} = 8.62\times10^{22}/cm^3 = 8.62\times10^{28}/m^3$$

(b) $$n = \frac{2\left(7.1g/cm^3\right)\left(6.022\times10^{23}/mole\right)}{65.37g/mole} = 13.1\times10^{22}/cm^3 = 13.1\times10^{28}/m^3$$

Both are in good agreement with the values in Table 10-3, $8.61\times10^{28}/m^3$ for *Mg* and $13.2\times10^{28}/m^3$ for *Zn*.

10-15. (a) $\rho = \dfrac{m_e\langle v\rangle}{ne^2\lambda}$ (Equation 10-13) $\sigma = \dfrac{1}{\rho} = \dfrac{ne^2\lambda}{m_e\langle v\rangle}$ (Equation 10-13)

$$\rho = \frac{\left(9.11\times10^{-31}kg\right)\left(1.08\times10^5 m/s\right)}{\left(8.47\times10^{28}m^{-3}\right)\left(1.602\times10^{-19}C\right)^2\left(0.37\times10^{-9}m\right)} = 1.22\times10^{-7}\,\Omega\bullet m$$

$$\sigma = \frac{1}{\rho} = \frac{1}{1.22\times10^{-7}\,\Omega\bullet m} = 8.17\times10^6\left(\Omega\bullet m\right)^{-1}$$

(Problem 10-15 continued)

(b) $\langle v \rangle = \sqrt{\dfrac{8kT}{\pi m_e}}$ (Equation 10-9)

$$\rho(200K) = \rho(300K)\langle v(200K)\rangle / \langle v(300K)\rangle$$

$$= \rho(300K)(200K/300K)^{1/2}$$

$$= (1.22\times10^{-7}\,\Omega\bullet m)(200K/300K)^{1/2} = 9.96\times10^{-8}\,\Omega\bullet m$$

$$\sigma(200K) = \frac{1}{\rho(200K)} = \frac{1}{9.96\times10^{-8}\,\Omega\bullet m} = 1.00\times10^{7}\,(\Omega\bullet m)^{-1}$$

(c) $\rho(100K) = \rho(300K)\langle v(100K)\rangle / \langle v(300K)\rangle$

$$= (1.22\times10^{-7}\,\Omega\bullet m)(100K/300K)^{1/2} = 7.04\times10^{-8}\,\Omega\bullet m$$

$$\sigma(100K) = \frac{1}{\rho(100K)} = \frac{1}{7.04\times10^{-8}\,\Omega\bullet m} = 1.42\times10^{7}\,(\Omega\bullet m)^{-1}$$

10-16. $\langle E \rangle = \dfrac{3}{5}E_F$ (Equation 10-22)

(a) for *Cu:* $\langle E \rangle = \dfrac{3}{5}(7.06eV) = 4.24eV$

(b) for *Li:* $\langle E \rangle = \dfrac{3}{5}(4.77eV) = 2.86eV$

$$E_F = \frac{(hc)^2}{2mc^2}\left(\frac{3}{8\pi V}\times\frac{N}{V}\right)^{2/3}$$

$$= \frac{(1240eV\bullet nm)^2}{2(5.11\times10^5 eV)}\left[\frac{3(5.90\times10^{28}m^{-3})}{8\pi}\left(\frac{10^{-9}m}{1nm}\right)^3\right]^{1/3} = 5.53eV$$

10-17. A long, thin wire can be considered one-dimensional.

$$E_F = \frac{h^2}{32m}\left(\frac{N}{L}\right)^2 = \frac{(hc)^2}{32mc^2}\left(\frac{N}{L}\right)^2 \quad \text{(Equation 10-15)}$$

For *Mg:* $N/L = \left(8.61\times10^{28}/m^2\right)^{1/3}$

$$E_F = \frac{\left(1240eV\bullet nm\times10^{-9}m/nm\right)^2\left(8.61\times10^{28}/m^3\right)^{2/3}}{32\left(0.511\times10^6 eV\right)} = 1.87eV$$

10-18. (a) For *Ag:*

$$E_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3} = \frac{\left(1240eV\bullet nm\times10^{-9}m/nm\right)^2}{2\left(0.511\times10^6 eV\right)}\left(\frac{3\times5.86\times10^{28}m^{-3}}{8\pi}\right)^{2/3} = 5.50eV$$

For *Fe:* Similarly, $E_F = 11.2eV$

(b) For *Ag:* $E_F = kT_F$ (Equation 10-23)

$$T_F = \frac{E_F}{k} = \frac{5.50eV}{8.617\times10^{-5}eV/K} = 6.38\times10^4 K$$

For *Fe:* Similarly, $T_F = 13.0\times10^4 K$

Both results are in close agreement with the values given in Table 10-3.

10-19. Note from Figure 10-11 that most of the excited electrons are within about $2kT$ above the Fermi energy E_F, i.e., $\Delta E \approx 2kT$. Note, too, that $kT \ll E_F$, so the number ΔN of excited electrons is: $\Delta N \approx N(E_F)n(E_F)\Delta E \approx N(E_F)(1/2)(2kT) \approx N(E_F)kT$ and

$$N = \frac{8\pi V}{3}\left(\frac{2m}{h^2}\right)^{3/2} E_F^{3/2} \quad \text{(from Equation 10-20)}$$

Differentiating Equation 10-19 gives: $\Delta N(E_F) = \frac{\pi V}{2}\left(\frac{8m}{h^2}\right)^{3/2} E_F^{1/2}$

Then, $$\frac{\Delta N}{N} = \frac{\frac{\pi V}{2}\left(\frac{8m}{h^2}\right)^{3/2} E_F^{1/2}kT}{\frac{8\pi V}{3}\left(\frac{2m}{h^2}\right)^{3/2} E_F^{3/2}} = \frac{3}{2}kT\Big/E_F$$

(Problem 10-19 continued)

$$E_F \text{ for } Ag = 5.35eV, \text{ so } \frac{\Delta N}{N} = \frac{3}{2}\left(8.617\times10^{-5}eV/K\right)\left(300K\right)\Big/5.53eV = 0.0070 = 0.1\%$$

10-20. $u_F = \left(\frac{2E_F}{m_e}\right)^{1/2} = c\left(\frac{2E_F}{m_e c^2}\right)^{1/2}$ (Equation 10-24)

(a) for *Na*: $u_F = \left(3.00\times10^{8}m/s\right)\left[\frac{2\left(3.26eV\right)}{5.11\times10^{5}eV}\right]^{1/2} = 1.07\times10^{6}m/s$

(b) for *Au*: $u_F = \left(3.00\times10^{8}m/s\right)\left[\frac{2\left(5.55eV\right)}{5.11\times10^{5}eV}\right]^{1/2} = 1.40\times10^{6}m/s$

(c) for *Sn*: $u_F = \left(3.00\times10^{8}m/s\right)\left[\frac{2\left(10.3eV\right)}{5.11\times10^{5}eV}\right]^{1/2} = 1.90\times10^{6}m/s$

10-21. $\rho = \frac{m_e u_F}{ne^2\lambda}$ (Equation 10-25) $\qquad \lambda = \frac{m_e u_F}{ne^2\rho}$

(a) for *Na:* $\lambda = \frac{\left(9.11\times10^{-31}kg\right)\left(1.07\times10^{6}m/s\right)}{\left(2.65\times10^{28}m^{-3}\right)\left(1.609\times10^{-19}C\right)^2\left(4.2\times10^{-8}\Omega\bullet m\right)}$

$= 3.42\times10^{-8}m = 34.2nm$

(b) for *Au*: $\lambda = \frac{\left(9.11\times10^{-31}kg\right)\left(1.40\times10^{6}m/s\right)}{\left(5.90\times10^{28}m^{-3}\right)\left(1.609\times10^{-19}C\right)^2\left(2.04\times10^{-8}\Omega\bullet m\right)}$

$= 4.14\times10^{-8}m = 41.4nm$

(c) for *Sn*: $\lambda = \frac{\left(9.11\times10^{-31}kg\right)\left(1.90\times10^{6}m/s\right)}{\left(14.8\times10^{28}m^{-3}\right)\left(1.609\times10^{-19}C\right)^2\left(10.6\times10^{-8}\Omega\bullet m\right)}$

$= 4.31\times10^{-8}m = 43.1nm$

10-22. $C_v(electrons) = \frac{\pi^2}{2} R \frac{T}{T_F}$ (Equation 10-30)

$C_v(electrons) = \frac{\pi^2}{2} R \frac{kT}{E_F}$ because $E_F = kT_F$. C_v due to the lattice vibrations is 3*R*, assuming $T \gg T_E$ (rule of Dulong and Petit): $\frac{\pi^2}{2} R \frac{kT}{E_F} = 0.10(3R)$

$$T = \frac{0.10(3)(2)E_F}{\pi^2 k} = \frac{(0.60)(7.06eV)}{\pi^2 (8.617 \times 10^{-5} eV/K)} = 4.98 \times 10^3 K$$

This temperature is much higher than the Einstein temperature for a metal such as copper.

10-23. $U = \frac{3}{5} NE_F + \alpha N \left(\frac{kT}{E_F}\right) kT$ (Equation 10-29)

Average energy/electron = $U/N = \frac{3}{5} E_F + \alpha \left(\frac{kT}{E_F}\right) kT = \frac{3}{5} E_F + \frac{\pi^2}{4} \frac{(kT)^2}{E_F}$

For copper: $E = 7.06eV$, so

At *T* = 0*K*: $U/N = \frac{3}{5}(7.06eV) = 4.236eV$

At *T* = 300*K*: $U/N = \frac{3}{5}(7.06eV) + \frac{\pi^2}{4} \frac{(8.61 \times 10^{-5} eV/K)^2 (300K)^2}{7.06} = 4.236eV$

The average energy/electron at 300*K* is only 0.0002*eV* larger than at 0*K*, a consequence of the fact that 300*K* is very small compared to the T_F for *Cu* (81,600*K*). The classical value of $U/N = (3/2)kT = 0.039eV$, is far too small.

10-24. $C_v(electrons) = \frac{\pi^2}{2} R \frac{T}{T_F}$ (Equation 10-30)

Melting temperature of *Fe* = 1811*K* (from Table 10-1)

T_F for $Fe = 13 \times 10^4 K$ (from Table 10-3)

The maximum C_v for the *Fe* electrons, which is just before *Fe* melts, is:

(Problem 10-24 continued)

$$C_v\left(electrons\right)=\frac{\pi^2}{2}R\left(\frac{1811K}{13\times10^4K}\right)=0.0219R$$

The heat capacity of solids, including *Fe*, is 3*R* (rule of Dulong and Petit, see Section 8-1).

$$\frac{C_v\left(electrons\right)}{C_v}=\frac{0.0291R}{3R}=0.0073$$

10-25. $P=\dfrac{\rho_+-\rho_-}{\rho}=\dfrac{M}{\mu\rho}=\dfrac{\mu B}{kT}$ (Equation 10-35)

$$P=\frac{\left(9.285\times10^{-24}J/T\right)\left(2.0T\right)}{\left(1.38\times10^{-23}J/K\right)\left(200K\right)}=6.7\times10^{-3}$$

10-26. $\chi=\dfrac{\mu_0M}{B}=\dfrac{\mu_0\rho\mu^2}{kT}$

$$\chi\text{ units }=\left(\frac{N}{A^2}\right)\left(\frac{1}{m^3}\right)\left(\frac{J}{T}\right)^2\left(\frac{1}{J}\right)$$

$$=\frac{NJ^2}{A^2m^3T^2J}=\frac{NJ}{A^2m^3\left(Wb/m\right)^2}=\frac{NJ}{A^2m^3\left(N/Am\right)^2}=\frac{NJA^2m^2}{A^2m^3N^2}$$

$$=\frac{J}{Nm}=\frac{Nm}{Nm}=1\text{ dimensionless}$$

10-27. $E=hc/\lambda$

(a) For *Si*: $\lambda=hc/E=1240eV{\bullet}nm/1.14eV=1.088\times10^3nm=1.09\times10^{-6}m=1.09\times10^3nm$

(b) For *Ge*: Similarly, $\lambda=1.722\times10^3nm=1.72\times10^{-6}m=1.72\times10^3nm$

(c) For diamond: Similarly, $\lambda=1.77\times10^2nm=1.72\times10^{-7}m$

10-28. (a) For Ge: All visible light frequencies (wavelengths) can excite electrons across the 0.72eV band gap, being absorbed in the process. No visible light will be transmitted through the crystal.

(Problem 10-28 continued)

(b) For insulator: No visible light will be absorbed by the crystal since no visible frequencies can excite electrons across the 3.6eV band gap. The crystal will be transparent to visible light.

(c) Visible light wavelengths are 380-720*nm* corresponding to photon energies

$E = hf = hc/\lambda = 1240 eV \bullet nm/\lambda$

$\lambda = 380nm \rightarrow E = 3.3eV$

$\lambda = 720nm \rightarrow E = 1.7eV$

Visible light photons will be absorbed, exciting electrons for band gaps $\leq 3.3eV$.

10-29. (a) $E = hc/\lambda = 1240 eV \bullet nm \Big/ \left(3.35\mu m \times 10^3 nm/\mu m\right) = 0.37eV$

(b) $E = kT = 0.37eV \therefore T = 0.37eV/k = 0.37eV/8.617 \times 10^{-5} eV/K = 4300K$

10-30. (a) $N = \dfrac{mN_A}{M} = \dfrac{\rho V N_A}{M} = \dfrac{\left(2.33g/cm^3\right)\left(100nm \times 10^{-7} cm/nm\right)^3 \left(6.02 \times 10^{23}/mol\right)}{28g/mol}$

$= 5.01 \times 10^7$ Si atoms

(b) $\Delta E = 13eV \Big/ \left(4 \times 5.01 \times 10^7\right) = 6.5 \times 10^{-8} eV$

10-31. (a) $E_1 = -\dfrac{1}{2}\left(\dfrac{ke^2}{\hbar}\right)^2 \dfrac{m^*}{\kappa^2}\dfrac{1}{(1)^2}$ (Equation 10-43)

$$E_1 = -\frac{1}{2}\frac{\left[\left(9 \times 10^9 N \bullet m^2/C^2\right)\left(1.60 \times 10^{-19} C\right)^2\right]^2}{\left(1.055 \times 10^{-34} J \bullet s\right)^2} \times \frac{(0.2)\left(9.11 \times 10^{-31} kg\right)}{(11.8)^2}$$

$= -3.12 \times 10^{-21} J = -0.0195eV$

Ionization energy = 0.0195*eV*

(b) $\langle r_1 \rangle = a_0 (1)^2 \left(m_e/m^*\right)\kappa$ (Equation 10-44)

$= 0.0529nm(1/0.2)(11.8) = 3.12nm$

(c) $E_g(Si) = 1.11eV$ at $293K$

(Problem 10-31 continued)

$E_1 / E_g = 0.0195 / 1.11 = 0.0176$ or about 2%.

10-32. (a) $E_1 = -\frac{1}{2}\left(\frac{ke^2}{\hbar}\right)^2 \frac{m^*}{\kappa^2}\frac{1}{(1)^2}$ (Equation 10-43)

$$E_1 = -\frac{1}{2}\frac{\left[\left(9\times10^9 N\bullet m^2/C^2\right)\left(1.60\times10^{-19}C\right)^2\right]^2}{\left(1.055\times10^{-34}J\bullet s\right)^2}\times\frac{(0.34)\left(9.11\times10^{-31}kg\right)}{(15.9)^2}$$

$$= -2.92\times10^{-21}J = -0.0182eV$$

(b) $\langle r_1\rangle = a_0(1)^2\left(m_e/m^*\right)\kappa$ (Equation 10-44)

$$= 0.0529nm(1/0.34)(15.9) = 2.48nm$$

10-33. Electron configuration of *Si*: $1s^2\,2s^2\,2p^6\,3s^2\,3p^2$

(a) *Al* has a $3s^2\,3p$ configuration outside the closed *n* = 2 shell (3 electrons), so a *p*-type semiconductor will result.

(b) *P* has a $3s^2\,3p^3$ configuration outside the closed *n* = 2 shell (5 electrons), so an *n*-type semiconductor results.

10-34. $E = kT = 0.01eV \quad \therefore \quad T = 0.01eV / 8.617\times10^{-5}eV/K = 116K$

10-35. (a) $V_H = v_d Bw = \frac{dBw}{nq} = \frac{iB}{qnt}$ (Equation 10-45 and Example 10-9)

The density of charge carriers *n* is:

$$n = \frac{iB}{qtV_H} = \frac{(20A)(0.25T)}{\left(1.60\times10^{-19}C\right)\left(0.2\times10^{-3}m\right)\left(2.2\times10^{-6}V\right)} = 7.10\times10^{28}\ \text{carriers/m}^3$$

(b) $N = \frac{\rho N_A}{M} = \frac{\left(5.75\times10^3 kg/m^3\right)\left(6.02\times10^{26}/mol\right)}{118.7kg/mol} = 2.92\times10^{28}$

Each *Sn* atom contributes $n/N = 7.10\times10^{28}/2.92\times10^{28} = 2.4$ charge carriers

10-36. $I_{net} = I_0\left(e^{eV_b/kT} - 1\right)$ (Equation 10-49)

(a) $e^{eV_b/kT} = 10$, so $eV_b/kT = \ln(10)$. Therefore,

$$\left(1.60\times10^{-19}C\right)V_b \Big/ \left(1.381\times10^{-23}J/K\right)\left(300K\right) = \ln(10)$$

$$V_b = \left(1.38\times10^{-23}J/K\right)\left(300K\right)\ln(10)\Big/\left(1.60\times10^{-19}C\right) = 0.0596V = 59.6mV$$

(b) $e^{eV_b/kT} = 0.1$

$$V_b = 0.0596V\left(\ln 0.1/\ln 10\right) = -0.0596 = -59.6mV$$

10-37. $I_{net} = I_0\left(e^{eV_b/kT} - 1\right)$ (Equation 10-49)

(a) $e^{eV_b/kT} = 5$, so $eV_b/kT = \ln(5)$. Therefore,

$$V_b = \frac{kT\ln(5)}{e} = \frac{\left(1.38\times10^{-23}J/K\right)\left(200K\right)\ln(5)}{\left(1.60\times10^{-19}C\right)} = 0.0278V = 27.8mV$$

(b) $e^{eV_b/kT} = 0.5$

$$eV_b/kT = \ln(0.5)$$

$$V_b = \frac{kT\ln(0.5)}{e} = \frac{\left(1.38\times10^{-23}J/K\right)\left(200K\right)\ln(0.5)}{\left(1.60\times10^{-19}C\right)} = -0.0120V = -12.0mV$$

10-38. $I_{net} = I_0\left(e^{eV_b/kT} - 1\right)$ (Equation 10-49)

Assuming $T = 300K$,

$$\frac{I(0.2V) - I(0.1V)}{I(0.1V)} = \frac{I_0\left(e^{e(0.2V)/kT} - 1\right) - I_0\left(e^{e(0.1V)/kT} - 1\right)}{I_0\left(e^{e(0.1V)/kT} - 1\right)} = \frac{e^{e(0.2V)/kT} - e^{e(0.1V)/kT}}{\left(e^{e(0.1V)/kT} - 1\right)} = 47.7$$

10-39. (a) From Equation 10-49, $\exp\left(eV_b/kT\right) = 10$

Taking *ln* of both sides and solving for V_b,

$$V_b = \left(kT/e\right)\ln(10) = \frac{\left(1.38\times10^{-23}J/K\right)\left(77K\right)\ln(10)}{\left(1.60\times10^{-19}C\right)} = 0.0153\text{ volts} = 15.3mV$$

(Problem 10-39 continued)

(b) Similarly, for $exp\left(eV_b / kT\right) = 1; \quad V_b = 0$

(c) For (a): $I_{net} = I_0\left(e^{eV_b/kT} - 1\right) \quad \therefore \quad I_{net} = 1mA(10-1) = 9mA$

For (b): $I_{net} = 0$

10-40. $E = hc/\lambda$ For $\lambda = 484nm$ $E = E_{gap}$

$$E_{gap} = hc/484nm = 1240eV \bullet nm/484nm = 2.56eV$$

10-41. $M^{\alpha}T_c =$ constant (Equation 10-55)

First, we find the constant for *Pb* using the mass of natural *Pb* from Appendix A, T_c for *Pb* from Table 10-6, and α for *Pb* from Table 10-7.

$$\text{constant} = (207.19u)^{0.49}(7.196K) = 98.20$$

For $^{206}Pb: T_c = \text{constant}/M^{\alpha} = 98.20 \big/ (205.974u)^{0.49} = 7.217K$

For $^{207}Pb: T_c = \text{constant}/M^{\alpha} = 98.20 \big/ (206.976u)^{0.49} = 7.200K$

For $^{208}Pb: T_c = \text{constant}/M^{\alpha} = 98.20 \big/ (207.977u)^{0.49} = 7.183K$

10-42. (a) $E_g = 3.5kT_c$ (Equation 10-56)

T_c for I is $3.408K$, so, $E_g = 3.5\left(8.617\times10^{-5}eV/K\right)\left(3.408K\right) = 1.028\times10^{-3}eV$

(b) $E_g = hc/\lambda$

$$\lambda = hc/E_g = 1240eV \bullet nm/1.028\times10^{-3}eV = 1.206\times10^{6}nm$$

$$= 1.206\times10^{-3}m = 1.206mm$$

10-43. (a) $E_g = 3.5kT_c$ For Sn: $T_c = 3.722K$

$$E_g = 3.5\left(8.617\times10^{-5}eV/K\right)\left(3.722K\right) = 0.0011eV$$

(Problem 10-43 continued)

(b) $E_g = hc/\lambda$

$$\lambda = hc/E_g = 1240 eV \bullet nm / 6\times10^{-4} eV = 2.07\times10^{6} nm = 2.07\times10^{-3} m$$

10-44. At $T/T_c = 0.5$ $E_g = (T)/E_g(0) = 0.95$ where $E_g(0) = 3.5kT_c$ (Equation 10-56)

So, $E_g(T) = 0.95(3.5)kT_c = 3.325kT_c$

(a) For Sn: $E_g(T) = 3.325\left(8.617\times10^{-5} eV/K\right)(3.722K) = 1.07\times10^{-3} eV$

(b) For Nb: $E_g(T) = 3.325\left(8.617\times10^{-5} eV/K\right)(9.25K) = 2.65\times10^{-3} eV$

(c) For Al: $E_g(T) = 3.325\left(8.617\times10^{-5} eV/K\right)(1.175K) = 3.37\times10^{-4} eV$

(d) For Zn: $E_g(T) = 3.325\left(8.617\times10^{-5} eV/K\right)(0.85K) = 2.44\times10^{-4} eV$

10-45. $B_C(T)/B_C(0) = 1-(T/T_C)^2$

(a) $B_C(T)/B_C(0) = 0.1 = 1-(T/T_C)^2$

$$(T/T_C)^2 = 1-0.1 = 0.9$$

$$T/T_C = 0.95$$

(b) Similarly, for $B_C(T)/B_C(0) = 0.5$

$$T/T_c = 0.71$$

(c) Similarly, for $B_C(T)/B_C(0) = 0.9$

$$T/T_c = 0.32$$

10-46. 1. Referring to Figure 10-56, notice that the length of the diagonal of a face of the cube is r; therefore, $a^2 + a^2 = (4r)^2 \Rightarrow a = \sqrt{8}\, r$

2. Volume of the unit cube $V_{cube} = a^3 = 8^{3/2} r^3$

(Problem 10-46 continued)

3. The cube has 8 corners, each occupied by ⅛ of an atom's volume. The cube has 6 faces, each occupied by ½ of an atom's volume. The total number of atoms in the unit cube is then 8×⅛ + 6×½ = 4. Each atom has volume $4\pi r^3/3$. The total volume occupied by atoms is then $V_{atoms} = 4\times(4\pi r^3/3)$.

4. The fraction of the cube's volume occupied by atoms is then:

$$\frac{V_{atoms}}{V_{cube}} = \frac{4\times(4\pi r^3)}{8^{3/2}\,r^3} = 0.74$$

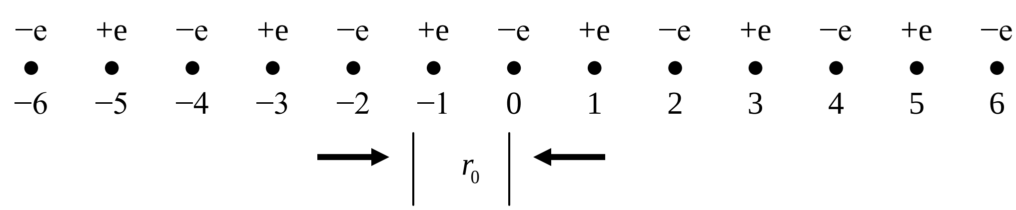
10-47. T_F for *Cu* is 81,700*K*, so only those electrons within $E_F - kT$ of the Fermi energy could be in states above the Fermi level. The fraction *f* excited above E_F is approximately:

$f = kT/E_F = T/T_F$

(a) $f = 300K/81{,}700K = 3.7\times10^{-3}$

(b) $f = 1000K/81{,}700K = 12.2\times10^{-3}$

10-48.



(a) For the negative ion at the origin (position 0) the attractive potential energy is:

$$V = -\frac{2ke^2}{r_0}\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots\right)$$

(b) $V = -\alpha\dfrac{ke^2}{r_0}$, so the Madelung constant is:

$$\alpha = 2\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots\right)$$

(Problem 10-48 continued)

Noting that $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots,\qquad \ln(2) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$

and $\alpha = 2\ln(2) = 1.386$

10-49. $C_v = \frac{\pi^2}{2} R \frac{T}{T_F}$ (Equation 10-30)

$$T_F = \frac{\pi^2}{2} R \frac{T}{\left(3.74\times10^{-4}\,J/mol\bullet K\right)T}$$

and $E_F = kT_F = \frac{\pi^2}{2} R \frac{kT}{\left(3.74\times10^{-4}\,J/mol\bullet K\right)T}$

$$= \frac{\pi^2\left(8.314J/mol\bullet K\right)\left(1.38\times10^{-23}\,J/K\right)}{2\left(3.74\times10^{-4}\,J/mol\bullet K\right)}$$

$$= 1.51\times10^{-18}\,J\left(1/1.60\times10^{-19}\,J/eV\right) = 9.45eV$$

10-50. (a) $N = \int_0^{E_F} g(E)\,dE = \int_0^{E_F} AE^{1/2}dE = A(2/3)E^{3/2}\Big|_0^{E_F} = (2A/3)E_F^{3/2}$

(b) $N' = \int_{E_F-kT}^{E_F} AE^{1/2}dE = (2A/3)\left[E_F^{3/2} - \left(E_F - kT\right)^{3/2}\right]$

$$= (2A/3)\left[E_F^{3/2} - E_F^{3/2}\left(1 - kT/E_F\right)^{3/2}\right]$$

Because $kT \ll E_F$ for most metals,

$$\left(1 - kT/E_F\right)^{3/2} \approx 1 - (3/2)kTE_F^{1/2}$$

$$N' = (2A/3)\left[E_F^{3/2} - E_F^{3/2} + (3/2)kTE_F^{1/2}\right] = AkTE_F^{1/2}$$

The fraction within *kT* of E_F is then $f = \frac{N'}{N} = \frac{AkTE_F^{1/2}}{(2A/3)E_F^{3/2}} = \frac{3kT}{2E_F}$

(c) For *Cu* $E_F = 7.04eV$; at $300K$, $\quad f = \frac{3(0.02585eV)}{2(7.04eV)} = 0.0055$

10-51. (a) $N_1 = \left(1.17 \times 10^6 eV / photon\right) \big/ \left(0.72 eV / pair\right) = 1.63 \times 10^6 \, pairs / photon$

$$N_2 = \left(1.33 \times 10^6 eV / photon\right) \big/ \left(0.72 eV / pair\right) = 1.85 \times 10^6 \, pairs / photon$$

(b) $\Delta N_1 = \sqrt{N_1} = 1.23 \times 10^3 \rightarrow \Delta N_1 / N_1 = 7.8 \times 10^{-4}$

$$\Delta N_2 = \sqrt{N_2} = 1.36 \times 10^3 \rightarrow \Delta N_2 / N_2 = 7.4 \times 10^{-4}$$

(c) Energy resolution $\dfrac{\Delta E}{E} \approx \dfrac{\Delta N}{N}$

$$\therefore \quad \Delta E_1 / E_1 \approx 0.078\%$$

$$\Delta E_2 / E_2 \approx 0.074\%$$

10-52. $\lambda = \dfrac{m_{eff} \langle v \rangle}{n \rho e^2}$ (Equation 10-13)

$$\langle v \rangle = \left(3kT / m_{eff}\right)^{1/2} = \left[\frac{3\left(1.38 \times 10^{-23} J/K\right)\left(300K\right)}{0.2\left(9.11 \times 10^{-31} kg\right)}\right]^{1/2} = 2.61 \times 10^5 m/s$$

Substituting into λ:

$$\lambda = \frac{0.2\left(9.11 \times 10^{-31} kg\right)\left(2.61 \times 10^5 m/s\right)}{\left(10^{22} m^{-3}\right)\left(5 \times 10^{-3} \Omega \bullet m\right)\left(1.60 \times 10^{-19} C\right)^2} = 3.7 \times 10^{-8} m = 37 nm$$

For *Cu*: $u_F = \left(2E_F / m_e\right)^{1/2} = \left[\dfrac{2\left(7.06 eV\right)}{9.11 \times 10^{-31} kg}\right]^{1/2} = 1.57 \times 10^6 m/s$

$n = 8.47 \times 10^{28} m^{-3}$ and $\rho = 1.7 \times 10^{-8} \Omega \bullet m$ (Example 10-5)

Substituting as above, $\lambda = 3.9 \times 10^{-8} m = 39 nm$

The mean free paths are approximately equal.

10-53. Compute the density of Cu (1) as if it were an fcc crystal and (2) as if it were a bcc crystal. The result closest to the actual measured density is the likely crystalline form.

1. The unit cube of an fcc crystal of length a on each side composed of hard, spherical atoms each of radius r contains 4 atoms. The side and radius are related by $a = \sqrt{8}\,r$ and the volume of the cube is $V_{cube} = a^3 = 8^{3/2} r^3$. (See problem 10-46 and Appendix B3.) The density of fcc Cu would be:

$$d = \frac{m_{Cu}}{V_{cube}} = \frac{4(M / N_A)}{8^{3/2} r^3}$$

where M = molar mass (atomic weight) and N_A = Avagadro's number.

$$d = \frac{4(63.55\,\text{g/mol})}{8^{3/2}(1.28\times10^{-8}\,\text{cm})^3(6.022\times10^{23}\,/\,\text{mol})} = 8.90\,\text{g/cm}^3$$

2. For a bcc crystal (refer to Appendix B3), let the length of a side = a, the diagonal of a face = f, and the diagonal through the body = b. Then from geometry we have $a^2 + f^2 = b^2 = (4r)^2$ and $a^2 + a^2 = f^2$. Combing these yields:

$$a^2 + f^2 = 3a^2 = (4r)^2 \Rightarrow a = \frac{4r}{\sqrt{3}}$$

The unit cube has 8 corners with ⅛ of a Cu atom at each corner plus 1 Cu atom at the center, or 8×⅛ + 1 = 2 atoms. The density of bcc Cu is then:

$$d = \frac{m}{V_{cube}} = \frac{2(M / N_A)}{(4r/\sqrt{3})^3} = \frac{2(63.55\,\text{g/mol})\times 3^{3/2}}{4^3(1.28\times10^{-8}\,\text{cm})^3(6.022\times10^{23}\,/\text{mol})} = 8.17\,\text{g/cm}^3$$

Based on the above density calculations, metallic Cu is most likely a face-centered cubic crystal.

10-54. (a) For small V_b (from Equation 10-49)

$$e^{eV_b/kT} \approx 1 + eV_b/kT, \text{ so } I = I_0 eV_b/kT = V_b/R$$

$$R = V_b\big/\left(I_0 eV_b/kT\right) = kT\big/eI_0 = 0.025eV\big/\left(e\times10^{-9}A\right) = 25.0M\Omega$$

(b) For $V_b = -0.5V$; $\quad R = V_b/I = 0.5\big/10^{-9}A = 500M\Omega$

(c) For $V_b = +0.5V$; $\quad I = 10^{-9}A\left(e^{0.5/0.025} - 1\right) = 0.485A$

Thus, $R = V_b/I = 0.5/0.485 = 1.03\Omega$

(d) $\dfrac{dI}{dV_b} = \dfrac{eI_0}{kT =} e^{eV_b/kT} \qquad R_{ac} = \dfrac{dV_b}{dI} = \dfrac{kT}{eI_0}e^{-eV_b/kT} = 25M\Omega e^{-20} = 0.0515\Omega$

10-55. $a_0 = \dfrac{\varepsilon_0 h^2}{\pi m_e e^2} = \dfrac{\kappa\varepsilon_o h^2}{\pi\left(m_e\right)_{eff}e^2} = \dfrac{\kappa\hbar^2}{\left(m_e\right)_{eff}ke^2}$

silicon: $a_0 = \dfrac{12\left(1.055\times10^{-34}J\bullet s\right)^2}{0.2\left(9.11\times10^{-31}kg\right)\left(9\times10^9 N\bullet m^2/C^2\right)\left(1.602\times10^{-19}C\right)^2}$

$$= 3.17\times10^{-9}m = 3.17nm$$

This is about 14 times the lattice spacing in silicon (0.235nm)

germanium: : $a_0 = \dfrac{16\left(1.055\times10^{-34}J\bullet s\right)^2}{0.10\left(9.11\times10^{-31}kg\right)\left(9\times10^9 N\bullet m^2/C^2\right)\left(1.602\times10^{-19}C\right)^2}$

$$= 8.46\times10^{-9}m = 8.46nm$$

This is nearly 35 times the lattice spacing in germanium (0.243nm)

10-56. (a) $E_n = -\dfrac{1}{2}\left(\dfrac{ke^2}{\hbar}\right)^2\dfrac{m^*}{\kappa^2}\dfrac{1}{n^2}$ (Equation 10-44)

$$= -\frac{1}{2}\left[\frac{1.440eV\bullet nm}{6.582\times10^{-16}eV\bullet s}\right]^2\left[\frac{0.015\left(0.511\times10^6 eV/c^2\right)}{18^2}\right]\frac{1}{n^2}$$

or

(Problem 10-56 continued)

$$= -\frac{1}{2}\left[\frac{2\pi\left(9\times10^{9}N\bullet m^{2}/C^{2}\right)\left(1.60\times10^{-19}C\right)^{2}}{6.63\times10^{-34}J\bullet s}\right]^{2}\left[\frac{0.015\left(9.11\times10^{-31}kg\right)}{18^{2}}\right]\frac{1}{n^{2}}$$

$$= -\frac{1.01\times10^{-22}}{n^{2}}J = -6.28\times10^{-4}eV/n^{2}$$

$\therefore$ donor ionization energy = $6.28\times10^{-4}eV$

(b) $r_1 = a_0\dfrac{m_e}{m^*}\kappa$ (Equation 10-45)

$$= 0.0529nm\left(1/0.015\right)18$$

$$r_1 = 63.5nm$$

(c) Donor atom ground states will begin to overlap when atoms are $2r_1 = 127nm$ apart.

donor atom concentration $= \left(\dfrac{10^{9}nm/m}{127nm/atom}\right)^{3} = 4.88\times10^{20}m^{-3}$

10-57. (a) $\rho = \dfrac{m_e u_F}{ne^2\lambda}$ (Equation 10-25)

So the equation in the problem can be written as $\rho = \rho_m + \rho_i$. Because the impurity increases ρ_m by $1.1\times10^{-8}\Omega\bullet m$, $\rho_i = 1.1\times10^{-8}\Omega\bullet m$ and

$\lambda_i = \dfrac{m_e u_F}{ne^2\left(1.18\times10^{-8}\Omega\bullet m\right)}$ where $n = 8.47\times10^{28}electrons/m^3$ (from Table 10-3)

and $u_F = \left(2E_F/m_e\right)^{1/2} = 1.57\times10^{6}m/s$. Therefore,

$$\lambda_i = \frac{\left(9.11\times10^{-31}kg\right)\left(1.57\times10^{6}m/s\right)}{\left(8.47\times10^{28}/m^{3}\right)\left(1.60\times10^{-19}C\right)^{2}\left(1.1\times10^{-8}\Omega\bullet m\right)} = 6.00\times10^{-8}m = 60.0nm$$

(b) $\lambda = 1/n_a\pi r^2$ and $d = 2r$ (Equation 10-12)

So we have $d^2 = 4/n_i\pi\lambda_i$ where n_i = 1% of $n = 8.47\times10^{26}/m^3$

$$d^2 = 4/\left(8.47\times10^{28}/m^{3}\right)\pi\left(6.60\times10^{-8}m\right) = 2.28\times10^{-22}m^{2} \rightarrow d = 0.0151nm$$

10-58. (a) The modified Schrödinger equation is:

$$-\frac{\hbar^2}{2m^* r^2}\frac{d}{dr}\left(r^2\frac{dR(r)}{dr}\right)+\left[-\frac{ke^2}{r\kappa}+\frac{\hbar^2\ell(\ell+1)}{2m^* r^2}\right]R(r)=ER(r)$$

The solution of this equation, as indicated following Equation 7-24, leads to solutions of the form: $R_{n\ell}(r)=a_0' e^{-r/a_0' n} r^{-1}\mathcal{L}_{n\ell}(r/a_0')$, where $a_0'=(\hbar^2\kappa^2/ke^2 m)$

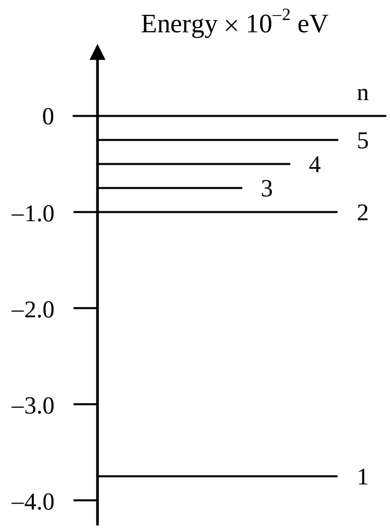
(b) By substitution into Equation 7-25, the allowed energies are:

$$E_n=-\frac{1}{2}\left(\frac{ke^2}{\hbar\kappa}\right)\frac{m^*}{n^2}=-\frac{E_1}{n^2}\text{ where }E_1=\frac{1}{2}\left(\frac{ke^2}{\kappa}\right)^2 m^*$$

(c) For *As* electrons in *Si*: $m^*=0.2m_e$ (see Problem 10-31) and $\kappa(Si)=11.8$,

$$E_1=-\frac{1}{2}\frac{\left[(9\times10^9 N\bullet m^2/C^2)(1.60\times10^{-19}C)^2\right]^2}{(1.055\times10^{-34}J\bullet s)^2}\times\frac{0.2(9.11\times10^{-31}kg)}{(11.8)^2}$$

$$=-3.12\times10^{-21}J=-0.0195eV$$



10-59. $U=-\alpha\frac{ke^2}{r_0}\left[\frac{r_0}{r}-\frac{1}{n}\left(\frac{r_0}{r}\right)^n\right]$ (Equation 10-5)

$$F=-\frac{dU}{dr}=-Kr\text{ yields }K=\alpha\frac{(n-1)ke^2}{r_0^3}$$

(a) For *NaCl*: $\alpha=1.7476$, $n=9.35$, and $r_0=0.282nm$ and

(Problem 10-59 continued)

$$\mu = \frac{m(Na)m(Cl)}{m(Na)+m(Cl)} = \frac{(22.99u)(35.45u)}{(22.99u)+(35.45u)} = 13.95u$$

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{M}} = \frac{1}{2\pi}\left[\frac{\alpha(n-1)ke^2}{(13.95u)r_0^3}\right]^{1/2}$$

$$= \frac{1}{2\pi}\left[\frac{(1.7476)(8.35)(9\times10^9\,N\bullet m^2/C^2)(1.60\times10^{-19}C)^2}{(13.95u\times1.66\times10^{-27}kg/u)(0.282\times10^{-9}m)^3}\right]^{1/2} = 1.28\times10^{13}Hz$$

(b) $\lambda = c/f = (.00\times10^8 m/s)/1.28\times10^{13}Hz = 23.4\mu m$

This is of the same order of magnitude as the wavelength of the infrared absorption bands in *NaCl*.

10-60. (a) Electron drift speed is reached for:

$$\frac{dv}{dt} = 0 \rightarrow v_d = -e\mathcal{E}\tau/m$$

(b) Writing Ohm's law as $j = \sigma\mathcal{E}$ and $j = |v_d|ne$ (from Equation 10-11)

$j = e\mathcal{E}\tau ne/m = \mathcal{E}\tau ne^2/m,$ which satisfies Ohm's law because $j \propto \mathcal{E}.$ Thus,

$\sigma = \tau ne^2/m$ and $\rho = 1/\sigma = m/\tau ne^2.$

10-61. (a) For *r*, *s*, and *t* all even $(-1)^{r+s+t} = +1$ and the ion's charge at that location is:

$-1(1.60\times10^{-19}C) = 1.60\times10^{-19}C.$

Similarly, for any permutation of

r, s even; *t* odd: $(-1)^{r+s+t} = -1,$ ion charge = $-1.60\times10^{-19}C.$

r even; *s, t* odd: $(-1)^{r+s+t} = +1,$ ion charge = $-1.60\times10^{-19}C.$

r, s, and *t* all odd: $(-1)^{r+s+t} = +1,$ ion charge = $-1.60\times10^{-19}C.$

(Problem 10-61 continued)

(b) $U = -\alpha \dfrac{ke^2}{r}$

If the interatomic distance $r = a$, then a cube *2a* on each side

$$U = -ke^2\left(\frac{4}{a} - \frac{4}{\sqrt{2}a} + \frac{2}{a} - \frac{4}{\sqrt{2}a} - \frac{4}{\sqrt{2}a} + \frac{4}{\sqrt{3}a} + \frac{4}{\sqrt{3}a}\right)$$

$$U = -\frac{ke^2}{a}(2.1335) \quad \text{where } \alpha = 2.1335.$$

Similarly, for larger cubes (using spreadsheet). The value of α is approaching 1.7476 slowly.

10-62. (a) $M = \mu(\rho_+ - \rho_-) \rightarrow \dfrac{M}{\rho} = \mu \dfrac{(\rho_+ - \rho_-)}{\rho}$

$$\frac{M}{\rho} = \mu \frac{e^{\mu B/kT} - e^{-\mu B/kT}}{e^{\mu B/kT} + e^{-\mu B/kT}} = \mu \tanh(\mu B/kT)$$

and $M = \mu\rho \tanh(\mu B/kT)$

(b) For $\mu B \ll kT$, $T \gg 0$ and $\tanh(\mu B/kT) \approx \mu B/kT$

$$\chi = \frac{\mu_0 M}{B} = \frac{\mu_0 \mu\rho\mu B}{BkT} = \frac{\mu_0 \rho\mu^2}{kT}$$

Chapter 11 – Nuclear Physics

11-1.

Isotope	Protons	Neutrons
^{18}F	9	9
^{25}Na	11	14
^{51}V	23	28
^{84}Kr	36	48
^{120}Te	52	68
^{148}Dy	66	82
^{175}W	74	101
^{222}Rn	86	136

11-2. The momentum of an electron confined within the nucleus is:

$$\Delta p \approx \hbar / \Delta x = 1.055\times10^{-34}\, J\bullet s / 10^{-14}\, m$$

$$\approx 1.055\times10^{-20}\, J\bullet s/m \times \left(1/1.602\times10^{-13}\, J/MeV\right)$$

$$\approx 6.59\times10^{-8}\, MeV\bullet s/m$$

The momentum must be at least as large as Δp, so $p_{\min} \geq 6.59\times10^{-8}\, MeV\bullet s/m$ and the electron's kinetic energy is:

$$E_{\min} = p_{\min}c = \left(6.59\times10^{-8}\, MeV\bullet s/m\right)\left(3.00\times10^{8}\, m/s\right) = 19.8 MeV.$$

This is twenty times the observed maximum beta decay energy, precluding the existence of electrons in the nucleus.

11-3. A proton-electron model of ^{6}Li would consist of 6 protons and 3 electrons. Protons and electrons are spin −1/2 (Fermi-Dirac) particles. The minimum spin for these particles in the lowest available energy states is $1/2\hbar$, so $^{6}Li\ (S=0)$ cannot have such a structure.

11-4. A proton-electron model of ^{14}N would have 14 protons and 7 electrons. All are Fermi-Dirac spin-1/2 particles. In the ground state the proton magnetic moments would add to a small fraction of the proton magnetic moment of $2.8\mu_N$, but the unpaired electron would give the system a magnetic moment of the order of that of an electron, about $1\mu_B$. Because μ_B is approximately 2000 times large than μ_N, the ^{14}N magnetic moment would be about 1000 times the observed value, arguing against the existence of electrons in the nucleus.

11-5. The two proton spins would be antiparallel in the ground state with $S = 1/2 - 1/2 = 0$. So the deuteron spin would be due to the electron and equal to $1/2\hbar$. Similarly, the proton magnetic moments would add to zero and the deuteron's magnetic moment would be $1\mu_B$. From Table 11-1, the observed spin is $1\hbar$ (rather than $1/2\hbar$ found above) and the magnetic moment is $0.857\mu_N$, about 2000 times smaller than the value predicted by the proton-electron model.

11-6.

	Isotopes			Isotones	
(a)	^{18}F	^{17}F	^{19}F	^{16}N	^{17}O
(b)	^{208}Pb	^{206}Pb	^{210}Pb	^{207}Tl	^{209}Bi
(c)	^{120}Sn	^{119}Sn	^{118}Sn	^{121}Sb	^{122}Te

11-7.

	Nuclide	Isobars		Isotopes
(a)	${}^{14}_{8}O_6$	${}^{14}_{6}C_8$	${}^{14}_{7}N_7$	${}^{16}_{8}O_8$
(b)	${}^{63}_{28}Ni_{35}$	${}^{63}_{29}Cu_{34}$	${}^{63}_{30}Zn_{33}$	${}^{60}_{28}Ni_{32}$
(c)	${}^{236}_{93}Np_{143}$	${}^{236}_{92}U_{144}$	${}^{236}_{94}Pu_{142}$	${}^{235}_{93}Np_{142}$

11-8. mass $= A\text{u} = A\left(1.66\times10^{-27}\,kg/\text{u}\right)$ volume $=(4/3)\pi R^3 = (4/3)\pi\left(R_0 A^{1/3}\right)^3$

where $R_0 = 1.2\,fm = 1.2\times10^{-15}\,m$

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{A\left(1.66\times10^{-27}\,kg/\text{u}\right)}{(4/3)\pi\left(1.2\times10^{-15}\,m\right)^3 A} = 2.29\times10^{17}\,kg/m^3$$

11-9. $B = ZM_H c^2 + Nm_N c^2 - M_A c^2$ (Equation 11-11)

(a) ${}^{9}_{4}Be_5$

$$B = 4\left(1.007825uc^2\right) + 5\left(1.008665uc^2\right) - 9.012182uc^2$$
$$= 0.062443uc^2 = \left(0.062443uc^2\right)\left(931.5MeV/uc^2\right)$$
$$= 58.2MeV$$
$$B/A = 58.2MeV/9\ nucleons = 6.46MeV/nucleon$$

(b) ${}^{13}_{6}C_7$

$$B = 6\left(1.007825uc^2\right) + 7\left(1.008665uc^2\right) - 13.003355uc^2$$
$$= 0.104250uc^2 = \left(0.104250uc^2\right)\left(931.5MeV/uc^2\right)$$
$$= 91.1MeV$$
$$B/A = 91.1MeV/13\ nucleons = 7.47\ MeV/nucleon$$

(c) ${}^{57}_{26}Fe_{31}$

$$B = 26\left(1.007825uc^2\right) + 31\left(1.008665uc^2\right) - 56.935396uc^2$$
$$= 0.536669uc^2 = \left(0.536669uc^2\right)\left(931.5MeV/uc^2\right)$$
$$= 499.9MeV$$
$$B/A = 499.9MeV/57\ nucleons = 8.77\ MeV/nucleon$$

11-10. $R = R_0 A^{1/3}$ where $R_0 = 1.2\,fm$ (Equation 11-3)

(a) ${}^{16}O \rightarrow R = 1.2\,fm(16)^{1/3} = 3.02\,fm$

(b) ${}^{56}Fe \rightarrow R = 1.2\,fm(56)^{1/3} = 4.58\,fm$

(c) ${}^{197}Au \rightarrow R = 1.2\,fm(197)^{1/3} = 6.97\,fm$

(d) ${}^{238}U \rightarrow R = 1.2\,fm(238)^{1/3} = 7.42\,fm$

11-11. (a) $B = M\left({}^{3}He\right)c^2 + m_n c^2 - M\left({}^{4}He\right)c^2$

$= 3.016029uc^2 + 1.008665uc^2 - 4.002602uc^2$

$= 0.022092uc^2\left(931.5MeV/uc^2\right) = 20.6MeV$

(b) $B = M\left({}^{6}Li\right)c^2 + m_n c^2 - M\left({}^{7}Li\right)c^2$

$= 6.015121uc^2 + 1.008665uc^2 - 7.016003uc^2$

$= 0.007783uc^2\left(931.5MeV/uc^2\right) = 7.25MeV$

(c) $B = M\left({}^{13}N\right)c^2 + m_n c^2 - M\left({}^{14}N\right)c^2$

$= 13.005738uc^2 + 1.008665uc^2 - 14.003074uc^2$

$= 0.011329uc^2\left(931.5MeV/uc^2\right) = 10.6MeV$

11-12. $B = \left[+a_1 A - a_2 A^{2/3} - a_3 Z^2 A^{-1/3} - a_4\left(A - 2Z\right)^2 A^{-1} \pm a_5 A^{-1/2}\right]c^2$ (This is Equation 11-13 on the Web page www.whfreeman.com/tiplermodernphysics6e.) The values of the a_i in MeV/c^2 are given in Table 11-3 (also on the Web page).

For ${}^{23}Na$:

$$B = \left[15.67(23) - 17.23(23)^{2/3} - 0.75(11)^2(23)^{-1/3} - 93.2(23 - 2\times 11)^2(23)^{-1} + 0(23)^{-1/2}\right]c^2$$
$$= 184.9MeV$$

$M\left({}^{23}Na\right)c^2 = 11m_p c^2 + 12m_n c^2 - B$ (Equation 11-14 on the Web page)

$$= \left[11\left(1.007825uc^2\right) + 12\left(1.008665uc^2\right)\right] - 184.9MeV$$

$$M\left({}^{23}Na\right) = 23.190055u - 184.9MeV/c^2\left(1/931.5MeV/c^2 \bullet u\right)$$

$$= 23.190055u - 0.198499u = 22.991156u$$

This result differs from the measured value of 22.989767u by only 0.008%.

11-13. $R = \left(1.07 \pm 0.02\right)A^{1/3}\,fm$ (Equation 11-5) $R = 1.4A^{1/3}\,fm$ (Equation 11-7)

(a) ${}^{16}O$: $R = 1.07A^{1/3} = 2.70\,fm$ and $R = 1.4A^{1/3} = 3.53\,fm$

(b) ${}^{63}Cu$: $R = 1.07A^{1/3} = 4.26\,fm$ and $R = 1.4A^{1/3} = 5.57\,fm$

(Problem 11-13 continued)

(c) ^{208}Pb: $R = 1.07A^{1/3} = 6.34\,fm$ and $R = 1.4A^{1/3} = 8.30\,fm$

11-14. $\Delta U = \frac{3}{5}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R}\left(Z^2 - (Z-1)^2\right)$ (Equation 11-2)

where Z = 20 for *Ca* and $\Delta U = 5.49 MeV$ from a table of isotopes (e.g., Table of Isotopes 8th ed., Firestone, et al, Wiley 1998).

$$R = \frac{3}{5}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R\Delta U}\left(Z^2 - (Z-1)^2\right)$$

$$= 0.6\left(8.99\times10^9 N\bullet m^2/C^2\right)\left(1.60\times10^{-19}C\right)\left(2.0^2 - 19^2\right)/\left(5.49\times10^6 eV\right)$$

$$= 6.13\times10^{-15}m = 6.13\,fm$$

11-15. (a) $R = R_0 e^{-\lambda t} = R_0 e^{-(\ln 2)t/t_{1/2}}$ (Equation 11-19)

at $t = 0$: $R = R_0 = 4000\,counts/s$

at $t = 10s$: $R = R_0 e^{-(\ln 2)(10s)/t_{1/2}}$

$$1000 = 4000 e^{-(\ln 2)(10s)/t_{1/2}}$$

$$1/4 = e^{-(\ln 2)(10s)/t_{1/2}}$$

$$\ln(1/4) = -(\ln 2)(10s)/t_{1/2} \Rightarrow t_{1/2} = 5.0s$$

(b) at $t = 20s$: $R = (4000 counts/s)e^{-(\ln 2)(20s)/5s} = 200 counts/s$

11-16. $R = R_0 e^{-(\ln 2)t/2min}$ at $t = 0$: $R = R_0 = 2000 counts/s$

(a) at $t = 4\,min$: $R = (2000 counts/s)e^{-(\ln 2)(4\,min)/2\,min} = 500 counts/s$

(b) at $t = 6\,min$: $R = (2000 counts/s)e^{-(\ln 2)(6\,min)/2\,min} = 250 counts/s$

(c) at $t = 8\,min$: $R = (2000 counts/s)e^{-(\ln 2)(8\,min)/2\,min} = 125 counts/s$

11-17. $R = R_0 e^{-\lambda t} = R_0 e^{-(\ln 2)t/t_{1/2}}$ (Equation 11-19)

(a) at $t = 0$: $R = R_0 = 115.0\,decays/s$

at $t = 2.25h$: $R = 85.2\,decays/s$

$$85.2\,decays/s = (115.0\,decays/s)e^{-\lambda(2.25h)}$$

$$(85.2/115.0) = e^{-\lambda(2.25h)}$$

$$\ln(85.2/115.0) = -\lambda(2.25h)$$

$$\lambda = -\ln(85.2/115.0)/2.25h = 0.133h^{-1}$$

$$t_{1/2} = \ln 2/\lambda = \ln 2/0.133h^{-1} = 5.21h$$

(b) $\left|\frac{dN}{dt}\right| = \lambda N \quad \rightarrow \quad \left|\frac{dN_0}{dt}\right| = R_0 = \lambda N_0$ (from Equation 11-17)

$$N_0 = R_0/\lambda = (15.0\,atoms/s)/(0.133h^{-1})(1h/3600s)$$

$$= 3.11 \times 10^6\,atoms$$

11-18. (a) ^{226}Ra $\qquad t_{1/2} = 1620y$

$$R = -\frac{dN}{dt} = \lambda N = \frac{\ln 2}{t_{1/2}} N = \frac{\ln 2}{t_{1/2}} \frac{N_A m}{M}$$

$$= \frac{\ln 2(6.022 \times 10^{23}/mole)(1g)}{(1620y)(3.16 \times 10^7 s/y)(26.025g/mole)} = 3.61 \times 10^{10} s^{-1}$$

$1Ci = 3.7 \times 10^{10} s^{-1}$, or nearly the same.

(b) $Q = M(^{226}Ra)c^2 - \left[M(^{222}Rn)c^2 + M(^{4}He)c^2\right]$

$$= 226.025402uc^2 - \left[222.017571uc^2 + 4.002602uc^2\right]$$

$$= 0.005229uc^2 = (0.005229uc^2)(931.5MeV/uc^2)$$

$$= 4.87MeV$$

11-19. (a) $R = -\dfrac{dN}{dt} = R_0 e^{-(\ln 2)t/t_{1/2}}$ (from Equation 11-19)

when $t = 0$, $R = R_0 = 8000\,counts/s$

when $t = 10\,min$, $R = 1000\,counts/s = 8000\,counts/s \times e^{-10(\ln 2)/t_{1/2}}$

$$e^{-10(\ln 2)/t_{1/2}} = 1000/8000 = 1/8$$

$$-10\ln(2)/t_{1/2} = \ln(1/8)$$

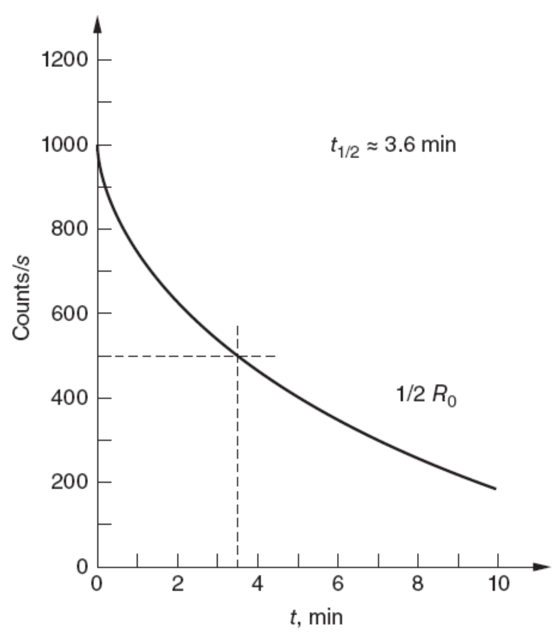
$$t_{1/2} = \frac{-10\ln(2)}{\ln(1/8)} = 3.33\,min$$

Notice that this time interval equals three half-lives.

(b) $\lambda = \ln(2)/t_{1/2} = \ln(2)/3.33\,min = 0.208\,min^{-1}$

(c) $R = R_0 e^{-t(\ln 2)/t_{1/2}} = R_0 e^{-t}$ Thus, $R = (8000\,counts/s)e^{-0.208(1)} = 6500\,counts/s$

11-20. (a) and (b)



(c) Estimating from the graph, the next count (at 8 min) will be approximately 220 counts.

11-21. ^{62}Cu is produced at a constant rate R_0, so the number of ^{62}Cu atoms present is:

$N = R_0/\lambda\left(1-e^{-\lambda t}\right)$ (from Equation 11-26). Assuming there were no ^{62}Cu atoms initially present. The maximum value *N* can have is $R_0/\lambda = N_0$,

$$N = N_0\left(1-e^{-\lambda t}\right)$$
$$0.90N_0 = N_0\left(1-e^{-t(\ln 2)/t_{1/2}}\right)$$
$$e^{-t(\ln 2)/t_{1/2}} = 1-0.90 = 0.10$$
$$-t\ln(2)/t_{1/2} = \ln(0.10)$$
$$t = -10\ln(0.10)/\ln(2) = 33.2\,min$$

11-22. (a) $t_{1/2} = \ln(2)/\lambda = \ln(2)/9.8\times10^{-10}\,y^{-1} = 7.07\times10^{8}\,y$ (Equation 11-22)

(b) Number of ^{235}U atoms present is:

$$N = \frac{1.0\mu g N_A}{M} = \frac{\left(10^{-6}\,g\right)\left(6.02\times10^{23}\,atoms/mol\right)}{235\,g/mol} = 2.56\times10^{15}\,atoms$$

$$-\frac{dN}{dt} = \lambda N = 9.8\times10^{-10}\,y^{-1}\left(1/3.16\times10^{7}\,s/y\right)\left(2.56\times10^{15}\,atoms\right) \quad \text{(Equation 11-17)}$$
$$= 0.079\,decays/s$$

(c) $N = N_0 e^{-\lambda t}$ (Equation 11-18)

$$N = \left(2.56\times10^{15}\right)e^{-\left(9.8\times10^{-10}\,y^{-1}\right)\left(10^{6}\,y\right)} = 2.558\times10^{15}$$

11-23. (a) $t_{1/2} = \ln(2)/\lambda = \ln(2)/0.266y^{-1} = 2.61y$ (Equation 11-22)

(b) Number of *N* atoms in 1 *g* is:

$$N = \frac{1.0g N_A}{M} = \frac{(1g)\left(6.02\times10^{23}\,atoms/mol\right)}{22\,g/mol} = 2.74\times10^{22}\,atoms$$

$$-\frac{dN}{dt} = \lambda N = \left(0.266y^{-1}\right)\left(1/3.16\times10^{7}\,s/y\right)\left(2.74\times10^{22}\,atoms\right)$$
$$= 2.3\times10^{14}\,decays/s = 2.3\times10^{14}\,Bq$$

(Problem 11-23 continued)

(c) $-\dfrac{dN}{dt} = \lambda N_0 e^{-\lambda t}$ (Equation 11-19)

$$= \left(0.266 y^{-1}\right)\left(1/3.16\times10^{7} s/y\right)\left(2.74\times10^{22}\right)e^{-(0.266y)(3.5y)}$$

$$= 9.1\times10^{13} decays/s = 9.1\times10^{13} Bq$$

(d) $N = N_0 e^{-\lambda t}$ (Equation 11-18)

$$N = \left(2.74\times10^{22}\right)e^{-\left(0.266y^{-1}\right)(3.5y)} = 1.08\times10^{22}$$

11-24. (a) ^{22}Na has an excess of protons compared with ^{23}Na and would be expected to decay by β^+ emission and/or electron capture. (It does both.)

(b) ^{24}Na has an excess of neutrons compared with ^{23}Na and would be expected to decay by β^- emission. (It does.)

11-25. $\log t_{1/2} = AE_\alpha^{-1/2} + B$ (Equation 11-30)

for $t_{1/2} = 10^{10} s$, $E_\alpha = 5.4 MeV$
for $t_{1/2} = 1 s$, $E_\alpha = 7.0 MeV$ } from Figure 11-16

$$\log\left(10^{10}\right) = A(5.4)^{-1/2} + B$$

(i) $10 = 0.4303A + B$

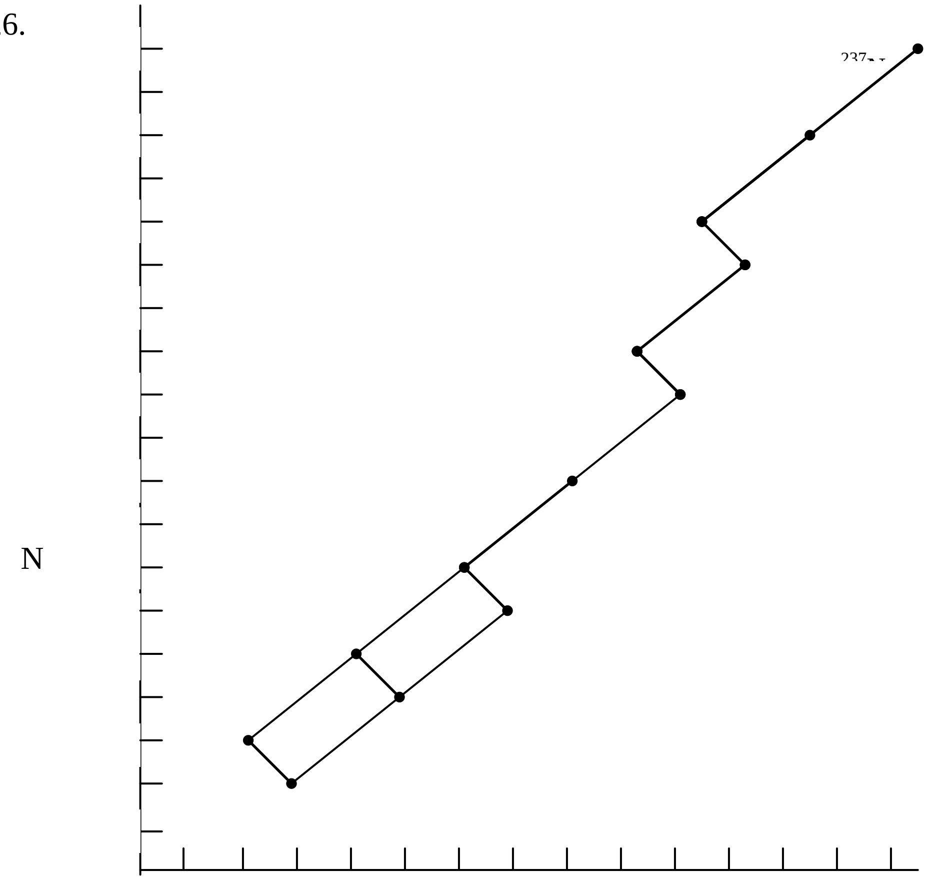
$$\log(1) = A(7.0)^{-1/2} + B$$

(ii) $0 = 0.3780A + B \rightarrow B = -0.3780A$

Substituting (ii) into (i),

$$10 = 0.4303A = 0.3780A - 0.0523A, \quad A = 191, \quad B = -0.3780A = -72.2$$

11-26.



11-27. $^{232}_{90}Th \rightarrow ^{228}_{88}Ra + \alpha$

$$Q = M\left(^{232}Th\right)c^2 - M\left(^{228}Ra\right)c^2 - M\left(^{4}He\right)c^2$$

$$= 232.038051uc^2 - 228.031064uc^2 - 4.002602uc^2$$

$$= 0.004385uc^2\left(931.50MeV/uc^2\right) = 4.085MeV$$

The decay is a 2-particle decay so the *Ra* nucleus and the α have equal and opposite momenta.

$$\rho_\alpha = \sqrt{2m_\alpha E_\alpha} = \rho_{Ra} = \sqrt{2m_{Ra}E_{Ra}} \text{ where } E_\alpha + E_{Ra} = 4.085MeV$$

$$2m_\alpha E_\alpha = 2M_{Ra}E_{Ra} = 2M_{Ra}\left(4.085 - E_\alpha\right)$$

$$E_\alpha = \frac{M_{Ra}}{M_{Ra} + m_\alpha}\left(4.085MeV\right) = \frac{228.031064\left(4.085MeV\right)}{228.031064 + 4.002602} = \left(0.983\right)\left(4.085MeV\right) = 4.01MeV$$

11-28. ${}^{7}_{4}Be_3 \rightarrow {}^{7}_{3}Li_4 + \nu_e$

(a) Yes, the decay would be altered. Under very high pressure the electrons are "squeezed" closer to the nucleus. The probability density of the electrons, particularly the K electrons, is increased near the nucleus making electron capture more likely, thus decreasing the half-life.

(b) Yes, the decay would be altered. Stripping all four electrons from the atom renders electron capture impossible, lengthening the half-life to infinity.

11-29. ${}^{67}Ga \xrightarrow{E.C.} {}^{67}Zn + \nu_e$

$$Q = M\left({}^{67}Ga\right)c^2 - M\left({}^{67}Zn\right)c^2$$

$$= 66.9282uc^2 - 66.972129uc^2$$

$$= 0.001075uc^2\left(931.50MeV/uc^2\right) = 1.00MeV$$

11-30. ${}^{72}Zn \rightarrow {}^{72}Ga + \beta^- + \overline{\nu}_e$

$$Q = M\left({}^{72}Zn\right)c^2 - M\left({}^{72}Ga\right)c^2$$

$$= 71.926858uc^2 - 71.926367uc^2$$

$$= 0.000491uc^2\left(931.50MeV/uc^2\right) = 0.457MeV = 457keV$$

11-31. ${}^{233}Np \rightarrow {}^{232}Np + n$ and ${}^{233}Np \rightarrow {}^{232}U + p$

For n emission: $Q = M\left({}^{233}Np\right)c^2 - M\left({}^{232}Np\right)c^2 - m_n c^2$

$$= 233.040805uc^2 - 232.040022uc^2 - 1.008665uc^2$$

$$= -0.007882uc^2$$

$\left\{Q < 0 \text{ means } M\left(products\right) > M\left({}^{233}Np\right);\ \text{prohibited by conservation of energy.}\right\}$

For p emission: $Q = M\left({}^{233}Np\right)c^2 - M\left({}^{232}U\right)c^2 - m_n c^2$

$$= 233.040805uc^2 - 232.037131uc^2 - 1.008665uc^2$$

$$= -0.004991uc^2$$

(Problem 11-31 continued)

$\{Q < 0$ means $M(products) > M(^{233}Np)$; prohibited by conservation of energy.$\}$

11-32.

	286	280	247	235	174	124	80	61	30	0
286	–									
280	6	–								
247	39	33	–							
235	50	45	12	–						
174	112	106	73	61	–					
124	162	156	123	111	50	–				
80	206	200	167	155	94	44	–			
61	225	219	186	174	113	63	19	–		
30	256	250	217	205	144	91	50	31	–	
0	286	280	247	235	174	124	80	61	30	–

Tabulated γ energies are in *keV*. Higher energy α levels in Figure 11-19 would add additional columns of γ rays.

11-33. $^{8}Be \rightarrow 2\alpha$

$$Q = M(^{8}Be)c^2 - M(^{4}He)c^2$$

$$= 8.005304uc^2 - 2(4.002602)uc^2$$

$$= 0.000100uc^2(931.50MeV/uc^2) = 0.093MeV = 93keV$$

11-34. $^{80}Br \rightarrow {}^{80}Kr + \beta^- + \bar{\nu}_e$ and $^{80}Br \rightarrow {}^{80}Se + \beta^+ + \nu_e$ and $^{80}Br \xrightarrow{E.C.} {}^{80}Se + \nu_e$

For β^- decay: $Q = M(^{80}Br)c^2 - M(^{80}Kr)c^2$

$$= 79.918528uc^2 - 79.916377uc^2$$

$$= 0.002151uc^2(931.50MeV/uc^2) = 2.00MeV$$

(Problem 11-34 continued)

For β^+ decay: $Q = M\left({}^{80}Br\right)c^2 - M\left({}^{80}Se\right)c^2 - 2m_e c^2$

$$= 79.918528uc^2 - 79.916519uc^2 - 2\left(0.511MeV\right)$$

$$= 0.002009uc^2\left(931.50MeV/uc^2\right) - 1.022MeV = 0.85MeV$$

For E.C.: $Q = M\left({}^{80}Br\right)c^2 - M\left({}^{80}Se\right)c^2$

$$= 79.918528uc^2 - 79.916519uc^2$$

$$= 0.002009uc^2\left(931.50MeV/uc^2\right) = 1.87MeV$$

11-35. $R = R_0 A^{1/3}$ where $R_0 = 1.2\,fm$ (Equation 11-3)

For ${}^{12}C: R = 1.2\left(12\right)^{1/3} = 2.745\,fm = 2.745\times10^{-15}m$ and the diameter = $5.490\times10^{-15}m$

Coulomb force: $F_C = \dfrac{ke^2}{r^2} = \dfrac{\left(9.00\times10^9\right)\left(1.6\times10^{-19}C\right)^2}{\left(5.490\times10^{-15}m\right)^2} = 7.65N$

Gravitational force: $F_G = G\dfrac{m_p^2}{r^2} = \dfrac{6.67\times10^{-11}\left(1.67\times10^{-27}kg\right)^2}{\left(5.490\times10^{-15}m\right)^2} = 6.18\times10^{-36}N$

The corresponding Coulomb potential is: $U_C = F_C \times r = 7.65N\left(5.490\times10^{-15}m\right)$

$$= 4.20\times10^{-14}J \big/ 1.60\times10^{-13}J/MeV$$

$$= 0.26MeV$$

The corresponding gravitational potential is:

$$U_G = F_G \times r = \left(6.18\times10^{-36}N\right)\left(5.490\times10^{-15}m\right)$$

$$= 3.39\times10^{-50}J \Big/ \left(1/1.60\times10^{-13}J/MeV\right) = 2.12\times10^{-37}MeV$$

The nuclear attractive potential exceeds the Coulomb repulsive potential by a large margin (50*MeV* to 0.26*MeV*) at this separation. The gravitational potential is not a factor in nuclear structure.

11-36. The range R of a force mediated by an exchange particle of mass m is:

$R = \hbar / mc$ (Equation 11-50)

$mc^2 = \hbar c / R = 197.3 MeV \bullet fm / 5 fm = 39.5 MeV$

$m = 39.5 MeV / c^2$

11-37. The range R of a force mediated by an exchange particle of mass m is:

$R = \hbar / mc$ (Equation 11-50)

$mc^2 = \hbar c / R = 197.3 MeV \bullet fm / 0.25 fm = 789 MeV$

$m = 789 MeV / c^2$

11-38.

Nuclide	Last proton(s)	Last neutron(s)	ℓ	j
${}^{29}_{14}Si_{15}$	$\cdots(1d_{5/2})^6$	$\cdots(2s_{1/2})$	0	1/2
${}^{37}_{17}Cl_{20}$	$\cdots(1d_{3/2})$	$\cdots(1d_{3/2})^4$	2	3/2
${}^{71}_{31}Ga_{40}$	$\cdots(2p_{3/2})^3$	$\cdots(2p_{1/2})^2$	1	3/2
${}^{59}_{27}Co_{32}$	$\cdots(1f_{7/2})^7$	$\cdots(2p_{3/2})^4$	3	7/2
${}^{73}_{32}Ge_{41}$	$\cdots(2p_{3/2})^4$	$\cdots(1g_{9/2})$	4	9/2
${}^{33}_{16}S_{17}$	$\cdots(2s_{1/2})^2$	$\cdots(1d_{3/2})$	2	3/2
${}^{81}_{38}Sr_{49}$	$\cdots(2p_{3/2})^6$	$\cdots(1g_{9/2})^9$	4	9/2

The nucleon configurations are taken directly from Figure 11-35, and the ℓ and j values are those of the unpaired nucleon.

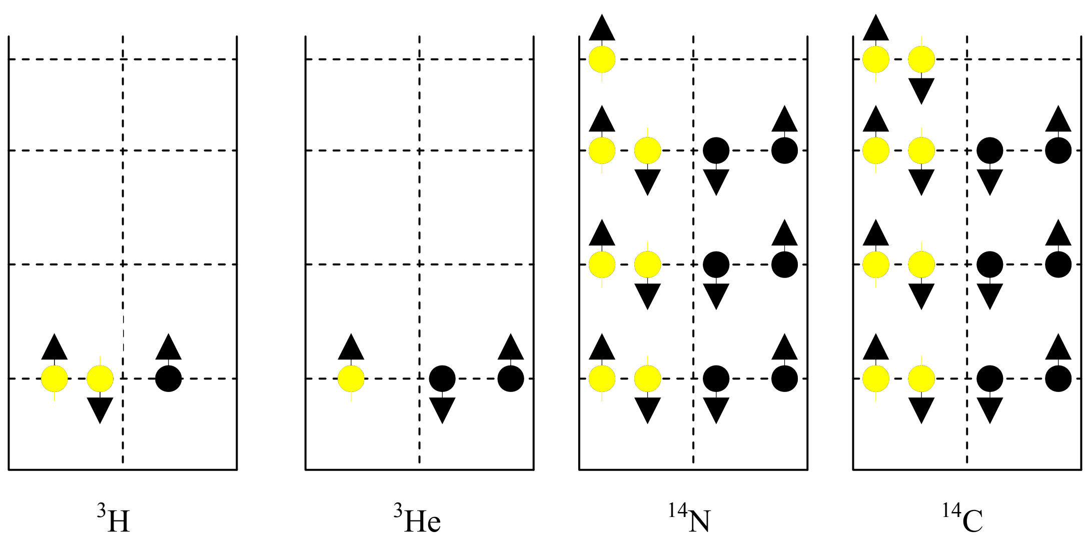
11-39.

Isotope	Odd nucleon	Predicted μ/μ_N
$^{29}_{14}Si$	neutron	-1.91
$^{27}_{17}Cl$	proton	$+2.29$
$^{71}_{31}Ga$	proton	$+2.29$
$^{59}_{27}Co$	proton	$+2.29$
$^{73}_{32}Ge$	neutron	-1.91
$^{33}_{16}S$	neutron	-1.91
$^{87}_{38}Sr$	neutron	-1.91

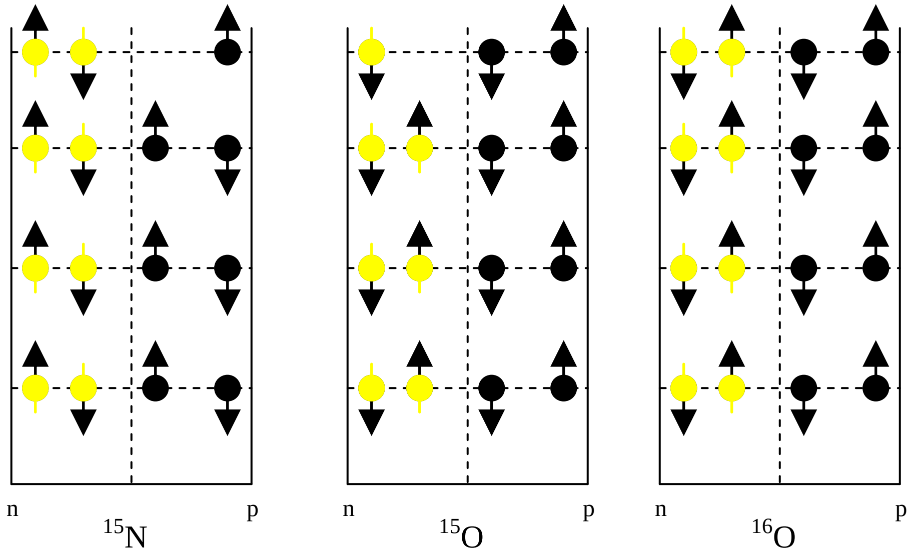
11-40. Nuclear spin of ^{14}N must be $1\hbar$ because there are 3 m_I states, +1, 0, and 1.

11-41. ^{36}S, ^{53}Mn, ^{82}Ge, ^{88}Sr, ^{94}Ru, ^{131}In, ^{145}Eu

11-42.

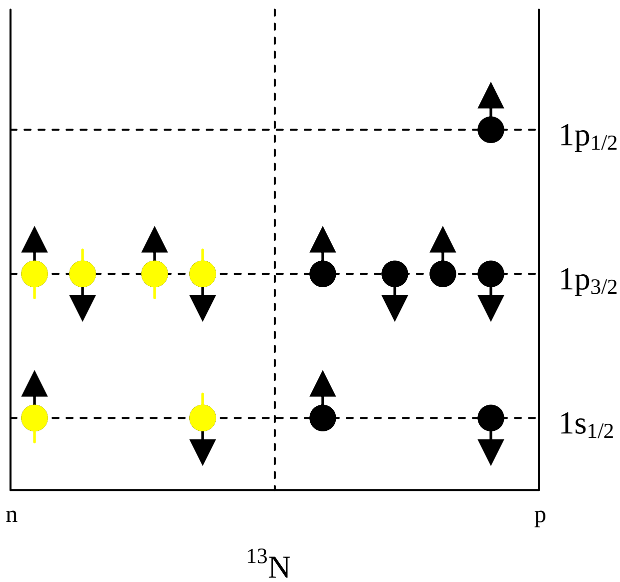


(Problem 11-42 continued)



11-43. ${}^{3}_{2}He$, ${}^{40}_{20}Ca$, ${}^{60}_{28}Ni$, ${}^{124}_{50}Sn$, ${}^{204}_{82}Pb$

11-44. (a)

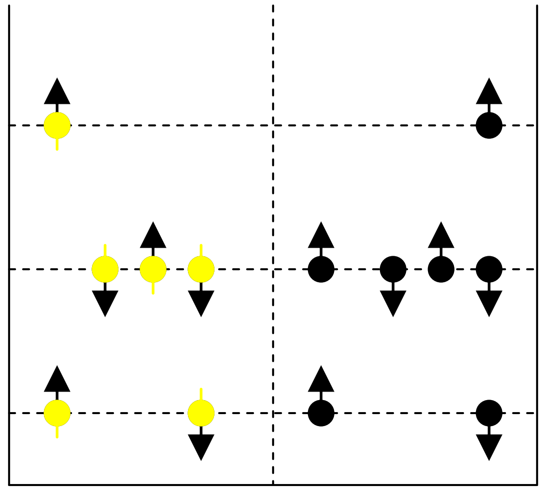


(b) j = ½ due to the single unpaired proton.

(c) The first excited state will likely be the jump of a neutron to the empty neutron level, because it is slightly lower than the corresponding proton level. The j = 1/2 or 3/2, depending on the relative orientations of the unpaired nucleon spins.

(Problem 11-44 continued)

(d)



First excited state. There are several diagrams possible.

11-45. $^{30}_{14}Si \quad j = 0$

$^{37}_{17}Cl \quad j = 3/2$

$^{55}_{27}Co \quad j = 7/2$

$^{90}_{40}Zr \quad j = 0$

$^{107}_{49}In \quad j = 9/2$

11-46 (a) $Q = M\left(^2H\right)c^2 + M\left(^2H\right)c^2 - M\left(^3H\right)c^2 - M\left(^1H\right)c^2$

$$= 2\left(2.014102uc^2\right) - 3.016049uc^2 - 1.007825uc^2$$

$$= 0.004330uc^2\left(931.5MeV/uc^2\right) = 4.03MeV$$

(b) $Q = M\left(^3He\right)c^2 + M\left(^2H\right)c^2 - M\left(^4He\right)c^2 - M\left(^1H\right)c^2$

$$= 3.016029uc^2 + 2.014012uc^2 - 4.002602uc^2 - 1.007825uc^2$$

$$= 0.019704uc^2\left(931.5MeV/uc^2\right) = 18.35MeV$$

(Problem 11-46 continued)

(c) $Q = M\left({}^{6}Li\right)c^2 + m_n c^2 - M\left({}^{3}H\right)c^2 - M\left({}^{4}He\right)c^2$

$= 6.01512uc^2 + 1.008665uc^2 - 3.016049uc^2 - 4.002602uc^2$

$= 0.005135uc^2\left(931.5MeV/uc^2\right) = 4.78MeV$

11-47. (a) $Q = M\left({}^{3}H\right)c^2 + M\left({}^{1}H\right)c^2 - M\left({}^{3}He\right)c^2 - m_N c^2$

$= 3.016049uc^2 + 1.007825uc^2 - 3.016029uc^2 - 1.0078665uc^2$

$= -0.000820uc^2\left(931.5MeV/uc^2\right) = -0.764MeV$

(b) The threshold for this endothermic reaction is:

$E_{th} = \dfrac{m+M}{M}\left|Q\right|$ (Equation 11-61)

$= \dfrac{3.016049uc^2 + 1.007825uc^2}{1.007825uc^2}\left|0.764MeV\right| = 3.05MeV$

(c) $E_{th} = \dfrac{\left|1.007825uc^2 + 3.016049uc^2\right|}{3.016049uc^2}\left|0.764MeV\right| = 1.02MeV$

11-48. ${}^{14}N + {}^{2}H \rightarrow {}^{16}O^*$

Possible products: ${}^{16}O^* \rightarrow {}^{14}N + {}^{2}H$ ${}^{16}O^* \rightarrow {}^{16}O + \gamma$ ${}^{16}O^* \rightarrow {}^{15}O + n$

${}^{16}O^* \rightarrow {}^{15}N + p$ ${}^{16}O^* \rightarrow {}^{12}C + \alpha$

11-49. (a) ${}^{12}C\left(\alpha,\, p\right){}^{15}N$

$Q = M\left({}^{12}C\right)c^2 + M\left({}^{4}He\right)c^2 - M\left({}^{15}N\right)c^2 - m_p c^2$

$= 12.000000uc^2 + 4.002602uc^2 - 15.000108uc^2 - 1.007825uc^2$

$= -0.005331uc^2\left(931.5MeV/uc^2\right) = -4.97MeV$

(b) ${}^{16}O\left(d,\, p\right){}^{17}O$

$Q = M\left({}^{16}O\right)c^2 + M\left({}^{2}H\right)c^2 - M\left({}^{17}O\right)c^2 - m_p c^2$

$= 15.994915uc^2 + 2.014102uc^2 - 16.999132uc^2 - 1.007825uc^2$

$= 0.002060uc^2\left(931.5MeV/uc^2\right) = 1.92MeV$

11-50. The number of ^{75}As atoms in sample N is:

$$N = \frac{V\rho N_A}{M} = \frac{\left(1cm\times 2cm\times 30\mu m\times 10^{-4}cm/\mu m\right)\left(5.73g/cm^3\right)\left(6.02\times 10^{23}atoms/mol\right)}{74.9216g/mol}$$

$$= 2.76\times 10^{20} \quad {}^{75}As \text{ atoms}$$

The reaction rate R per second per ^{75}As atoms is:

$R = \sigma I$ (Equation 11-62

$$= \left(4.5\times 10^{-24}cm^2/{}^{75}As\right)\left(0.95\times 10^{13}neutrons/cm^2\bullet s\right)$$

$$= 4.28\times 10^{-11}s^{-1}$$

Reaction rate = NR

$$= \left(.76\times 10^{20}atoms\right)\left(4.28\times 10^{-11}/s\bullet atom\right) = 1.18\times 10^{10}/s$$

11-51. (a) $^{23}Ne(p,n)\,^{23}Na$ $\quad$ $^{22}Ne(d,n)\,^{23}Na$ $\quad$ $^{20}F(\alpha,n)\,^{23}Na$

(b) $^{11}B(\alpha,p)\,^{14}C$ $\quad$ $^{14}N(n,p)\,^{14}C$ $\quad$ $^{13}C(d,p)\,^{14}C$

(c) $^{29}Si(\alpha,d)\,^{31}P$ $\quad$ $^{32}P(p,d)\,^{31}P$ $\quad$ $^{32}Si(n,d)\,^{31}P$

11-52. (a) ^{14}C $\quad$ (b) n $\quad$ (c) ^{58}Ni $\quad$ (d) α

(e) ^{14}N $\quad$ (f) ^{160}Er $\quad$ (g) ^{3}H $\quad$ (i) p

11-53. $\frac{Q}{c^2} = m_p + m_n - m_d$

$$= 1.007276u + 1.008665u - 2.013553u$$

$$= 0.002388u \quad \text{(See Table 11-1.)}$$

$$Q = \left(0.002388u\right)\left(931.5MeV/uc^2\right)c^2 = 224MeV$$

11-54. $P = \frac{dW}{dt} = E\frac{dN}{dt}$

$$\frac{dN}{dt} = \frac{P}{E} = \frac{500\times10^{6}\,J/s}{200\times10^{6}\,eV/\,fission}\left(\frac{1eV}{1.60\times10^{-19}\,J}\right) = 1.56\times10^{19}\,fissions/s$$

11-55. The fission reaction rate is:

$R(N) = R(0)k^{N}$ (see Example 11-22 in More section)

$$k^{N} = R(N)/R(0)$$

$$N\log k = \log\left[R(N)/R(0)\right]$$

$$N = \frac{\log\left[R(N)/R(0)\right]}{\log k}$$

(a) For the reaction rate to double $R(N) = 2R(0)$: $N = \frac{\log 2}{\log 1.1} = 7.27$

(b) For $R(N) = 10R(0)$: $N = \frac{\log 10}{\log 1.1} = 24.2$

(c) For $R(N) = 100R(0)$: $N = \frac{\log 100}{\log 1.1} = 48.3$

(d) Total time $t = N(1ms) = N\ ms$: (a) 7.27ms (b) 24.2ms (c) 48.3ms

(e) Total time $t = N(100ms) = 100N\ ms$: (a) 0.727ms (b) 2.42ms (c) 4.83ms

11-56.

$$^{235}_{92}U_{143} + n \rightarrow \begin{cases} ^{101}_{40}Zr_{61} + ^{134}_{52}Te_{82} + n \\ ^{101}_{41}Nb_{60} + ^{133}_{51}Sb_{82} + 2n \\ ^{101}_{43}Tc_{58} + ^{132}_{49}In_{83} + 3n \\ ^{102}_{45}Rh_{57} + ^{130}_{47}Ag_{83} + 4n \end{cases}$$

11-57. $500MW = \left(500\frac{J}{s}\right)\left(\frac{1MeV}{1.60\times10^{-13}J}\right)\left(\frac{1\,fusion}{17.6MeV}\right) = 1.78\times10^{14}\ fusions/s$

Each fusion requires one ^2H atom (and one ^3H atom; see Equation 11-67) so ^2H must be provided at the rate of $1.78\times10^{14}\,atoms/s$.

11-58. The reactions per ^{238}U atom is:

$R = \sigma I$ (Equation 11-62)

$$= \left(0.02\times10^{-24}cm^2/atom\right)\left(5.0\times10^{11}n/m^2\right)\left(\frac{1m^2}{10^4cm^2}\right) = 1.00\times10^{-18}\ /atom$$

The number N of ^{238}U atoms is:

$$N = \frac{(5.0g)\left(6.02\times10^{23}atoms/mol\right)}{238.051g/mol} = 1.26\times10^{22}\quad {}^{238}U \text{ atoms}$$

Total ^{239}U atoms produced = RN

$$= \left(1.00\times10^{-18}\ /atom\right)\left(1.26\times10^{22}atoms\right) = 1.26\times10^4\quad {}^{239}U \text{ atoms}$$

11-59. $Q_1 = M\left({}^1H\right)c^2 + M\left({}^1H\right)c^2 - M\left({}^2H\right)c^2$

$$= 1.007825uc^2 + 1.007825uc^2 - 2.014102uc^2$$

$$= 0.001548uc^2\left(931.50MeV/uc^2\right) = 1.4420MeV$$

$$Q_2 = M\left({}^2He\right)c^2 + M\left({}^1H\right)c^2 - M\left({}^3He\right)c^2$$

$$= 2.014102uc^2 + 1.007825uc^2 - 3.016029uc^2$$

$$= 0.005898uc^2\left(931.50MeV/uc^2\right) = 5.4940MeV$$

$$Q_3 = M\left({}^3He\right)c^2 + M\left({}^3He\right)c^2 - M\left({}^4He\right)c^2 - 2m\left({}^1H\right)c^2$$

$$= 2\left(3.016029uc^2\right) - 4.002602uc^2 - 2\left(1.007825uc^2\right)$$

$$= 0.013806uc^2\left(931.50MeV/uc^2\right) = 12.8603MeV$$

$$Q = Q_1 + Q_2 + Q_3 = 1.4420MeV + 5.4940MeV + 12.8603MeV = 19.80MeV$$

11-60. Total power = $1000 MWe / 0.30 = 3333 MW$

$$= 3.33\times10^{9}\, J/s\left(1MeV/1.60\times10^{-13}\,J\right) = 2.08\times10^{22}\, MeV/s$$

(a) 1 day $= 8.64\times10^{4}\,s$

Energy/day $= 2.08\times10^{22}\, MeV/s\left(8.64\times10^{4}\, s/day\right) = 1.80\times10^{27}\, MeV/day$

The fission of 1*kg* of ^{235}U provides $4.95\times10^{26}\,MeV$ (from Example 11-19)

$1kg\ ^{235}U/day = 1.80\times10^{27}\,MeV \big/ \left(4.95\times10^{26}\,MeV/kg\right) = 3.64kg/day$

(b) $1kg\ ^{235}U/year = 3.64\,kg\ ^{235}U/day\left(365 days/year\right) = 1.33\times10^{3}\,kg/year$

(c) Burning coal produces

$$3.15\times10^{7}\, J/kg\left(1MeV/1.60\times10^{-13}\,J\right) = 1.97\times10^{20}\,MeV/kg\ coal$$

Ratio of *kg* coal needed per *kg* of ^{235}U is: $\dfrac{4.95\times10^{26}\,MeV/kg\ ^{235}U}{1.97\times10^{20}\,MeV/kg\,coal} = 2.51\times10^{6}$

For 1 day: $3.64kg\ ^{235}U\left(2.51\times10^{6}\right) = 9.1\times10^{6}\,kg$ This is about 10,000 tons/day, the approximate capacity of 100 railroad coal hopper cars.

For 1 year: $9.12\times10^{6}\,kg/day\left(365 days/year\right) = 3.33\times10^{9}\,kg/year$

11-61. $\rho\left(H_2O\right) = 1000kg/m^3$, so

(a) $1000kg$: $\dfrac{10^{6}\,g\left(6.02\times10^{23} molecules\ H_2O/mol\right)\left(2H/molecule\right)\left(0.00015\ ^{2}H\right)}{18.02g/mol}$

$$= 1.00\times10^{25}\ ^{2}H\ atoms$$

Each fusion releases 5.49*MeV*.

Energy release = $\left(1.00\times10^{25}\right)\left(5.49MeV\right) = 5.49\times10^{25}\,MeV$

$$= \left(5.49\times10^{25}\,MeV\right)\left(1.60\times10^{-13}\,J/MeV\right) = 8.78\times10^{12}\,J$$

(b) Energy used/person (in 1999) = $3.58\times10^{20}\,J \big/ 5.9\times10^{9}\ people$

$$= 6.07\times10^{10}\,J/person\bullet y$$

(Problem 11-61 continued)

Energy used per person per hour = $6.07\times10^{10}\, J/person\bullet y\times\frac{1y}{8760h}$

$$=6.93\times10^{6}\, J/person\bullet h$$

At that rate the deuterium fusion in $1m^3$ of water would last the "typical" person

$$\frac{8.80\times10^{12}\, J}{6.93\times10^{6}\, J/person\bullet h}=1.27\times10^{6}\, h\approx145y$$

11-62. (a) $Q=M\left({}^{235}U\right)c^2+m_nc^2-M\left({}^{120}Cd\right)c^2-M\left({}^{110}Ru\right)c^2+5m_nc^2$

$$=235.043924uc^2+1.008665uc^2-119.909851uc^2-109.913859uc^2$$
$$-5\left(1.008665uc^2\right)$$
$$=1.186uc^2\left(931.5MeV/uc^2\right)=1.10\times10^3\, MeV$$

(b) This reaction is not likely to occur. Both product nuclei are neutron-rich and highly unstable.

11-63. The original number N_0 of ${}^{14}C$ nuclei in the sample is:

$N_0=\left(15g\right)\left(6.78\times10^{10}\, nuclei/g\right)=1.017\times10^{12}$ where the number of ${}^{14}C$ nuclei per gram of C was computed in Example 11-27. The number N of ${}^{14}C$ present after 10,000y is:

$$N=N_0e^{-\lambda t}=N_0e^{-(\ln 2)(t/t_{1/2})} \quad \text{(Equation 11-18)}$$
$$=\left(1.017\times10^{12}\right)e^{-(\ln 2)(10000/5730)}=3.034\times10^{11}$$
$$R=\lambda N=\left(\ln 2/t_{1/2}\right)N \quad \text{(Equation 11-19)}$$
$$=\left(\ln 2/5730y\right)\left(1y/3.16\times10^{7}s\right)\left(3.034\times10^{11}\right)$$
$$=1.16\, decays/s$$

11-64. If from a living organism, the decay rate would be:

$\left(15.6decays/g\bullet min\right)\left(175g\right)=20.230decays/min$ (from Example 11-27)

The actual decay rate is: $\left(8.1decays/s\right)\left(60s/min\right)=486decays/min$

(Problem 11-64 continued)

$$\left(\frac{1}{2}\right)^2 = \frac{486 decays/min}{20,230 decays/min} \quad \text{(from Example 11-27)}$$

$$2^n = 20,230/486$$

$$n\ln(2) = \ln(20,230/486)$$

$$n = \ln(20,230/486)/\ln(2) = 5.379 \text{ half lives}$$

Age of bone = $(5.379\, half\ \ lives)(5730\, y/half\ \ life) = 30,800\, y$

11-65. $$t = \frac{t_{1/2}}{\ln(2)}\ln(1 + N_D/N_P) \qquad \text{(Equation 11-92)}$$

$t_{1/2}\left({}^{87}Rb\right) = 4.88\times10^{10}\, y$ and $N_P/N_D = 36.5$

$$t = \frac{4.88\times10^{10}\, y}{\ln(2)}\ln\left[1 + (1/36.5)\right] = 1.90\times10^9\, y$$

11-66. The number of X rays counted during the experiment equals the number of atoms of interest in the same, times the cross section for activation σ_x, times the particle beam intensity, where

I = proton intensity = $250nA\times(eC/proton)^{-1} = 1.56\times10^{12}\, protons/s$

$\sigma_x = 650b = 650\times10^{-24}\, cm^2$

$m = \text{mass} = 0.35 mg/cm^2 \times 0.00001 = 3.5\times10^{-6}\, mg/cm^2$

$n =$ number of atoms of interest $= mN_A/A$

$t =$ exposure time

detector efficiency $= 0.0035$

$\in =$ overall efficiency $= 0.60\times$ detector efficiency

$$N = I\sigma_x \frac{mN_A}{A} t \in$$

$$N = \left(\frac{250\times10^{-9}\, C/s}{1.60\times10^{-19}\, C/proton}\right)\left(650\times10^{-24}\, cm^2\right)$$

(Problem 11-66 continued)

$$\times\left(\frac{\left(0.35\times10^{-3}\,g/cm^2\right)\left(0.00001\right)\left(6.02\times10^{23}\,mol^{-1}\right)}{80\,g/mol}\right)$$

$$\times\left(15\,min\times 60s/min\right)\left(0.60\times0.0035\right)$$

$$=5.06\times10^{4}\text{ counts in 15 minutes}$$

11-67. $t=\dfrac{t_{1/2}}{\ln(2)}\ln\left(1+N_D/N_P\right)$ (Equation 11-92)

$$t_{1/2}\left({}^{232}Th\right)=1.40\times10^{10}\,y$$

$$N_P\left({}^{232}Th\right)=\frac{4.11g\left(6.02\times10^{23}\,atoms/mol\right)}{232.04g/mol}=1.066\times10^{22}\,atoms$$

$$N_D\left({}^{208}Pb\right)=\frac{0.88g\left(6.02\times10^{23}\,atoms/mol\right)}{208g/mol}=2.547\times10^{21}\,atoms$$

$$N_D/N_P=2.547\times10^{21}/1.066\times10^{22}=0.2389$$

$$t=\frac{1.40\times10^{10}\,y}{\ln(2)}\ln\left[1+0.2389\right]=4.33\times10^{9}\,y$$

11-68. $f=\dfrac{\Delta E}{h}=\dfrac{2\left(\mu_z\right)_p B}{h}$

(a) For Earth's field: $f=\dfrac{2\left(2.79\mu_N\right)\left[\left(3.15\times10^{-8}\,eV/T\right)/1\mu_N\right]\left(0.5\times10^{-4}\,T\right)}{4.14\times10^{-15}\,eV\bullet s}$

$$=2.13\times10^{3}\,Hz=2.12\,kHz$$

(b) For $B = 0.25T$: $f=2.12\times10^{3}\,Hz\left(0.25T/0.5\times10^{-4}\,T\right)=1.06\times10^{7}\,Hz=10.6\,MHz$

(c) For $B= 0.5T$: $f=2.12\times10^{3}\,Hz\left(0.5T/0.5\times10^{-4}\,T\right)=2.12\times10^{7}\,Hz=21.2\,MHz$

11-69. (a) $N\left({}^{12}C^{+3}\right) = \dfrac{\left(12\times10^{-6}C/s\right)\left(10\,min\right)\left(60s/min\right)}{3\left(1.60\times10^{-19}C\right)} = 1.50\times10^{16}$

$${}^{14}C/{}^{12}C \text{ ratio} = 1500/1.50\times10^{16} = 10^{-13}$$

(b) mass ${}^{12}C = \dfrac{\left(1.50\times10^{15}atoms/min\right)\left(75\,min\right)\left(12\right)\left(1.66\times10^{-27}kg\right)}{0.015}$

$$= 1.49\times10^{-7}kg = 1.49\times10^{-4}g = 0.149mg$$

(c) The ${}^{14}C/{}^{12}C$ ratio in living C is 1.35×10^{-12}.

$$\frac{\text{sample } {}^{14}C/{}^{12}C}{\text{living } {}^{14}C/{}^{12}C} = \frac{10^{-13}}{1.35\times10^{-12}} = \frac{0.10}{1.35} = \left(\frac{1}{2}\right)^n$$

where n = # of half-lives elapsed. Rewriting as (see Example 11-28)

$$2^n = \frac{1.35}{0.10} = 13.5$$

$$n\ln(2) = \ln(13.5) \qquad \therefore n = \ln(13.5)/\ln(2) = 3.75$$

age of sample = $3.75t_{1/2} = 3.75(5730y) = 2.15\times10^4\,y$

11-70. If from live wood, the decay rate would be 15.6 disintegrations/g•min. The actual rate is 2.05 disintegrations/g•min.

$$\left(\frac{1}{2}\right)^2 = \frac{2.05decays/g\bullet min}{15.6decays/g\bullet min} \quad \text{(from Example 12-13)}$$

$$2^n = 15.6/2.05$$

$$n\ln(2) = \ln(15.6/2.05)$$

$$n = \ln(15.6/2.05)/\ln(2) = 2.928 \text{ half lives of } {}^{14}C$$

Age of spear thrower = $nt_{1/2} = (2.928)(5730y) = 16{,}800y$

11-71. Writing Equation 11-14 as:

$$M(Z,A)c^2 = Zm_p^2 + (A-Z)m_n c^2 - \left[a_1 A - a_2 A^{2/3} - a_3 A^{-1/3} z^2 - a_4 (A-2Z)^2 A^{-1} + a_5 A^{-1/2}\right]c^2$$

and differentiating,

$$\frac{\partial M}{\partial Z} = m_p - m_n - \left[-2a_3 A^{-1/3} Z - 2a_4 (A-2Z)(-2)A^{-1}\right]$$

$$0 = (m_p - m_n) + 2a_3 A^{-1/3} Z - 4a_4 + 8a_4 A^{-1} Z$$

$$0 = (m_p - m_n) - 4a_4 + (2a_3 A^{-1/3} + 8a_4 A^{-1})Z$$

$$Z = \frac{(m_p - m_n) + 4a_4}{2a_3 A^{-1/3} + 8a_4 A^{-1}} \quad \text{where } a_3 = 0.75 \text{ and } a_4 = 93.2$$

(a) For $A = 27$: $Z = \dfrac{(1.008665 - 1.007825)(931.5 MeV/uc^2) + 4(93.2)}{2(0.75)(27)^{-1/3} + 8(93.2)(27)^{-1}} = 13.2$

Minimum $Z = 13$

(b) For $A = 65$: Computing as in (a) with $A = 65$ yields $Z = 31.5$. Minimum $Z = 29$.

(c) For $A = 139$: Computing as in (a) with $A = 139$ yields $Z = 66$. Minimum $Z = 57$.

11-72. (a) $R = (0.31)E^{3/2} = (0.31)(5MeV)^{3/2} = 3.47cm$

(b) $R(g/cm^2) = R(cm)\rho = (3.47cm)(1.29\times10^{-3} g/cm^3) = 4.47\times10^{-3} g/cm^2$

(c) $R(cm) = R(g/cm^2)/\rho = (4.47\times10^{-3} g/cm^2)/(2.70 g/cm^3) = 1.66\times10^{-3} cm$

11-73. For one proton, consider the nucleus as a sphere of charge e and charge density $\rho_c = 3e/4\pi R^3$. The work done in assembling the sphere, i.e., bringing charged shell dq up to r, is: $dU_c = k\rho_c\left(\dfrac{4\pi r^3}{3}\right)(\rho_c 4\pi r^2 dr)\left(\dfrac{1}{r}\right)$ and integrating from 0 to R yields:

$$U_c = \frac{k\rho_c^2 16\pi^2 R^5}{15} = \frac{3}{5}\frac{ke^2}{R}$$

For two protons, the coulomb repulsive energy is twice U_c, or $6ke^2/5R$.

11-74. The number N of ^{144}Nd atoms is: $N = \dfrac{(53.94g)(6.02\times10^{23}atoms/mol)}{144g/mol} = 2.25\times10^{23}atoms$

$$-\frac{dN}{dt} = \lambda N \quad \rightarrow \quad \lambda = (-dN/dt)/N$$

$$= (2.36s^{-1})/(2.25\times10^{23}) = 1.05\times10^{-23}s^{-1}$$

$$t_{1/2} = \ln(2)/\lambda = \ln(2)/1.05\times10^{-23}s^{-1} = 6.61\times10^{23}s = 2.09\times10^{15}y$$

11-75. $R = R_0 e^{-t(\ln 2)/t_{1/2}}$ (from Equation 11-19)

(a) At t = 0: $R = R_0$ = 115.0 *decays/min*

At $t = 4d\ 5h = 4.21d$: R = 73.5 *decays/min*

$$73.5 decays/min = (115.0 decays/min)e^{-(\ln 2)(4.21d)/t_{1/2}}$$

$$(73.5/115.0) = e^{-(\ln 2)(4.21d)/t_{1/2}}$$

$$\ln(73.5/115.0) = -(\ln 2)(4.21d)/t_{1/2}$$

$$t_{1/2} = -(\ln 2)(2.41d)/\ln(73.5/115.0) = 6.52d$$

(b) $R = 10 decays/min = (115.0 decays/min)e^{-(\ln 2)t/6.52d}$

$$t = -\ln(10/115.0)(6.52d)/\ln(2) = 23.0d$$

(c) $R = 2.5 decays/min = (115.0 decays/min)e^{-(\ln 2)t/6.52d}$

$$t = -\ln(2.5/115.0)(6.52d)/\ln(2) = 36.0d$$ (because t = 0)

This time is 13 days (= $2t_{1/2}$) after the time in (b).

11-76. For ^{227}Th: $t_{1/2} = 18.72d$ (nucleus A)

For ^{223}Ra: $t_{1/2} = 11.43d$ (nucleus B)

At t = 0 there are 10^6 *Th* atoms and 0 *Ra* atoms

(a) $N_A = N_{0A}e^{-\lambda_A t}$ (Equation 11-18)

$$N_B = \frac{N_{0A}\lambda_A}{\lambda_B - \lambda_A}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right) + N_{0B}e^{-\lambda_B t}$$ (Equation 11-26 on the Web page)

(Problem 11-76 continued)

$$N_A = 10^6 e^{-(\ln 2)(15d)/18.72d} = 5.74 \times 10^5$$

$$N_B = \frac{10^6 (\ln 2)/18.72d}{(\ln 2)(1/11.43d - 1/18.72d)}\left(e^{-(\ln 2)(15)/18.72} - e^{-(\ln 2)(15)/11.43}\right) + 0 = 2.68 \times 10^5$$

(b) $N_A = N_B$ means $N_{0A} e^{-\lambda_A t} = \dfrac{N_{0A}\lambda_A}{\lambda_B - \lambda_A}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right)$

Cancelling N_{0A} and rearranging,

$$-\frac{(\lambda_B - \lambda_A)}{\lambda_A} + 1 = e^{(\lambda_A - \lambda_B)t}$$

$$\lambda_A = \frac{\ln 2}{18.82d} = 0.0370d^{-1} \qquad \lambda_B = \frac{\ln 2}{11.43d} = 0.0606d^{-1}$$

$$\ln\left(-\frac{(\lambda_B - \lambda_A)}{\lambda_A} + 1\right) = (\lambda_B - \lambda_A)t$$

$$t = \ln\left(-\frac{(\lambda_B - \lambda_A)}{\lambda_A} + 1\right)\Big/(\lambda_A - \lambda_B) = \ln\left(-\frac{(0.0606 - 0.0370)}{0.0370} + 1\right)\Big/(0.0370 - 0.0606)$$

$$= 43.0d$$

11-77. (a) $\Gamma = \hbar/\tau = 6.582 \times 10^{-16} eV \bullet s \big/ 0.13 \times 10^{-9} s = 5.06 \times 10^{-6} eV$

(b) $E_r = \dfrac{(hf)^2}{2Mc^2} = \dfrac{(0.12939MeV)^2}{2M(^{191}I)c^2}$ (Equation 11-47)

$$= 4.71 \times 10^{-8} MeV = 4.71 \times 10^{-2} eV$$

(c) (See Section 1-5)

The relativistic Doppler shift Δf for either receding or approaching is:

$$\frac{\Delta f}{f_0} \approx \beta = \frac{v}{c} \qquad h\Delta f = \Gamma \qquad hf_0 = E$$

$$\frac{\Gamma}{E} = \frac{v}{c} \rightarrow v = \frac{\Gamma_c}{E} = \frac{(5.06 \times 10^{-6} eV)(3.00 \times 10^8 m/s)}{(0.12939MeV)(10^6 eV/MeV)} = 0.0117m/s = 1.17cm/s$$

11-78. $B = ZM\ \ {}^{1}H\ \ c^2 + Nm_n c^2 - M_A c^2$

For ${}^{3}He$: $B = 2\ \ 1.007825uc^2\ \ + 1.008665uc^2 - 3.016029uc^2$

$= 0.008826uc^2\ \ 931.5MeV/uc^2\ \ = 7.72MeV$

For ${}^{3}H$: $B = 1.007825uc^2 + 2\ \ 1.008665uc^2\ \ - 3.016029uc^2$

$= 0.009106uc^2\ \ 931.5MeV/uc^2\ \ = 8.48MeV$

$R = R_0 A^{1/3} = 1.2\,fm 3^{1/3} = 1.730\,fm = 1.730 \times 10^{-15}\,m$

$U_c = ke^2/R = ke/R\ \ eV\ \ = 8.32 \times 10^5 eV = 0.832MeV$ or about 1/10 of the binding energy.

11-79. For ${}^{47}Ca$:

$B = M\ \ {}^{46}Ca\ \ c^2 + m_n c^2 - M\ \ {}^{47}Ca\ \ c^2$

$= 45.953687uc^2 + 1.008665uc^2 - 46.954541uc^2$

$= 0.007811uc^2\ \ 931.5MeV/uc^2\ \ = 7.28MeV$

For ${}^{48}Ca$:

$B = M\ \ {}^{47}Ca\ \ c^2 + m_n c^2 - M\ \ {}^{48}Ca\ \ c^2$

$= 45.954541uc^2 + 1.008665uc^2 - 47.952534uc^2$

$= 0.010672uc^2\ \ 931.5MeV/uc^2\ \ = 9.94MeV$

Assuming the even-odd ${}^{47}Ca$ to be the "no correction" nuclide, the average magnitude of the correction needed to go to either of the even-even nuclides ${}^{46}Ca$ or ${}^{48}Ca$ is approximately *B – average binding energy of the odd neutron*, $9.94MeV + 7.28MeV\ \ /2 = 8.61MeV$. So the correction for ${}^{46}Ca$ is 8.16 – 7.28 = 0.88 *MeV* and for ${}^{48}Ca$ is 9.94 – 8.16 = 1.78 *MeV*, an "average" of about 1.33 *MeV*. The estimate for a_5 is then: $a_5 A^{-1/2} = 1.33MeV \rightarrow a_5 = 1.33/48^{-1/2} = 9.2$. This value is about 30% below the accepted empirical value of $a_5 = 12$.

11-80. For a nucleus with $I > 0$ the α feels a centripetal force

$F_c = mv^2 / r = -dV / dr$ where $r =$ distance of the α from the nuclear center. The corresponding potential energy $V \propto -\ln\ r$ and becomes larger (i.e., more negative) as r increases. This lowers the total energy of the α near the nuclear boundary and results in a wider barrier, hence lower decay probability.

11-81. (a) $R = R_0 A^{1/3}$ where $R_0 = 1.2 f$ (Equation 11-3)

$$R\ ^{141}Ba = 1.2\,fm\ \ 10^{-15} m / fm\ \ 141\ ^{1/3} = 6.24 \times 10^{-15} m$$

$$R\ ^{92}Kr = 1.2\,fm\ \ 10^{-15} m / fm\ \ 92\ ^{1/3} = 5.42 \times 10^{-15} m$$

(b) $$V = kq_1 q_2 / r = \frac{8.998 \times 10^9 N \bullet m^2 / C^2\ \ 56\ \ 1.60 \times 10^{-19} C\ \ 36\ \ 1.60 \times 10^{-19} C}{6.24 \times 10^{-15} m + 5.42 \times 10^{-15} m\ \ 1.60 \times 10^{-19} J / eV}$$

$$= 2.49 \times 10^8 eV = 249 MeV$$

This value is about 40% larger than the measured value.

11-82. (a) In the lab, the nucleus (at rest) is at $x = 0$ and the neutron moving at v_L is at x.

$$x_{CM} = \frac{M\ \ 0\ + mx}{M + m} = \frac{mx}{M + m} \qquad v_L = \frac{x}{dt} \text{ and } V = \frac{dx_m}{dt} = \frac{m\ \ x / dt}{M + m}$$

$$V = \frac{mv_L}{M + m}$$

(b) The nucleus at rest in the lab frame moves at speed V in the *CM* frame before the collision. In an elastic collision in the *CM* system, the particles reverse their velocities, so the speed of the nucleus is still V, but in the opposite direction.

(c) In the *CM* frame in the nucleus velocity changes by *2V*. This is also the change in the lab system where the nucleus was initially at rest. It moves with speed *2V* in the lab system after the collision.

(d) $$\frac{1}{2} M\ \ 2V\ ^{1/2} = \frac{1}{2} M \left[\frac{2mv_L}{M + m} \right] = \frac{1}{2} mv_L^2 \left[\frac{4mM}{M + m\ ^2} \right]$$

(Problem 11-82 continued)

Before collision: $E_i = \frac{1}{2}mv_L^2$

After collision: $E = \frac{1}{2}mv_L^2 - \frac{1}{2}mv_L^2\left[\frac{4mM}{(M+m)^2}\right] = E_i\left(1 - \frac{4mM}{(M+m)^2}\right)$

11-83. At the end of the two hour irradiation the number of ^{32}P and ^{56}Mn atoms are given by

$N = \frac{R_0}{\lambda}(1 - e^{-\lambda t})$ from Equation 11-26 where $R_0 = \sigma I$ (Equation 11-62)

For ^{32}P:

$$R_0 = (0.180\times10^{-24}cm^2)(10^{12}neutrons/cm^2\bullet s) = 1.80\times10^{-13}\ (^{32}P\ atoms/s\ per\ ^{31}P)$$

$$N_0 = \frac{R_0 t_{1/2}}{\ln(2)}(1 - e^{-(\ln 2)(t/t_{1/2})}) = \frac{(1.80\times10^{-13}/s)(3600s/h)(342.2h)}{\ln(2)}(1 - e^{-(\ln 2)(2h/342.2h)})$$

$$= 1.29\times10^{-9}\ ^{32}P\ atoms/^{31}P\ atom$$

For ^{56}Mn:

$$R_0 = (13.3\times10^{-24}cm^2)(10^{12}neutrons/cm^2\bullet s) = 1.33\times10^{-11}\ (^{56}Mn\ atoms/s\ per\ ^{55}Mn)$$

$$N_0 = \frac{(1.33\times10^{-11}/s)(3600s/h)(2.58h)}{\ln(2)}(1 - e^{-(\ln 2)(2h/2.58h)})$$

$$= 7.42\times10^{-8}\ ^{56}Mn\ atoms/^{55}Mn\ atom$$

(a) Two hours after the irradiation stops, the activities are:

$$\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t} = -\frac{N_0 \ln(2)}{t_{1/2}} e^{-(\ln 2)(t/t_{1/2})}$$

For ^{32}P:

$$\left|\frac{dN}{dt}\right| = \frac{(1.29\times10^{-9})\ln(2)}{(14.26d)(8.64\times10^4 s/d)} e^{-(\ln 2)(2h/342.2h)} = 7.23\times10^{-16} decays/^{31}P\ atom$$

For ^{56}Mn:

(Problem 11-83 continued)

$$\left|\frac{dN}{dt}\right| = \frac{7.42\times10^{-8}\ \ln 2}{2.58h\ \ 3600s/h}e^{-\ln 2\ \ 48h/2.58h} = 1.39\times10^{-17}\,decays/^{56}Mn\ atom$$

The total activity is the sum of these, each multiplied by the number of parent atoms initially present.

11-84. $Q = 200MeV/\,fission.$

$$E = NQ = 7.0\times10^{19}\,J = N\ \ 200MeV/\,fission\ \ \ 1.60\times10^{-13}\,J/MeV$$

$$N = \frac{7.0\times10^{19}\,J}{200MeV/\,fission\ \ \ 1.60\times10^{-13}\,J/MeV} = 2.19\times10^{30}\ fissions$$

Number of moles of ^{235}U needed = $N/N_A = 2.19\times10^{30}/6.02\times10^{23} = 3.63\times10^{6}\,moles$

Fissioned mass/y = $3.63\times10^{6}\,moles\ \ \ 235g/mole\ = 8.54\times10^{8}\,g = 8.54\times10^{5}\,kg$

That is 3% of the mass of the ^{235}U atoms needed to produce the energy consumed.

Mass needed to produce $7.0\times10^{19}\,J = 8.54\times10^{5}\,kg/0.03 = 2.85\times10^{7}\,kg.$

Since the energy conversion system is 25% efficient:

Total mass of ^{235}U needed = $1.14\times10^{8}\,kg.$

11-85. The number of ^{87}Sr atoms present at any time is equal to the number of ^{87}Rb nuclei that have decayed, because ^{87}Sr is stable.

$$N\ \ Sr\ = N_0\ \ Rb\ - N\ \ Rb\ \rightarrow N\ \ Sr\ /N\ \ Rb\ = N_0\ \ Rb\ /N\ \ Rb\ - 1\mathrm{N}$$

$$N\ \ Sr\ /N\ \ Rb\ = 0.010$$

$$N_0\ \ Rb\ /N\ \ Rb\ = N\ \ Sr\ /N\ \ Rb\ + 1 = 1.010$$

and also

$$N\ \ Rb\ /N_0\ \ Rb\ = e^{-\ln 2\ \,t/t_{1/2}} = 1/1.010$$

$$\frac{-\ \ln 2\ \ t}{t_{1/2}} = \ln\ 1/1.010$$

$$t = -t_{1/2}\ln\ 1/1.010\ /\ln(2) = -\ 4.9\times10^{10}\,y\ \ln\ 1/1.010\ /\ln(2) = 7.03\times10^{8}\,y$$

11-86. (a) Average energy released/reaction is: $(3.27MeV + 4.03MeV)/2 = 3.65MeV$

$$P = \frac{E}{t} = 4W = 4J/s = N(3.65MeV)(1.60\times10^{-13}J/MeV)$$

$$N = \frac{4J/s}{(3.65MeV/reaction)(1.60\times10^{-13}J/MeV)} = 6.85\times10^{12}\ reactions/s$$

Half of the reactions produce neutrons, so $3.42\times10^{12}\ neutrons/s$ will be released.

(b) Neutron absorption rate $= 0.10(3.42\times10^{12}) = 3.42\times10^{11}\ neutrons/s$

Energy absorption rate =

$$(0.5MeV/neutron)(3.42\times10^{11}neutrons/s)(1.60\times10^{-13}J/MeV) = 2.74\times10^{-2}J/s$$

Radiation dose rate =

$$\left[(2.74\times10^{-2}J/s)/(80kg)\right]\left[100rad/(J/kg)\right] = 3.42\times10^{-3}rad/s$$

$$= (3.42\times10^{-2}rad/s)(4) = 0.137\,rem/s = 493\,rem/h$$

(c) 500 *rem*, lethal to half of those exposed, would be received in:

$$500rem/(492rem/h) = 1.02h$$

11-87. $R(t) = N_0\sigma I(1-e^{-\lambda t})$ (Equation 11-86)

For *Co*: $N_0 = \dfrac{35Bq}{(19\times10^{-24}cm^2)(3.5\times10^{12}/s\bullet cm^2)(1-e^{-1.319\times10^{-6}(2)})} = 2.00\times10^{17}$ atoms

For *Ti*: $N_0 = \dfrac{115Bq}{(0.15\times10^{-24}cm^2)(3.5\times10^{12}/s\bullet cm^2)(1-e^{-0.120(2)})} = 1.03\times10^{15}$ atoms

11-88. The net reaction is: $5\,{}^2H \rightarrow {}^3He + {}^4He + {}^1H + n + 25MeV$

Energy release / ${}^2H = 5MeV$ (assumes equal probabilities)

4ℓ water $\rightarrow 4000g/\left[2(1.007825)+15.994915\right]g/mol = 222.1\ moles$

4ℓ of water thus contains 2(222.1) moles of hydrogen, of which 1.5×10^{-4} is ${}^2H$, or

Number of ${}^2H$ atoms =

(Problem 11-88 continued)

$$\left[2\left(222.1\ moles\right)\right]\left(6.02\times10^{23}\,atoms/mol\right)\left(1.5\times10^{-4}\right)=4.01\times10^{22}$$

Total energy release = $\left(4.01\times10^{22}\right)\left(5MeV\right)=2.01\times10^{23}MeV=3.22\times10^{10}J$

Because the U.S. consumes about $1.0\times10^{20}J/y$, the complete fusion of the ^{2}H in 4ℓ of water would supply the nation for about $1.01\times10^{-2}s=10.1ms$

11-89. (a) $\Delta\lambda\le 2hc/Mc^2$

$$\Delta E\approx\frac{hc\Delta\lambda}{\lambda^2}=\frac{\left(hc\right)^2}{\lambda^2}\frac{\Delta\lambda}{hc}=\frac{E^2\Delta\lambda}{hc}$$

$$E_p=\Delta E\le\frac{E^2}{hc}\frac{2hc}{Mc^2}=\frac{2E^2}{Mc^2}$$

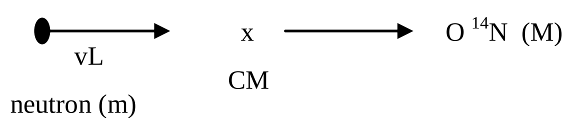
$$E^2\ge Mc^2E_p/2\quad\rightarrow\quad E\ge\left(Mc^2E_p/2\right)^{1/2}$$

$$\Delta E=E_f-E_i=E_i\left(1-\frac{4mM}{\left(M+m\right)^2}\right)-E_i=-E_i\left(\frac{4mM}{\left(M+m\right)^2}\right)$$

$$\frac{-\Delta E}{E_i}=\frac{4mM}{\left(M+m\right)^2}=\frac{4m/M}{\left(1+m/M\right)^2}$$ which is Equation 11-82 in More section.

(b) $E=\left[\left(5.7MeV\right)\left(938.28MeV\right)/2\right]^{1/2}=51.7MeV$

(c)



The neutron moves at v_L in the lab, so the *CM* moves at $v=v_Lm_N/\left(m_N+M\right)$ toward the right and the ^{14}N velocity in the *CM* system is *v* to the left before collision and *v* to the right after collision for an elastic collision. Thus, the energy of the nitrogen nucleus in the lab after the collision is:

$$E\left(^{14}N\right)=\frac{1}{2}M\left(2v\right)^2=2Mv^2=2M\left(\frac{mv_L}{m+M}\right)^2$$

(Problem 11-89 continued)

$$= \frac{2Mm\,(mv_L^2)}{(m+M)^2} = \frac{4Mm}{(m+M)^2}\left(\frac{1}{2}mv_L^2\right)$$

$$= \frac{4\,(14.003074u)(1.008665u)}{(1.008665u + 14.003074u)^2}\,(5.7MeV)$$

$= 1.43\ MeV$

(d) $E \geq \left[(14.003074uc^2)(931.5MeV/uc^2)(1.43MeV)/2\right]^{1/2} = 96.5MeV$

11-90. In the lab frame:

photon → ○ deuteron, M at rest

$E = hv = pc$

$p = hv/c = E/c$

In *CM* frame:

photon → ← ○ deuteron, M

$E \approx pc$ $\qquad$ $E_K \approx 1/2Mv^2 = p^2/2M$

$p = E/c$ $\qquad$ $p = \sqrt{2ME_K}$

For $E \approx pc$ in *CM* system means that a negligible amount of photon energy goes to recoil energy of the deuteron, i.e.,

$$\frac{p^2}{2M} \ll pc \approx E \qquad \text{or} \qquad \frac{(pc)^2}{2Mc^2} \ll pc \rightarrow pc \ll 2Mc^2$$

$$E \approx pc \ll 2Mc^2 = 2(1875.6MeV) = 3751.2MeV \quad \text{(see Table 11-1)}$$

In the lab, that incident photon energy must supply the binding energy $B = 2.22\ MeV$ plus the recoil energy E_K given by:

$$E_K = p^2/2M = (pc)^2/2Mc^2 \approx (B)^2/2Mc^2$$

$$= \frac{(2.22MeV)^2}{2(1875.6MeV)} = 0.0013MeV$$

(Problem 11-90 continued)

So the photon energy must be $E \geq 2.22MeV + 0.001MeV = 2.221MeV$, which is much less than 3751 *MeV*.

11-91. (a) $B = ZM\ \ {}^{1}H\ \ c^2 + Nm_n c^2 - M_A c^2$ (Equation 11-11)

For ${}^{7}Li$: $B = 3\ \ 1.007825uc^2\ \ + 4\ \ 1.008665uc^2\ \ - 7.016003uc^2$

$= 0.042132uc^2\ \ 931.5MeV / uc^2\ \ = 39.25MeV$

For ${}^{7}Be$: $B = 4\ \ 1.007825uc^2\ \ + 3\ \ 1.008665uc^2\ \ - 7.016928uc^2$

$= 0.040367uc^2\ \ 931.5MeV / uc^2\ \ = 37.60MeV$

$\Delta B = 1.65MeV$

For ${}^{11}B$: $B = 5\ \ 1.007825uc^2\ \ + 6\ \ 1.008665uc^2\ \ - 11.009305uc^2$

$= 0.0081810uc^2\ \ 931.5MeV / uc^2\ \ = 76.21MeV$

For ${}^{11}C$: $B = 6\ \ 1.007825uc^2\ \ + 5\ \ 1.008665uc^2\ \ - 11.011433uc^2$

$= 0.078842uc^2\ \ 931.5MeV / uc^2\ \ = 73.44MeV$

$\Delta B = 2.77MeV$

For ${}^{15}N$: $B = 7\ \ 1.007825uc^2\ \ + 8\ \ 1.008665uc^2\ \ - 15.000108uc^2$

$= 0.123987uc^2\ \ 931.5MeV / uc^2\ \ = 115.5MeV$

For ${}^{15}O$: $B = 8\ \ 1.007825uc^2\ \ + 7\ \ 1.008665uc^2\ \ - 15.003065uc^2$

$= 0.120190uc^2\ \ 931.5MeV / uc^2\ \ = 112.0MeV$

$\Delta B = 3.54MeV$

(b) $\Delta B = a_3 \Delta\ \ Z^2\ \ A^{-1/3} \quad \rightarrow \quad a_3 = \Delta B \big/ \Delta\ \ Z^2\ \ A^{-1/3}$

For $A = 7;\ Z = 4$: $a_3 = 1.65MeV \big/ 7\ \ 7\ \ ^{-1/3} = 0.45MeV$

For $A = 11;\ Z = 6$: $a_3 = 2.77\,MeV \big/ 11\ \ 11\ \ ^{-1/3} = 0.56MeV$

(Problem 11-91 continued)

For $A = 15;\ Z = 8$: $\quad a_3 = 3.54 MeV \big/ 15\ 15^{-1/3} = 0.58 MeV$

$\langle a_3 \rangle = 0.53 MeV$

These values differ significantly from the empirical value of $a_3 = 0.75 MeV$.

11-92. (a) Using $\partial M / \partial Z = 0$ from Problem 11-71.

$$Z = \frac{m_n - m_p \ + 4\alpha_4}{2a_3 A^{-1/3} + 8a_4 A^{-1}} \quad \text{where } a_3 = 0.75 MeV/c^2, \quad a_4 = 93.2 MeV/c^2$$

$$= \frac{1 + \ m_n - m_p \ \big/ 4a_4}{2A^{-1} + \ a_3 A^{-1/3} \big/ 2a_4} = \frac{A}{2} \frac{\left[1 + \ m_n - m_p \ \big/ 4a_4\right]}{\left[1 + a_3 A^{2/3} \big/ 4a_4\right]}$$

(b) & (c) For $A = 29$: $\quad Z = \dfrac{29}{2} \dfrac{\left[1 + \ 1.008665 - 1.007276 \quad 931.5 \ / \ 4 \quad 93.2 \ \right]}{\left[1 + 0.75 \ 29^{\ 2/3} \ / \ 4 \quad 93.2 \ \right]} = 14$

The only stable isotope with A = 29 is $^{29}_{14}Si$

For A =59: Computing as above with A = 59 yields Z = 29. The only stable isotope with A = 59 is $^{59}_{27}Co$

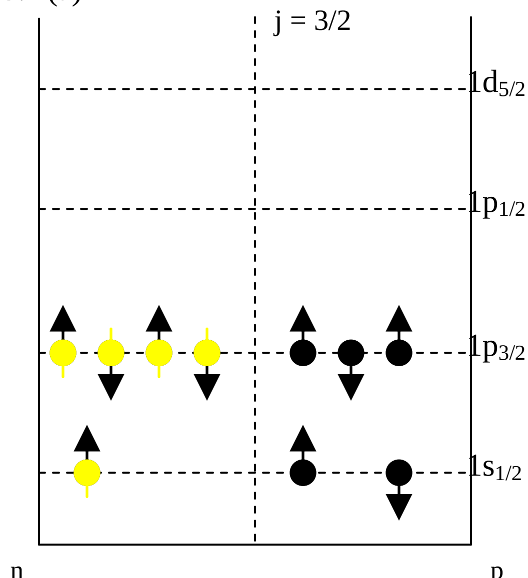
For A = 78: Computing as above with A = 78 yields Z = 38. $^{78}_{38}Sr$ is not stable. Stable isotopes with A = 78 are $^{78}_{34}Se$ and $^{78}_{38}Kr$.

The only stable isotope with A = 119 is $^{119}_{50}Sn$.

For A = 140: Computing as above with A = 140 yields Z = 69. $^{140}_{69}Tm$ is not stable. The only stable isotope with A = 140 is $^{140}_{58}Ce$.

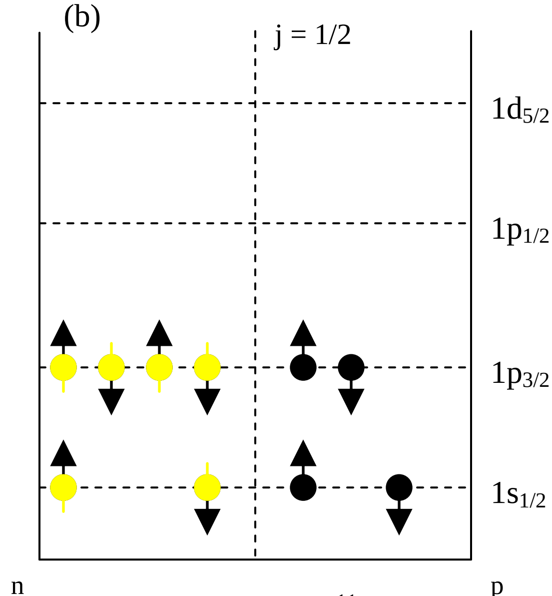
The method of finding the minimum Z for each A works well for $A \leq 60$, but deviates increasingly at higher A values.

11-93. (a)



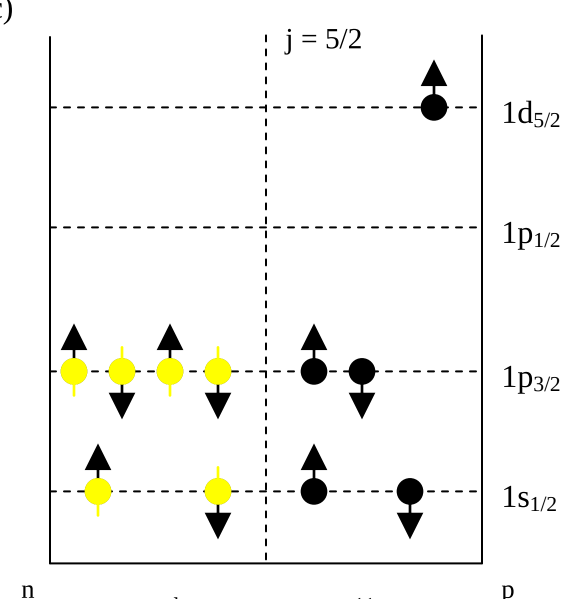
Ground state ^{11}B

(b)



1st excited state ^{11}B

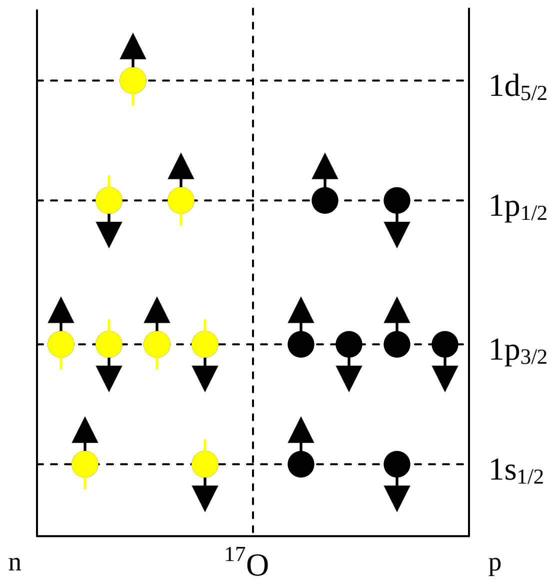
(c)



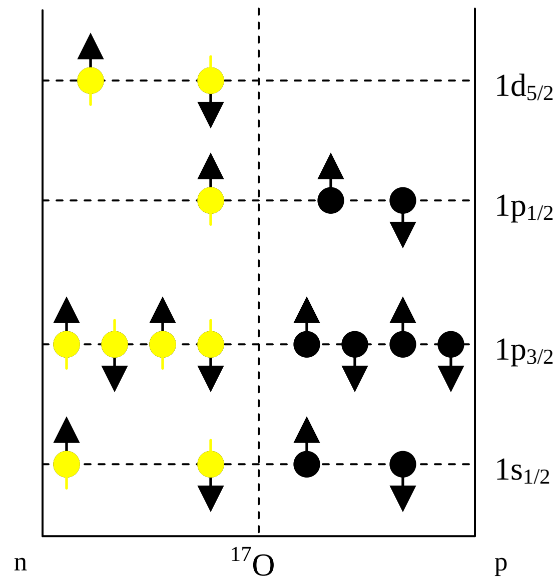
2nd excited state ^{11}B

(Problem 11-93 continued)

(d)

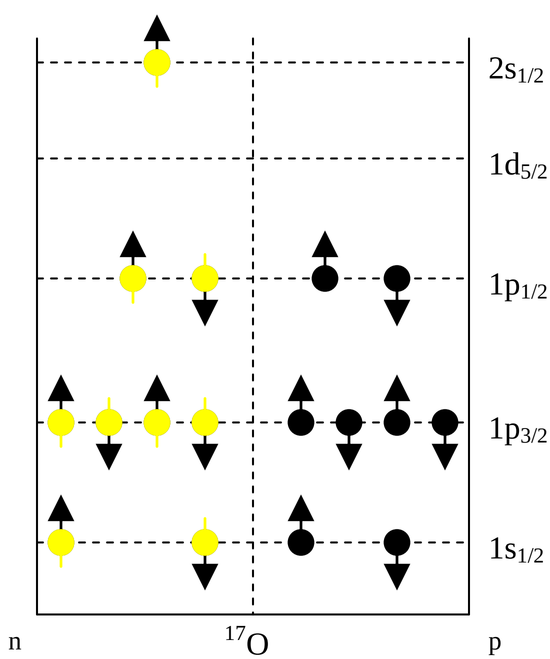


Ground state (j = 5/2)



1st excited state (j = 1/2)

(e)



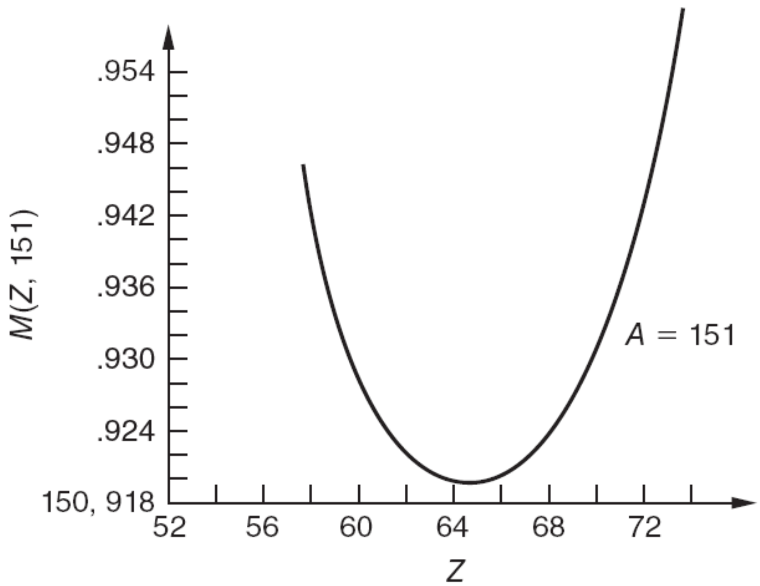
2nd excited state (j = 1/2)

11-94. (a) Data from Appendix A are plotted on the graph. For those isotopes not listed in Appendix A, data for ones that have been discovered can be found in the reference sources, e.g., *Table of Isotopes,* R.B. Firestone, Wiley – Interscience (1998). Masses for those not yet discovered or not in Appendix A are computed from Equation 11-14 (on the Web). Values of $M(Z, 151)$ computed from Equation 11-14 are listed below. Because values found from Equation 11-14 tend to overestimate the mass in the higher A regions, the calculated value was adjusted to the measured value for $Z = 56$, the lowest Z known for $A = 151$ and the lower Z values were corrected by a corresponding amount. The error introduced by this correction is not serious because the side of the parabola is nearly a straight line in this region. On the high Z side of the $A = 151$ parabola, all isotopes through $Z = 70$ have been discovered and are in the reference cited.

Z	N	$M(Z, 151)$ [Equation 11-14]	$M(Z, 151)$ [adjusted]
50	101	152.352638	151.565515
51	100	152.234612	151.447490
52	99	152.122188	151.335066
53	98	152.015365	151.228243
54	97	151.914414	151.127292
55	96	151.818525	151.031403
56	95	151.728507	150.941385*

* This value has been measured.

(Problem 11-94 continued)



(b) The drip lines occur for:

protons: $M(Z,151) - \left[M(Z-1,\ 150) - m_p\right] = 0$

neutrons: $M(Z,151) - \left[M(Z,\ 150) - m_n\right] = 0$

Write a calculator or computer program for each using Equation 11-14 (on Web page) and solve for *Z*.

11-95. (a) $M(Z,A) = Zm_p + Nm_n - \left[a_1A - a_2A^{2/3} - a_3A^{-1/3} - a_4(A-2Z)^2A^{-1} + a_5A^{-1/2}\right]$

from Equation 11-14 on the Web page.

$$M(126,310) = 126m_p + 184m_n - \begin{bmatrix} 15.67(310) - 17.23(310)^{2/3} \\ -0.75(126)^2(310)^{-1/3} \\ -93.2(310 - 2\times126)^2(310) \\ +12(310)^{-1} + 12(310)^{-1/2} \end{bmatrix}$$

$M(126,310) = 313.969022u$

(b) For β^- decay: (126, 310) → (127, 310) + β^- + $\overline{\nu_e}$

Computing *M*(127, 310) as in (a) yields 314.011614*u*.

(Problem 11-95 continued)

$$Q = M(126, 310)c^2 - M(127, 310)c^2$$

$$= 313.969022uc^2 - 314.011614uc^2$$

$$= 0.042592u(931.5MeV/uc^2) = -39.7MeV$$

$Q < 0$, so β^- decay is forbidden by energy conservation.

For β^+ decay: (126, 310) → (125, 310) + β^+ + ν_e

Computing M(125, 310) as in (a) yields 313.923610u.

$$Q = M(126, 310)c^2 - M(125, 310)c^2 - 2m_e c^2$$

$$= 313.969022uc^2 - 313.923610uc^2 - 1.022MeV$$

$$= 41.3MeV$$

β^+ decay and electron capture are possible decay modes.

For α decay: (126, 310) → (124, 310) + α

Computing M(124, 310) as in (a) yields 309.913540u.

$$Q = 313.969022uc^2 - 309.913540uc^2 - 4.002602uc^2$$

$$= 49.3MeV$$

α decay is also a possible decay mode.

11-96. (a) If the electron's kinetic energy is 0.782MeV, then its total energy is:

$$E = 0.782MeV + m_e c^2 = 0.782MeV + 0.511MeV = 1.293MeV$$

$$E^2 = (pc)^2 + (m_e c^2)^2 \quad \text{(Equation 2-32)}$$

$$p = \left[E^2 - (m_e c^2)^2\right]^{1/2} \Big/ c$$

$$= \left[(1.293MeV)^2 - (0.511MeV)^2\right]^{1/2} \Big/ c$$

$$= 1.189MeV/c$$

(b) For the proton $p = 1.189MeV/c$ also, so

$$E_{kin} = p^2/2m = (pc)^2 \Big/ 2mc^2$$

$$= (1.189MeV)^2 \Big/ 2(938.28MeV) = 7.53 \times 10^{-4} MeV = 0.753keV$$

(Problem 11-96 continued)

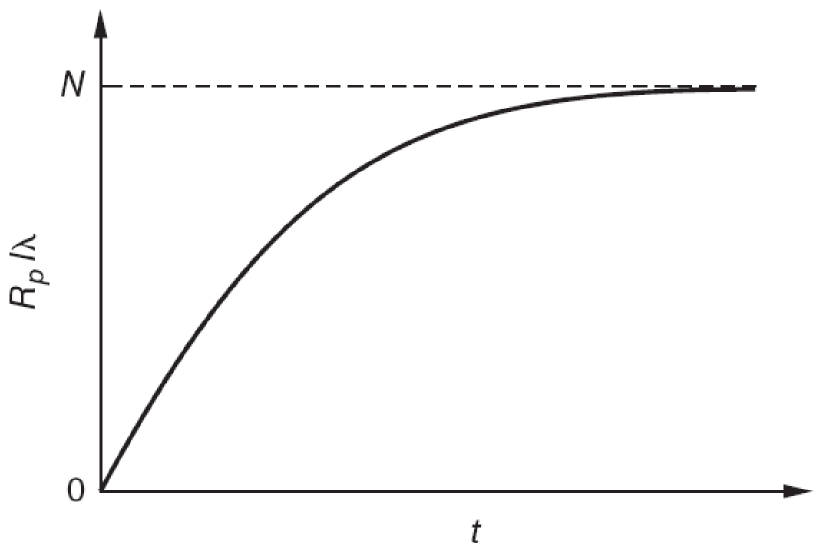
(c) $\dfrac{7.53\times10^{-4}\,MeV}{0.782\,MeV}\times100 = 0.0963\%$

11-97. $\dfrac{dN}{dt} = R_p = -\lambda N$ (Equation 11-17)

(a) $\lambda N = R_p - \lambda N_0 e^{-\lambda t} = R_p - R_p e^{-\lambda t} = R_p\left(1-e^{-\lambda t}\right)$

$N = \left(R_p/\lambda\right)\left(1-e^{-\lambda t}\right)$ (from Equation 11-17)

At $t = 0$, $N(0) = 0$. For large t, $N(t) \to R_p/\lambda$, its maximum value



(b) For $dN/dt \approx 0$

$R_p = \lambda N \to N = R_p/\lambda = R_p/\left(\ln 2/t_{1/2}\right)$

$N = 100s^{-1}/\left(\ln 2/10\,min\right) = \left(100s^{-1}\right)\left(60s/min\right)/\left(\ln 2/10\,min\right)$

$= 8.66\times10^4$ ^{62}Cu nuclei

11-98. (a) $4n + 3$ decays chain $^{235}_{92}U_{143} \to ^{207}_{82}Pb_{125}$ There are 12 α decays in the chain. (See graph below.)

(b) There are 9 β^- decays in the chain. (See graph below.)

(Problem 11-98 continued)

(c) $Q = M\left({}^{235}U\right)c^2 - M\left({}^{207}Pb\right)c^2 - 7M\left({}^{4}He\right)c^2$

$$= 235.043924uc^2 - 206.975871uc^2 - 7\left(4.002602\right)uc^2$$

$$= 0.049839uc^2\left(931.50MeV/uc^2\right) = 46.43MeV$$

(d) The number of decays in one year is:

$$-\frac{dN}{dt} = \lambda N_0 e^{-\lambda t} \text{ where } \lambda = \ln 2/t_{1/2} = \ln 2/7.04\times10^8 y = 9.85\times10^{-10} y^{-1}$$

$$N_0 = \frac{\left(1kg\right)\left(1000g/kg\right)\left(6.02\times10^{23} atoms/mol\right)}{235g/mol} = 2.56\times10^{24} atoms$$

$$-\frac{dN}{dt} = \left(9.85\times10^{-10} y^{-1}\right)\left(2.56\times10^{24}\right)\left(e^{-\lambda(1y)}\right) = 2.52\times10^{15} decays/y$$

Each decay results in the eventual release of 46.43 *MeV*, so the energy release per year Q is: $Q = \left(2.52\times10^{15} decays/y\right)\left(46.43MeV/decay\right)$

$$= \left(1.17\times10^{17} MeV/y\right)\left(1.60\times10^{-13} J/MeV\right)$$

$$= \left(1.87\times10^{4} J/y\right)\left(1cal/4.186J\right) = 4.48\times10^{3} cal/y$$

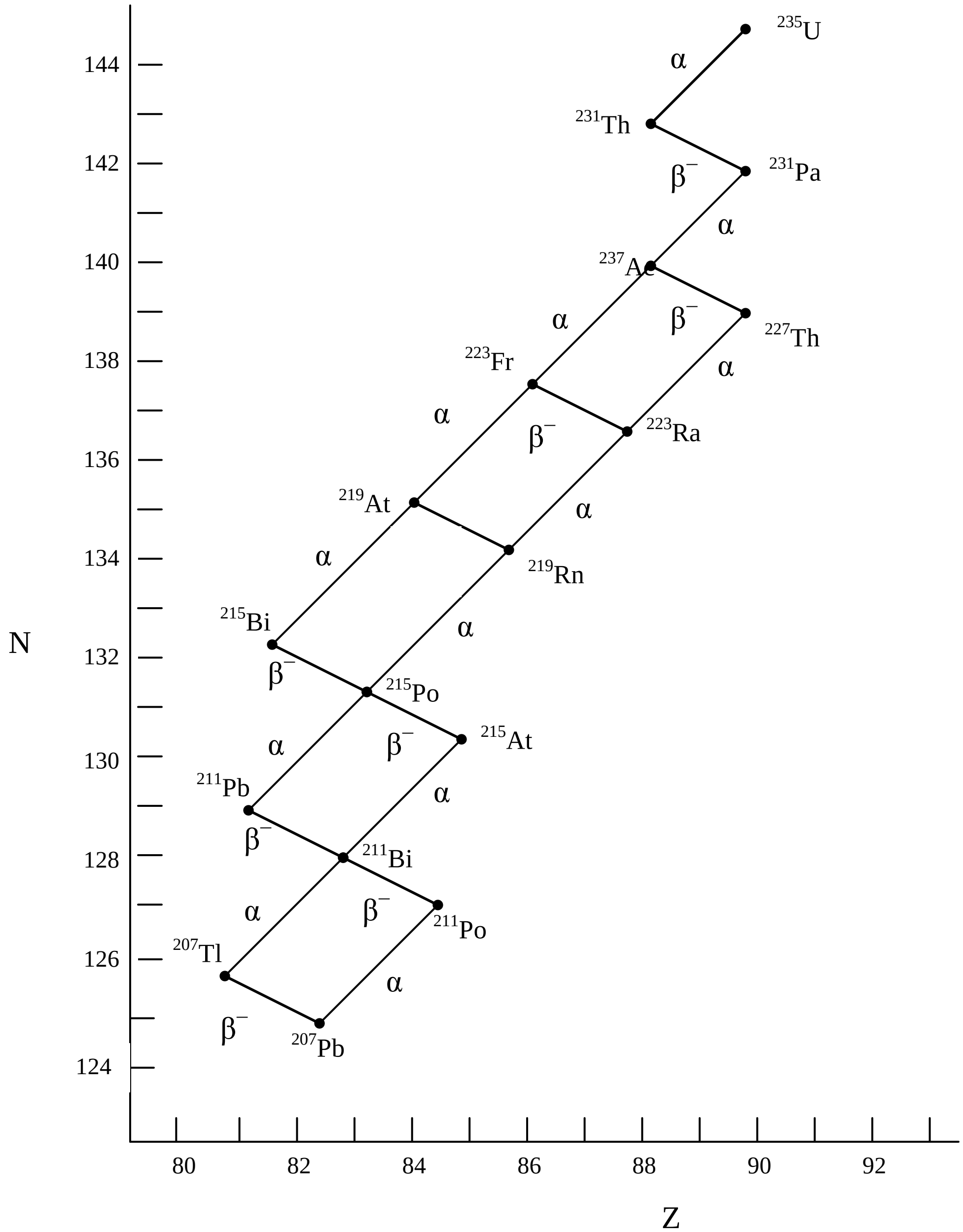
The temperature change ΔT is given by:

$Q = cm\Delta T$ or $\Delta T = Q/cm$ where $m = 1kg = 1000g$

and the specific heat of U is $c = 0.0276 cal/g\bullet°C$.

$$\Delta T = \left(4.48\times10^{3} cal/y\right)\Big/\left(0.0276 cal/g\bullet°C\right)\left(1000g\right) = 162°C$$

(Problem 11-98 continued)



11-99. The reactions are:

(1) $^{1}H + {}^{1}H \rightarrow {}^{2}H + \beta^{-} + \bar{\nu}_e$

(2) $^{1}H + {}^{2}H \rightarrow {}^{3}He + \gamma$

followed by

(3) $^{3}He + {}^{3}He \rightarrow {}^{4}He + {}^{1}H + {}^{1}H$ or

(Problem 11-99 continued)

(4) $^{1}H + {}^{3}He \rightarrow {}^{4}He + \beta^{+} + \nu_e$

(a) From 2(1) + 2(2) + (3):

$$6\,^{1}H + 2\,^{2}H + 2\,^{3}He \rightarrow 2\,^{2}H + 2\,^{3}He + {}^{4}He + 2\,^{1}H + 2\beta^{+} + 2\nu_e + 2\gamma$$

Cancelling $2\,^{1}H, 2\,^{2}H,$ and $2\,^{3}He$ on both sides of the sum,

$$4\,^{1}H \rightarrow {}^{4}He + 2\beta^{+} + 2\nu_e + 2\gamma$$

From (1) + (2) + (4):

$$4\,^{1}H + {}^{2}H + {}^{3}He \rightarrow {}^{2}H + {}^{3}He + {}^{4}He + 2\beta^{+} + 2\nu_e + \gamma$$

Cancelling $^{2}H,$ and ^{3}He on both sides of the sum,

$$4\,^{1}H \rightarrow {}^{4}He + 2\beta^{+} + 2\nu_e + \gamma$$

(b) $Q = 4m\left({}^{1}H\right)c^2 - M\left({}^{4}He\right)c^2 - 2m_e c^2$

$$= 4\left(938.280MeV\right) - 3727.409MeV - 2\left(0.511MeV\right)$$

$$= 24.7MeV$$

(c) Total energy release is 24.7 *MeV* plus the annihilation energy of the two β^{+}:

energy release $= 24.7MeV + 2\left(m_e c^2\right)$

$$= 24.7MeV + 2\left(1.022MeV\right) = 26.7MeV$$

Each cycle uses 4 protons, thus produces $26.7MeV/4 = 6.68MeV/proton.$

Therefore, ^{1}H (protons) are consumed at the rate:

$$\frac{dN}{dt} = \frac{P}{E} = \frac{4\times10^{26}\,J/s}{6.68\times10^{6}\,eV}\left(\frac{1eV}{1.60\times10^{-19}\,J}\right) = 3.75\times10^{38}\,protons/s$$

The number N of ^{1}H nuclei in the Sun is:

$$N = \frac{M_{\odot}}{M\left({}^{1}H\right)} = \frac{1/2\times2\times10^{30}\,kg}{1.673\times10^{-27}\,kg} = 5.98\times10^{56}\,protons$$

which will last at the present consumption rate for

$$t = \frac{N}{dN/dt} = \frac{5.98\times10^{56}\,protons}{3.75\times10^{38}\,protons/s} = 1.60\times10^{18}\,s$$

$$= 1.60\times10^{18}\,s\left(\frac{1y}{3.16\times10^{7}\,s}\right) = 5.05\times10^{10}\,y$$

11-100. At this energy, neither particle is relativistic, so

$$E_{He} = \frac{p_{He}^2}{2m_{He}} \qquad E_n = \frac{p_n^2}{2m_n} \qquad p_{He} = p_n \qquad E_{He} + E_n = 17.7MeV$$

$$2m_{He}E_{He} = 2m_nE_n = 2m_n\left(17.7MeV - E_{He}\right)$$

$$\left(m_{He} + m_n\right)E_{He} = \left(17.7MeV\right)m_n \qquad \text{Therefore, } E_{He} = \frac{m_n}{m_{He} + m_n}17.7MeV$$

$$E_{He} = \frac{\left(1.008665u\right)\left(17.7MeV\right)}{2.002602u + 1.008665u} = 3.56MeV$$

$$E_n = 17.7MeV - E_{He} = \left(17.7 - 3.56\right)MeV = 14.1MeV$$

11-101. (a) The number N of generations is: $N = \dfrac{5s}{0.08s/gen} = 62.5$ generations

Percentage increase in energy production = $\dfrac{R(N) - R(0)}{R(0)} \times 100$

$$= \left[\frac{R(N)}{R(0)} - 1\right] \times 100 \quad \text{where } R(N)/R(0) = k^N \quad \text{(from Example 11-22 in More section)}$$

$$= \left(k^N - 1\right) \times 100 = \left(\left(1.005\right)^{62.5} - 1\right) \times 100 = 137\%$$

(b) Because $k \propto neutron\ flux$, the fractional change in flux necessary is equal to the fractional change in k:

$$\frac{k-1}{k} = \frac{1.005 - 1}{1.005} = 0.00498$$

11-102. (a) For 5% enrichment:

$$\sigma_f = \left(0.05\right)\sigma_f\left(^{235}U\right) + \left(0.95\right)\sigma_f\left(^{238}U\right)$$

$$= \left(0.05\right)\left(584b\right) + \left(0.95\right)\left(0\right) = 29.2b$$

$$\sigma_a = \left(0.05\right)\sigma_a\left(^{235}U\right) + \left(0.95\right)\sigma_a\left(^{238}U\right)$$

$$= \left(0.05\right)\left(97b\right) + \left(0.95\right)\left(2.75b\right) = 7.46b$$

(Problem 11-102 continued)

$$k = 2.4\frac{\sigma_f}{\sigma_f+\sigma_a} = \frac{2.4\ \ 29.2b}{29.2b+2.46b} = 1.91 \quad \text{(Equation 11-68 in More section)}$$

(b) For 95% enrichment:

$$\sigma_f = 0.95\ \ \sigma_f\ \ {}^{235}U\ + 0.05\ \ \sigma_f\ \ {}^{238}U$$

$$= 0.95\ \ 584b\ + 0.05\ \ 0\ = 554.8b$$

$$\sigma_a = 0.95\ \ \sigma_a\ \ {}^{235}U\ + 0.05\ \ \sigma_a\ \ {}^{238}U$$

$$= 0.95\ \ 97b\ + 0.05\ \ 2.75b\ = 92.3b$$

The reaction rate after N generations is $R\ \ N\ = R\ \ 0\ \ k^N$.

For the rate to double $R\ \ N\ = 2R\ \ 0$ and $2 = k^N \rightarrow N = \ln\ 2\ /\ln\ k$.

$N\ \ 5\%\ = \ln\ 2\ /\ln\ 1.91\ = 1.07$ generations

$N\ \ 95\%\ = \ln\ 2\ /\ln\ 2.06\ = 0.96$ generations

Assuming an average time per generation of 0.01s

$t\ \ 5\%\ = 1.07\times10^{-2}s$ $\quad$ $t\ \ 95\%\ = 0.96\times10^{-2}s$

Number of generations/second = 1/seconds/generation

In 1s: $N\ \ 5\%\ = 93.5$ and $N\ \ 95\%\ = 104$

One second after the first fission:

$$R\ \ 5\%\ = R\ \ 0\ \ k^N = \ 1\ \ \ 1.91\ ^{93.5} = 1.9\times10^{26}$$

Energy rate = $1.9\times10^{26}\ fissions/s\ \ \ 200MeV/fission$

$$= 3.8\times10^{28}MeV/s\ \ 1.60\times10^{-13}J/MeV$$

$$= 6.1\times10^{15}J/s = 6.1\times10^{15}W$$

$$R\ \ 95\%\ = R\ \ 0\ \ k^N = \ 1\ \ \ 2.06\ ^{104} = 4.4\times10^{32}$$

Energy rate = $4.4\times10^{32}\ fissions/s\ \ \ 200MeV/fission$

$$= 8.8\times10^{34}MeV/s\ \ 1.60\times10^{-13}J/MeV$$

$$= 1.4\times10^{22}J/s = 1.4\times10^{22}W$$

Chapter 12 – Particle Physics

12-1. (a) Because the two pions are initially at rest, the net momentum of the system is zero, both before and after annihilation. For the momentum of the system to be zero after the interaction, the momentum of the two photons must be equal in magnitude and opposite in direction, i.e., their momentum vectors must add to zero. Because the photon energy is $E = pc$, their energies are also equal.

(b) The energy of each photon equals the rest energy of a π^+ or a π^-.

$E = m_\pi c^2 = 139.6 MeV$ (from Table 12-3)

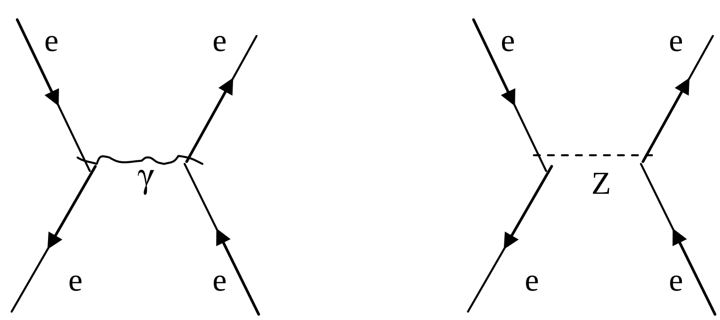
(c) $E = hf = hc/\lambda$ Thus, $\lambda = \frac{hc}{E} = \frac{1240 MeV \bullet fm}{139.6 MeV} = 8.88 fm$

12-2. (a) $E_\gamma = m_\Lambda c^2 + m_\pi c^2 = 2285 MeV + 139.6 MeV = 2424.6 MeV$

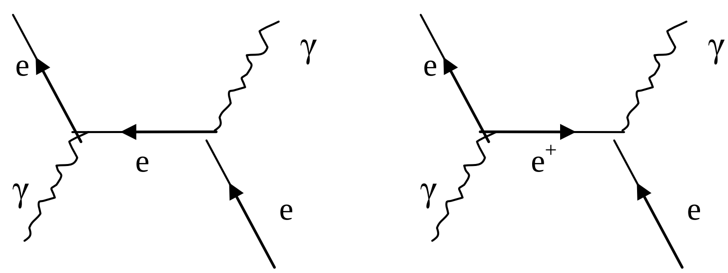
(b) $E_\gamma = 2m_p c^2 = 2(938.28 MeV) = 1876.56 MeV$

(c) $E_\gamma = 2m_\mu c^2 = 2(105.66 MeV) = 211.32 MeV$

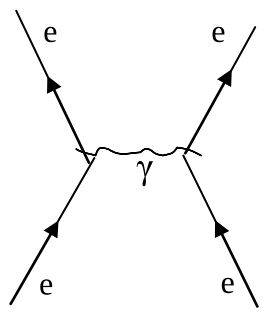
12-3. (a)



(b)

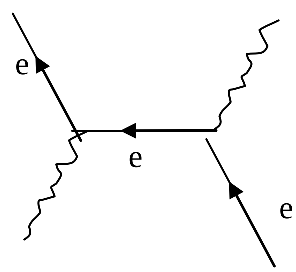


12-4. (a)



(b) See solution to Problem 12-3(a).

(c)



12-5. (a) ${}^{32}P \rightarrow {}^{32}S + e^-$ assuming no neutrino

$$Q = M\left({}^{32}P\right)c^2 - M\left({}^{32}S\right)c^2 \quad \text{(electron's mass is included in that of } {}^{32}S\text{)}$$

$$= 31.973908uc^2 - 31.972071uc^2$$

$$= \left(0.001837uc^2\right)\left(931.5MeV/uc^2\right) = 1.711MeV$$

To a good approximation, the electron has all of the kinetic energy

$$E_k \approx Q = 1.711MeV$$

(b) In the absence of a neutrino, the ${}^{32}S$ and the electron have equal and opposite momenta. The momentum of the electron is given by:

$$\left(pc\right)^2 = E^2 - \left(m_e c^2\right)^2 \quad \text{(Equation 2-32)}$$

$$= \left(E_k + m_e c^2\right)^2 - \left(m_e c^2\right)^2$$

$$= \left(Q + m_e c^2\right)^2 - \left(m_e c^2\right)^2 = Q^2 + 2Qm_e c^2$$

(Problem 12-5 continued)

The kinetic energy of the ^{32}S is then:

$$E_k = \frac{p^2}{2M} = \frac{(pc)^2}{2Mc^2} = \frac{Q^2 + 2Qm_e c^2}{2M\left(^{32}S\right)c^2}$$

$$= \frac{(1.711 MeV)^2 + 2(1.711 MeV)(0.511 MeV)}{2\left(31.972071 uc^2\right)\left(931.5 MeV/uc^2\right)}$$

$$= 7.85\times10^{-5} MeV = 78.5 eV$$

(c) As noted above, the momenta of the electron and ^{32}S are equal in magnitude and opposite direction.

$$(pc)^2 = Q^2 + 2Qm_e c^2 = (1.711 MeV)^2 + 2(1.711 MeV)(0.511 MeV)$$

$$p = \left[(1.711 MeV)^2 + 2(1.711 MeV)(0.511 MeV)\right]^{1/2} \Big/ c$$

$$= 2.16 MeV/c$$

12-6. (a) A single photon cannot conserver both energy and momentum.

(b) To conserver momentum each photon must have equal and opposite momenta so that the total momentum is zero. Thus, they have equal energies, each equal to the rest energy of a proton: $E_\gamma = m_p c^2 = 938.28 MeV$

(c) $E_\gamma = hv = hc/\lambda \quad \therefore \quad \lambda = \frac{hc}{E_\gamma} = \frac{1240 MeV\bullet fm}{938.28 MeV} = 1.32\, fm$

(d) $v = \frac{c}{\lambda} = \frac{3.00\times10^8 m/s}{1.32\times10^{-15} m} = 2.27\times10^{23} Hz$

12-7. (a) Conservation of charge: +1 +1 → +1 −1 +1 −1 = 0. Conservation of charge is violated, so the reaction is forbidden.

(b) Conservation of charge: +1 +1 → +1 −1 = 0. Conservation of charge is violated, so the reaction is forbidden.

12-8. (a) $\tau = 2/m\alpha^5 \rightarrow \tau = \left(2/m\alpha^5\right)\hbar^a c^b$

Since the units of τ must be seconds *s*, we have

$$s = kg^{-1} \times \left(\frac{N \bullet m^2}{C^2}\right)^{-5} \times \left(e^2\right)^{-5} \times \left(\frac{1}{\hbar c}\right)^{-5} \hbar^a c^b$$

having substituted the internal units of $\alpha = ke^2/\hbar c$. Re-writing the units, noting that

$J = N \bullet m$, gives: $s = \frac{1}{kg} \times \frac{C^{10}}{N^5 m^{10}} \times \frac{1}{C^{10}} \times \left(\frac{J^5 \bullet s^5 \bullet m^5}{s^5}\right) \times \hbar^a c^b$

Noting that $J = kg \bullet m^2/s^2$ and cancelling yields,

$$s = (kg)^{a-1} \times m^{2a+b} \times s^{-a-b}$$

Since $a-1$ must be 0, $a = 1$, and since $-a-b = 1$, $b = -2$.

$\therefore \quad \tau = 2\hbar/mc^2\alpha^5$

(b) $\tau = \dfrac{2\left(1.055 \times 10^{-34}\, J \bullet s\right)(137)^5}{\left(9.11 \times 10^{-31} kg\right)\left(3.00 \times 10^8\, m/s\right)^2} = 1.24 \times 10^{-10} s = 0.124 ns$

12-9. (a) Weak interaction

(b) Electromagnetic interaction

(c) Strong interaction

(d) Weak interaction

12-10. $\pi^0 \rightarrow \gamma + \gamma$ is caused by the electromagnetic interaction; $\pi^- \rightarrow \mu^- + \bar{v}_\mu$ is caused by the weak interaction. The electromagnetic interaction is the faster and stronger, so the π^0 will decay more quickly; the π^- will live longer.

12-11. (a) allowed; no conservation laws violated.

(b) allowed; no conservation laws violated.

(c) forbidden; doesn't conserve baryon number.

(d) forbidden; doesn't conserve muon lepton number.

12-12. (a) Electromagnetic interaction

(b) Weak interaction

(c) Electromagnetic interaction

(d) Weak interaction

(e) Strong interaction

(f) Weak interaction

12-13. For neutrino mass $m = 0$, travel time to Earth is $t = d/c$, where $d = 170{,}000c\bullet y$. For neutrinos with mass $m \neq 0$, $t' = d/v = d/\beta c$, where $\beta = v/c$.

$$\Delta t = t' - t = \frac{d}{c}\left(\frac{1}{\beta} - 1\right) = \frac{d}{c}\left(\frac{1-\beta}{\beta}\right)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \quad \text{(Equation 1-19)}$$

$$\gamma^2 = \frac{1}{1-\beta^2} = \frac{1}{(1-\beta)(1+\beta)}$$

$$1-\beta = \frac{1}{\gamma^2(1+\beta)} \approx \frac{1}{2\gamma^2} \quad \text{since } \beta \approx 1$$

Substituting into Δt,

$$\Delta t \approx \frac{d}{c}\left(\frac{1}{2\gamma^2}\right) \qquad E = \gamma mc^2 \rightarrow \gamma^2 = \left(E/mc^2\right)^2 \quad \text{(Equation 2-10)}$$

$$\Delta t \approx \frac{d}{2c}\left(\frac{mc^2}{E}\right)^2$$

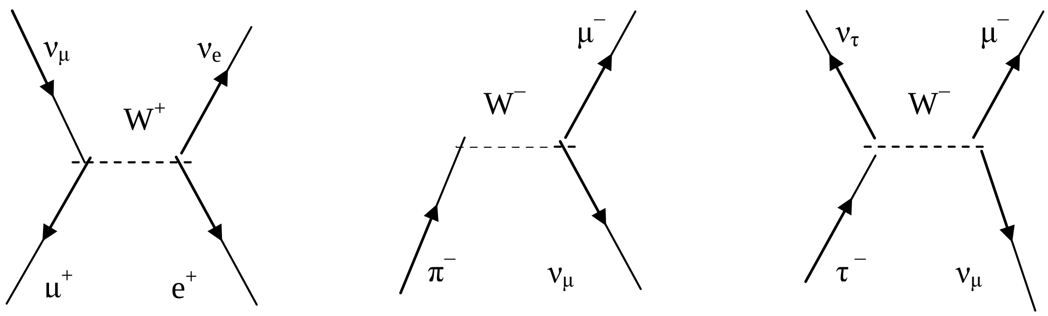
$$mc^2 = \left(\frac{(\Delta t)2cE^2}{d}\right)^{1/2} = \left(\frac{2\Delta t E^2}{d/c}\right)^{1/2}$$

$$= \left[\frac{2(12.5s)\left(10\times 10^6 eV\right)^2}{(170{,}000c\bullet y/c)\left(3.16\times 10^7 s/y\right)}\right]^{1/2} = 21.6eV$$

$$m \approx 22eV/c^2$$

12-14. The Σ^+ and Σ^- are members of a isospin multiplet, two charge states of the Σ hadron. Their mass difference is due to electromagnetic effects. The π^+ and π^- are a particle-antiparticle pair.

12-15. (a)



12-16. (See Table 12-8 in More Section.)

(a) 30 MeV

(b) 175 MeV

(c) 120 MeV

12-17. (a) $m_p c^2 < \left(m_n + m_e\right)c^2$ Conservation of energy and lepton number are violated.

(b) $m_n c^2 < \left(m_p + m_\pi\right)c^2$ Conservation of energy is violated.

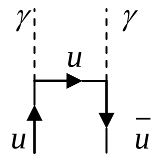
(c) Total momentum in the center of mass system is zero, so two photons (minimum) must be emitted. Conservation of linear momentum is violated.

(d) No conservation laws are violated. This reaction, $p\bar{p}$ annihilation, occurs.

(e) Lepton number before interaction is +1; that after interaction is −1. Conservation of lepton number is violated.

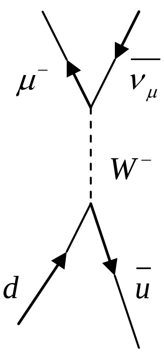
(f) Baryon number is +1 before the decay; after the decay the baryon number is zero. Conservation of baryon number is violated.

12-18. *(a)* The u and $\bar{u}$ annihilate via the EM interaction, creating photons.



(b) Two photons are necessary in order to conserve linear momentum.

(c)



12-19. (a) The strangeness of each of the particles is given in Table 12-6.

$\Delta S = +1$ The reaction can occur via the weak interaction.

(b) $\Delta S = -2$ This reaction is not allowed.

(c) $\Delta S = +1$ The reaction can occur via the weak interaction.

12-20. (a) The strangeness of each of the particles is given in Table 12-6.

$\Delta S = +2$ The reaction is not allowed.

(b) $\Delta S = +1$ This reaction can occur via the weak interaction.

(c) $\Delta S = 0$ The reaction can occur via either the strong, electromagnetic, or weak interaction.

12-21. (a) $n + n$ $\quad T_3 = -\frac{1}{2} - \frac{1}{2} = -1 \quad T = 1$

(b) $n + p$ $\quad T_3 = -\frac{1}{2} + \frac{1}{2} = 0 \quad T = 1$ or 0

(Problem 12-21 continued)

(c) $\pi^+ + p$ $\quad T_3 = 1 + \frac{1}{2} = \frac{3}{2} \quad T = \frac{3}{2}$

(d) $\pi^- + n$ $\quad T_3 = -1 - \frac{1}{2} = -\frac{3}{2} \quad T = \frac{3}{2}$

(e) $\pi^+ + n$ $\quad T_3 = 1 - \frac{1}{2} = \frac{1}{2} \quad T = \frac{1}{2}$ or $\frac{3}{2}$

12-22. (a) $\pi^- \rightarrow e^- + \gamma$ Electron lepton number changes from 0 to 1; violates conservation of electron lepton number.

(b) $\pi^0 \rightarrow e^- + e^+ + \nu_e + \bar{\nu}_e$ Allowed by conservation laws, but decay into two photons via electromagnetic interaction is more likely.

(c) $\pi^+ \rightarrow e^- + e^+ + \mu^+ + \nu_\mu$ Allowed by conservation laws but decay without the electrons is more likely.

(d) $\Lambda^0 \rightarrow \pi^+ + \pi^-$ Baryon number changes from 1 to 0; violates conservation of baryon number. Also violates conservation of angular momentum, which changes from 1/2 to 0.

(e) $n \rightarrow p + e^- + \bar{\nu}_e$ Allowed by conservation laws. This is the way the neutron decays.

12-23. (a) $\Omega^- \rightarrow \Lambda^0 + \text{K}^- \qquad \Omega^- \rightarrow \Xi^0 + \pi^-$

(b) $\Sigma^+ \rightarrow p + \pi^0 \qquad \Sigma^+ \rightarrow n + \pi^+$

(c) $\Lambda^0 \rightarrow p + \pi^- \qquad \Lambda^0 \rightarrow n + \pi^0$

(d) $\pi^0 \rightarrow \gamma + \gamma \qquad \pi^0 \rightarrow e^- + e^+ + e^- + e^+$

(e) $\text{K}^+ \rightarrow \mu^+ + \nu_\mu \qquad \text{K}^+ \rightarrow \pi^+ + \pi^0$

12-24. $\text{K}^- + p \rightarrow \text{K}^0 + \text{K}^+ + \Omega^-$

Because Ks have B = 0 and p has B = 1, conservation of B requires the Ω^- to have B = 1.

$\Omega^- \rightarrow \Xi^0 + \pi^-$

The π^- has B = 0, so conservation of B requires that the Ξ^0 have B = 1.

12-25. (a) $0+0 \rightarrow 0+0$ S is conserved.

(b) $-2 \rightarrow 0-1$ S is not conserved.

(c) $-1=-1+0$ S is conserved.

(d) $0+0 \rightarrow 0-1$ S is not conserved.

(e) $-3 \rightarrow -2+0$ S is not conserved.

12-26. Listed below are the baryon number, electric charge, strangeness, and hadron identity of the various quark combinations from Table 12-8 and Figure 12-21.

	Quark Structure	Baryon Number	Electric Charge (e)	Strangeness	Hadron
(a)	uud	+1	+1	0	p
(b)	udd	+1	0	0	n
(c)	uuu	+1	+2	0	Δ^{++}
(d)	uss	+1	0	−2	Ξ^0
(e)	dss	+1	−1	−2	Ξ^-
(f)	suu	+1	+1	−1	Σ^+
(g)	sdd	+1	−1	−1	Σ^-

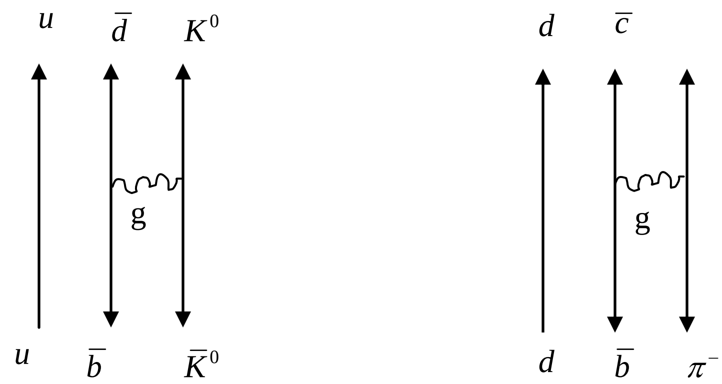
Note that 3-quark combinations are baryons.

12-27. Listed be below are the baryon number, electric charge, strangeness, and hadron identity of the various quark combinations from Table 12-9 and Figure 12-21.

	Quark Structure	Baryon Number	Electric Charge (e)	Strangeness	Hadron
(a)	$u\bar{d}$	0	+1	0	π^+
(b)	$\bar{u}d$	0	−1	0	π^-
(c)	$u\bar{s}$	0	+2	+1	K^+
(d)	$s\bar{s}$	0	0	+1	0
(e)	$\bar{d}s$	0	0	−1	K^{*0}

* forms η and η' along with $u\bar{u}$ and $d\bar{d}$

12-28.



12-29. (a) $T_3 = 0$ (from Figure 12-20a)

(b) $T = 1$ or 0 just as for ordinary spin.

(c) uds $B = 1/3 + 1/3 + 1/3 = 1$ $C = 2/3 - 1/3 - 1/3 = 0$

$S = 0 + 0 + -1 = -1$ The $T = 1$ state is the Σ^0. The $T = 0$ state is the Λ^0.

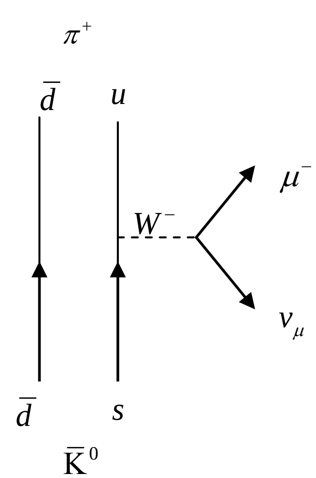
12-30. The +2 charge can result from either a *uuu*, *ccc*, or *ttt* quark configuration. Of these, only the *uuu* structure also has zero strangeness, charm, topness, and bottomness. (From Table 12-5.)

12-31. The range R is $R = \hbar c / mc^2$ (Equation 11-50). Substituting the mass of W^+ (from Table 12-4),

$$R = \frac{\left(1.055\times10^{-34} J\bullet s\right)\left(3.00\times10^{8} m/s\right)}{\left(81 GeV/c^2\right)\left(1.60\times10^{-10} J/GeV\right)} = 2.44\times10^{-18} m = 2.44\times10^{-3}\ fm$$

12-32.

$$\bar{\mathrm{K}}^0 \rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu$$
$$(\bar{d}s) \quad (\bar{d}u)$$

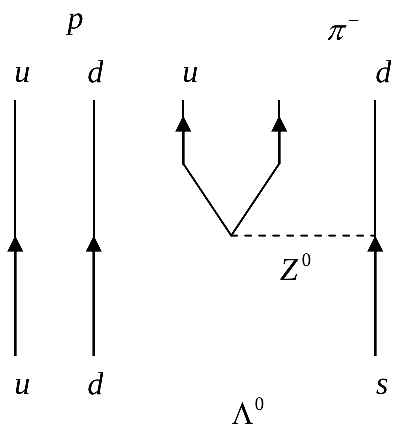


12-33.

$\Lambda^0 \rightarrow p + \pi^-$

$(uds) \quad (uud) \quad (\bar{u}d)$

Weak decay



12-34. $n \rightarrow p + \pi^-$

$$Q = m_n c^2 - m_p c^2 - m_\pi c^2$$

$$= (939.6 - 938.3 - 139.6)MeV$$

$$= -138.3MeV$$

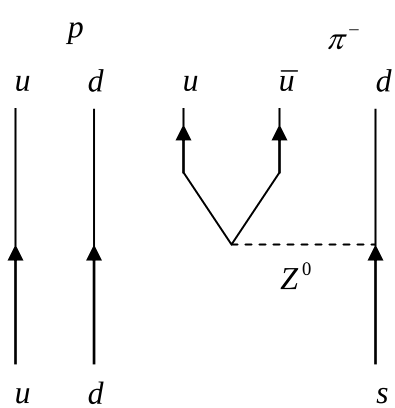
This decay does not conserve energy.

12-35.

$\Lambda^0 \rightarrow p + \pi^-$

$(uds) \quad (uud) \quad (\bar{u}d)$

Strong decay



12-36. (a) B = 1, S = −1, C = 0, $\tilde{B} = 0$

(b) Quark content is: *uds*

12-37. (a) The K^+ has charge +1, B = 0, and S = +1 from Table 12-6. It is a meson (quark-antiquark) structure. $u\bar{s}$ produces the correct set of quantum numbers. (From Table 12-5.)

(Problem 12-37 continued)

(b) The K^0 has charge 0, B = 0, and S = +1 from Table 12-6. The quark-antiquark structure tp produce these quantum numbers is $d\,\bar{s}$. (From Table 12-5.)

12-38. (a) Being a meson, the D^+ is constructed of a quark-antiquark pair. The only combination with *charge* = +1, *charm* = +1 and *strangeness* = 0 is the $c\bar{d}$. (See Table 12-5.)

(b) The D^-, antiparticle of the D^+, has the quark structure $\bar{c}\,d$.

12-39. The Σ^0 decays via the electromagnetic interaction whose characteristic time is $\sim 10^{-20}\,s$. The Σ^+ and Σ^- both decay via the weak interaction. The difference between these two being due to their slightly different masses.

12-40. If the proton is unstable, it must decay to less massive particles, i.e., leptons. But leptons have B = 0, so $p \rightarrow e^+ + v_e$ would have 1 = 0 + 0 = 0 and B is not conserved. The lepton numbers would not be conserved either; a "leptoquark" number would be conserved.

12-41. $V(H_2O) = 0.75\Delta V = 0.75\left(4\pi R^2 \Delta R\right)$, where *R*(Earth) = $6.37\times10^6\,m$ and $\Delta R = 1km = 10^3\,m.$

$$V(H_2O) = 0.75\left[4\pi\left(6.37\times10^6\,m\right)^2\left(10^3\right)\right] = 3.82\times10^{17}\,m^3$$

$$M(H_2O) = V(H_2O)\rho = \left(3.82\times10^{17}\,m^3\right)\left(1000kg/m^3\right) = 3.82\times10^{20}\,kg$$

Number of moles (H_2O) = $3.82\times10^{23}\,g/18.02g/mole = 2.12\times10^{22}\,moles$

Number of H_2O molecules = $N_A \times (\text{\# of moles})$

$$= \left(6.02\times10^{23}\,molecules/mole\right)\left(2.12\times10^{22}\,moles\right)$$

$$= 1.28\times10^{46}\,molecules\ \mathrm{H_2O}$$

Each molecule contains 10 protons (i.e., 2 in H atoms and 8 in the oxygen atom), so the number of protons in the world's oceans is $N = 1.28\times10^{47}$.

(Problem 12-41 continued)

The decay rate is $\left|\frac{dN}{dt}\right| = \lambda N$ where $\lambda = 1/\tau = 1/10^{32}\,y = 10^{-32}\,y^{-1}$

$$= \left(10^{-32}\,y^{-1}\right)\left(1.28\times 10^{47}\ protons\right)$$

$$= 1.28\times 10^{15}\ proton\ \ decays/y \approx 4\times 10^{7}\,decays/s$$

12-42. (a) $p \rightarrow e^{+} + \Lambda^{0} + \nu_e$

$$Q = \left(m_p c^2 - M\left(\Lambda^0\right)c^2 - m_e c^2\right)MeV$$

$$= \left(938.3 - 1116 - 0.511\right)MeV = -178MeV$$

Energy is not conserved.

(b) $p \rightarrow \pi^{+} + \gamma$

Spin (angular momentum) $\frac{1}{2} \rightarrow 0 + 1 = 1.$ Angular momentum is not conserved.

(c) $p \rightarrow \pi^{+} + \mathrm{K}^{0}$

Spin (angular momentum) $\frac{1}{2} \rightarrow 0 + 0 = 0.$ Angular momentum is not conserved.

12-43. n, B = 1, Q = 0, spin = 1/2, S = 0

Quark strucure	u	d	d	
B	1/3	+1/3	+1/3	= 1
Q	2/3	−1/3	−1/3	= 0
spin	1/2↑	1/2↑	1/2↓	= 1/2
S	0	+0	+0	= 0

$\bar{n}$, B = −1, Q = 0, spin = 1/2, S = 0

Quark strucure	$\bar{u}$	$\bar{d}$	$\bar{d}$	
B	−1/3	−1/3	−1/3	= −1
Q	−2/3	+1/3	+1/3	= 0
spin	1/2↑	1/2↑	1/2↓	= 1/2
S	0	+0	+0	= 0

(Problem 12-43 continued)

(b) Ξ^0, B = 1, Q = 0, spin = 1/2, S = −2

Quark strucure	u	s	s	
B	1/3	+1/3	+1/3	= 1
Q	2/3	−1/3	−1/3	= 0
spin	1/2↑	1/2↓	1/2↑	= 1/2
S	0	−1	−1	= −2

(c) Σ^+, B = 1, Q = 1, spin = 1/2, S = −1

Quark strucure	u	u	s	
B	1/3	+1/3	+1/3	= 1
Q	2/3	2/3	−1/3	= 0
spin	1/2↑	1/2↓	1/2↑	= 1/2
S	0	+0	−1	= −1

(d) Ω^-, B = 1, Q = −1, spin = 3/2, S = −3

Quark strucure	s	s	s	
B	1/3	+1/3	+1/3	= 1
Q	−1/3	−1/3	−1/3	= −1
spin	1/2↑	1/2↑	1/2↑	= 3/2
S	−1	−1	−1	= −3

(e) Ξ^-, B = 1, Q = −1, spin = 1/2, S = −2

Quark strucure	u	d	d	
B	1/3	+1/3	+1/3	= 1
Q	−1/3	−1/3	−1/3	= −1
spin	1/2↑	1/2↓	1/2↑	= 1/2
S	0	−1	−1	= −2

12-44. (a)

Quark strucure	d	d	d	
B	1/3	+1/3	+1/3	= 1
Q	−1/3	−1/3	−1/3	= −3/2
spin	1/2	1/2	1/2	= 3/2, 1/2
S	0	+0	+0	= 0

(b)

Quark strucure	u	$\overline{c}$	
B	1/3	−1/3	= 0
Q	2/3	−2/3	= 0
spin	1/2	1/2	= 1, 0
S	0	+0	= 0

(c)

Quark strucure	u	$\overline{b}$	
B	1/3	−1/3	= 0
Q	2/3	+1/3	= 1
spin	1/2	1/2	= 1, 0
S	0	+0	= 0

(d)

Quark strucure	$\overline{s}$	$\overline{s}$	$\overline{s}$	
B	−1/3	−1/3	−1/3	= −1
Q	1/3	+1/3	+1/3	= 1
spin	1/2	1/2	1/2	= 3/2, 1/2
S	1	+1	+1	= 3

12-45. The Z^0 has spin 1. Two identical spin 0 particles cannot have total spin 1.

12-46. (a) The final products (p, γ, e^-, neutrinos) are all stable.

(b) $\Xi^0 \rightarrow p + e^- + \overline{\nu}_e + \overline{\nu}_\mu + \nu_\mu$

(c) Conservation of charge: $0 \rightarrow +1 - 1 + 0 + 0 + 0 = 0$

Conservation of baryon number: $1 \rightarrow 1 + 0 + 0 + 0 + 0 = 1$

(Problem 12-46 continued)

Conservation of lepton number:

(i) for electrons: $0 \rightarrow 0+1-1+0+0=0$

(ii) for muons: $0 \rightarrow 0+0+0-1+1=0$

Conservation of strangeness: $-2 \rightarrow 0+0+0+0+0=0$

Even though the chain has $\Delta S=+2,$ no individual reaction in the chain exceeds $\Delta S=+1,$ so they can proceed via the weak interaction.

(d) No, because energy is not conserved.

12-47. $(2,1,0,1,0) \rightarrow c\,u\,u$

$(0,1,-2,1,0) \rightarrow c\,s\,s$

$(0,0,1,0,-1) \rightarrow b\,\bar{s}$

$(0,-1,1,0,0) \rightarrow \bar{s}\,\bar{d}\,\bar{u}$

$(0,1,-1,1,0) \rightarrow c\,s\,d$

$(-1,1,-3,0,0) \rightarrow s\,s\,s$

12-48. (a) $t_1 = x/u_1 \quad t_2 = x/u_2 \Rightarrow \Delta t = t_2 - t_1 = \dfrac{x}{u_2} - \dfrac{x}{u_1}$

$$\Delta t = x\left(\frac{u_1-u_2}{u_1u_2}\right) \approx \frac{x\,\Delta u}{c^2}$$

(b) $E = \dfrac{mc^2}{\sqrt{1-u^2/c^2}}$ (Equation 2-10)

$$E^2 = \frac{(mc^2)^2}{1-u^2/c^2} \Rightarrow 1-u^2/c^2 = \frac{(mc^2)^2}{E^2} \Rightarrow \frac{u}{c} = \left[1-\left(\frac{mc^2}{E}\right)^2\right]^{1/2}$$

Expanding the right side of the equation in powers of $(mc^2/E)^2$ and keeping only the first term yields

(Problem 12-48 continued)

$$\frac{u}{c} \approx 1 - \frac{1}{2}\left(\frac{mc^2}{E}\right)^2$$

(c) $$u_1 - u_2 = c\left[1 - \frac{1}{2}\left(\frac{mc^2}{E_1}\right)^2 - 1 + \frac{1}{2}\left(\frac{mc^2}{E_2}\right)^2\right]$$

$$= \frac{c(mc^2)^2}{2}\left(\frac{1}{E_2^2} - \frac{1}{E_1^2}\right) = \frac{c(mc^2)^2}{2}\left(\frac{E_1^2 - E_2^2}{E_1^2 E_2^2}\right)$$

$$\Delta u = u_1 - u_2 = \frac{c(2.2\,\text{eV}/c^2 \times c^2)^2}{2}\left[\frac{(20\,\text{MeV})^2 - (5\,\text{MeV})^2}{(20\,\text{MeV})^2(5\,\text{MeV})^2}\right]$$

$$= \frac{c(2.2\,\text{eV})^2 \times (10^{-6}\,\text{MeV/eV})^2}{2}\left[\frac{375}{(20)^2(5)^2(\text{MeV})^2}\right]$$

$$= c\,\frac{(2.2)^2(375)\times 10^{-12}}{2(20)^2(5)^2} = 2.7\times 10^{-5}\ \text{m/s}$$

And therefore, $$\Delta t = \frac{1.7\times 10^5\, c\cdot\text{y}\times 9.46\times 10^{15}\,\text{m}/c\cdot\text{y}\times 2.7\times 10^{-5}\,\text{m/s}}{(3.0\times 10^8\,\text{m/s})^2} = 0.48\,\text{s}$$

12-49. (a) $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$

Electron lepton number: $0 = -1 + 1 + 0 = 0$

Muon lepton number: $-1 = 0 + 0 - 1 = -1$

Tau lepton number: $0 = 0 + 0 + 0 = 0$

(b) $\tau^- \to \mu^- + \bar{\nu}_\mu + \nu_\tau$

Electron lepton number: $0 = 0 + 0 + 0 = 0$

Muon lepton number: $0 = 1 - 1 + 0 = 0$

Tau lepton number: $1 = 0 + 0 + 1 = 1$

(c) $\pi^- = \mu^- + \bar{\nu}_\mu$

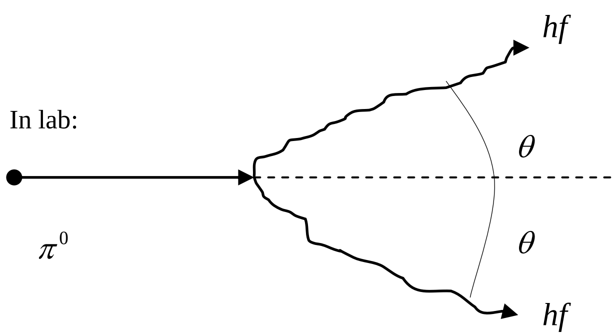
Electron lepton number: $0 = 0 + 0 = 0$

Muon lepton number: $0 = 1 - 1 = 0$

Tau lepton number: $0 = 0 + 0 = 0$

12-50. $\pi^0 \rightarrow \gamma + \gamma$

$$E^2 = (pc)^2 + (m_\pi c^2)^2$$



Conservation of momentum requires that each carry half of the initial momentum, hence the total energy: $2(hf/c)\cos\theta = p$ $\qquad hf = E/2 = \left[(pc)^2 + (m_\pi c^2)^2\right]^{1/2} \big/ 2$

$$\cos\theta = \frac{p}{2(hf/c)} = \frac{p}{2\left[(pc)^2 + (m_\pi c^2)^2\right]^{1/2} \big/ 2c}$$

$$= \frac{pc}{\left[(pc)^2 + (m_\pi c^2)^2\right]^{1/2}} = \frac{850MeV}{\left[(850MeV)^2 + (135MeV)^2\right]^{1/2}} = 0.9876$$

$$\theta = \cos^{-1}(0.9876) = 9.02°$$

12-51. (a) $\Lambda^0 \rightarrow p + \pi^-$

Energy: $1116MeV - (938 + 140)MeV = 38MeV$ conserved.

Electric charge: $0 \rightarrow +1 - 1 = 0$ conserved.

Baryon number: $1 \rightarrow 1 + 0 = 1$ conserved.

Lepton number: $0 \rightarrow 0 + 0 = 0$ conserved.

(b) $\Sigma^- \rightarrow n + p^-$

Energy: $1197MeV - (940 + 938)MeV = -681MeV$ not conserved.

Electric charge: $-1 \rightarrow 0 - 1 = -1$ conserved.

Baryon number: $1 \rightarrow 1 - 1 = 0$ not conserved.

Lepton number: $0 \rightarrow 0 + 0 = 0$ conserved.

This reaction is not allowed (energy and baryon conservation violated).

(Problem 12-51 continued)

(c) $\mu^- \to e^- + \overline{v}_e + v_\mu$

Energy: $105.6MeV - 0.511MeV = 105.1MeV$ conserved.

Electric charge: $-1 \to -1 + 0 + 0 = -1$ conserved.

Baryon number: $0 \to 0 + 0 + 0 = 0$ conserved.

Lepton number:

(i) electrons: $0 \to 1 - 1 + 0 = 0$ conserved.

(ii) muons: $1 \to 0 + 0 + 1 = 1$ conserved.

12-52. (a) The decay products in the chain are not all stable. For example, the neutron decays via $n \to p + e^- + \overline{v}_e$ Only the e^+ and e^- are stable.

(b) The net effect of the chain reaction is: $\Omega^- \to p + 3e^- + e^+ + 3\overline{v}_e + 2\overline{v}_\mu + 2v_\mu$

(c) Charge: $-1 \to +1 - 3 + 1 = -1$ conserved

Baryon number: $1 \to 1 + 0 + 0 + 0 + 0 + 0 + 0 = 1$ conserved.

Lepton number:

(i) electrons: $0 \to 0 + 3 - 1 - 3 + 1 + 0 + 0 = 0$ conserved

(ii) muons: $0 \to 0 + 0 + 0 + 0 + 0 - 2 + 2 = 0$ conserved

Strangeness: $-3 \to 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0$ not conserved

Overall reaction has $\Delta S = +3$; however, none of the individual reactions exceeds $\Delta S = +1$, so they can proceed via the weak interaction.

12-53. The proton and electron are free particles. The quarks are confined, however, and cannot be separated. The gluon clouds give the *u* and *d* effective masses of about 330 MeV/c^2, about 1/3 of the proton's mass.

12-54. (a) $\Lambda^0 \to p + \pi^-$

$$E_{kin} = \left[M\left(\Lambda^0\right) - m_p - m_\pi\right]c^2$$

$$= \left[1116MeV/c^2 - 938.3MeV/c^2 - 139.6MeV/c^2\right]c^2$$

$$= 38.1MeV$$

(Problem 12-54 continued)

(b) Because the Λ^0 decayed at rest, the *p* and π^- have momenta of equal magnitudes and opposite direction.

$$m_p v_p = m_\pi v_\pi \rightarrow m_p / m_\pi = v_\pi / v_p$$

$$\frac{E_{kin}(\pi)}{E_{kin}(p)} = \frac{\frac{1}{2} m_\pi v_\pi^2}{\frac{1}{2} m_p v_p^2} = \frac{m_\pi}{m_p}\left(\frac{m_p}{m_\pi}\right)^2 = \frac{m_p}{m_\pi} = \frac{938.3}{139.6} = 6.72$$

(c) $E_{kin} = E_{kin}(p) + E_{kin}(\pi) = E_{kin}(p) + 6.72E_{kin}(p) = 7.72E_{kin}(p) = 38.1MeV$

$$E_{kin}(p) = 38.1MeV / 7.71MeV = 4.94MeV$$

$$E_{kin}(\pi) = 6.72E_{kin}(p) = 33.2MeV$$

12-55. $\Sigma^0 \rightarrow \Lambda^0 + \gamma$

(a) E_T for decay products is the rest energy of the Σ^0, 1193*MeV*.

(b) The rest energy of $\Lambda^0 = 116MeV$, so $E_\gamma = 1193MeV - 1116MeV = 77MeV$ and $p_\gamma = E_\gamma / c = 77MeV / c$.

(c) The Σ^0 decays at rest, so the momentum of the Λ^0 equals in magnitude that of the photon.

$$E_{kin}(\Lambda^0) = p_\Lambda^2 / 2M(\Lambda) = (77MeV/c)^2 / \left[2\left(1116MeV/c^2\right)\right]$$

$$= 2.66MeV \quad \text{small compared to } E_\gamma$$

(d) A better estimate of E_γ and p_γ are then $E_\gamma = 77MeV - 2.66MeV = 74.3MeV$ and $p_\gamma = 74.3MeV / c.$

12-56. (a) $\Delta t = t_2 - t_1 = \dfrac{x}{u_2} - \dfrac{x}{u_1} = \dfrac{x(u_1 - u_2)}{u_1 u_2}$ Note that $u_1 u_2 \approx c^2$

$$\Delta t \approx \frac{x(u_1 - u_2)}{c^2} = \frac{x\Delta u}{c^2}$$

(Problem 12-56 continued)

(b) $E = \dfrac{mc^2}{\sqrt{1-u^2/c^2}}$ (Equation 2-10). Thus, $\dfrac{u}{c} = \left[\dfrac{1-\left(m_o c^2\right)^2}{E^2}\right]^{1/2} \approx 1-\dfrac{1}{2}\left(\dfrac{m_o c^2}{E}\right)^2$

(c) $u_1 - u_2 = c\left[1-\dfrac{1}{2}\left(\dfrac{m_o c^2}{E_1}\right)^2 - 1 + \dfrac{1}{2}\left(\dfrac{m_o c^2}{E}\right)^2\right]$

$$= \frac{c}{2}\left(\frac{m_o c^2}{E_2}\right)^2 - \frac{c}{2}\left(\frac{m_o c^2}{E_1}\right)^2 = \frac{c\left(m_o c^2\right)^2}{2}\left[\frac{E_1^2 - E_2^2}{E_1^2 E_2^2}\right]$$

$$= \frac{c\left(20eV\right)^2}{2}\left[\frac{\left(20\times 10^6 eV\right)^2 - \left(5\times 10^6 eV\right)^2}{\left(20\times 10^6 eV\right)^2\left(5\times 10^6 eV\right)^2}\right]$$

$$= \frac{c\left(20eV\right)^2}{2}\left[\frac{\left(20\right)^2 - \left(5\right)^2}{\left(20\right)^2\left(5\right)^2\left(10^6 eV\right)^2}\right] = 7.5\times 10^{-12} c$$

$$\Delta T \approx \frac{x\Delta u}{c^2}\frac{\left(170{,}000 c\bullet y\right)\left(7.5\times 10^{-12} C\right)}{c^2} = 1.28\times 10^{-6} y = 40.3s$$

(d) If the neutrino rest energy is $40eV$, then $\Delta u = 3.00\times 10^{-11} c$ and $\Delta t \approx 161s$. The difference in arrival times can thus be used to set an upper limit on the neutrino's mass.

12-57. $\tau^- \rightarrow e^- + \overline{\nu}_e + \nu_\tau$

$\tau^- \rightarrow \mu^- + \overline{\nu}_\mu + \nu_\tau$

$\tau^- \rightarrow d + \overline{u} + \nu_\tau$

The last decay is the most probable (three times as likely compared to each of the others) due to the three possible quark colors.

12-58. $(i)\ \dfrac{d\vec{p}}{dt} = e\vec{v} \times \vec{B}$

$(ii)\ \dfrac{dE}{dt} = 0$ which follows from the fact that the Lorentz force is $\perp \vec{v}$; therefore, $v = |\vec{v}| =$ constant and thus $\gamma(v) =$ constant.

Equation (i) then becomes: $\dfrac{d\vec{p}}{dt} = m\gamma \dfrac{d\vec{v}}{dt} = e\vec{v} \times \vec{B}$

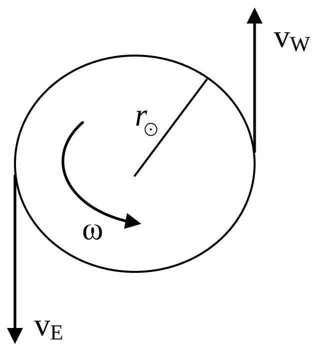
For circular orbits $\dfrac{m\gamma v^2}{R} = evB$ or, re-writing a bit,

$m\gamma v = p = eBR$ and $pc = ceBR \times \dfrac{1GeV}{1.60 \times 10^{-19} J} = 0.35BR\ GeV$

and finally, $p = 0.35BR\ GeV / c$

Chapter 13 – Astrophysics and Cosmology

13-1.



$|v_W - v_E| = 4km/s.$ Assuming Sun's rotation to be uniform, so that $v_W = -v_E$, then $|v_W| = |v_E| = 2km/s.$ Because $v = 2\pi/T$, $v_E = 2\pi r_\odot/T$ or

$$T = \frac{2\pi r_\odot}{v_E} = \frac{2\pi\left(6.96\times10^5 km\right)}{2km/s} = 2.19\times10^6 s = 25.3 days$$

13-2. $$|U| = \frac{2GM_\odot^2}{R_\odot} = \frac{2\left(6.67\times10^{-11}\right)\left(1.99\times10^{30}\right)^2}{6.96\times10^8} J = 7.59\times10^{41} J$$

The Sun's luminosity $L_\odot = 3.85\times10^{26} W$

$$\therefore\ t_L = \frac{|U|}{L_\odot} = \frac{7.59\times10^{41} J}{3.85\times10^{26} J/s} = 1.97\times10^{15} s = 6.26\times10^7 \, years$$

13-3. The fusion of ^1H to ^4He proceeds via the proton-proton cycle. The binding energy of ^4He is so high that the binding energy of two ^4He nuclei excees that of ^8Be produced in the fusion reaction: $^4He + {}^4He \rightarrow {}^8Be + \gamma$ and the ^8Be nucleus fissions quickly to two ^4He nuclei via an electromagnetic decay. However, at high pressures and temperatures a very small amount is always present, enough for the fusion reaction: $^8Be + {}^4He \rightarrow {}^{12}C + \gamma$ to proceed. This 3-^4He fusion to ^{12}C produces no net ^8Be and bypasses both *Li* and *B*, so their concentration in the cosmos is low.

13-4. The Sun is $28{,}000 c\bullet y$ from Galactic center = radius of orbit

$$\therefore \text{ time for 1 orbit} = \frac{2\pi r}{v} = \frac{2\pi\left(28{,}000 c\bullet y \times 9.45\times10^{15} m / c\bullet y\right)}{2.5\times10^5 m/s}$$

$$= 6.65\times10^{15} s = 2.11\times10^8 yr$$

13-5. Observed mass (average) $\approx 1\ H\ atom/m^3 = 1.67\times10^{-27}\ kg/m^3 = 10\%$ of total mass (a)

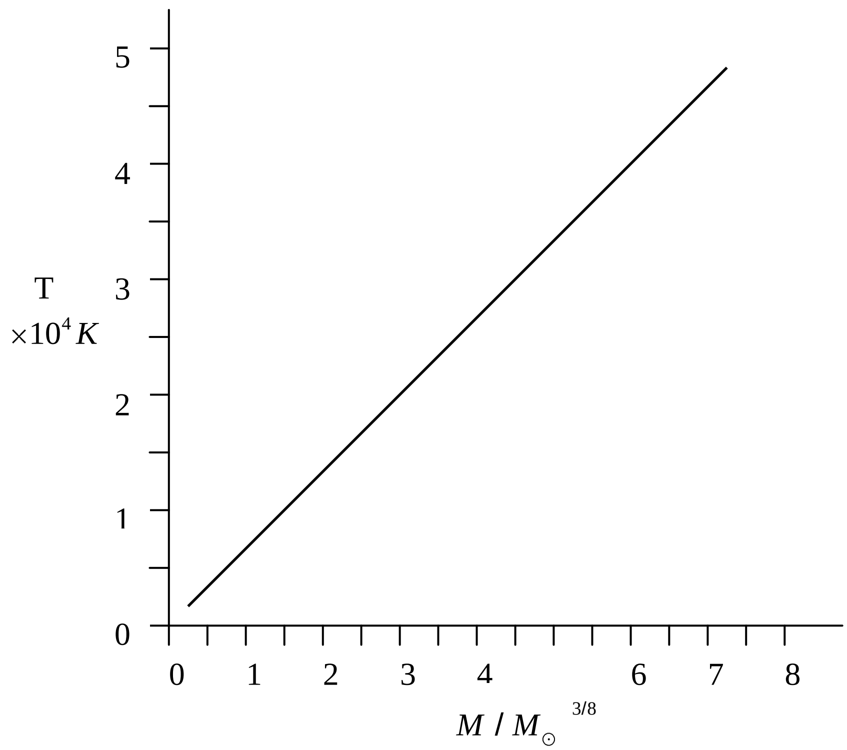
$\therefore$ missing mass $= 9\times1.67\times10^{-27}\ kg/m^3 = 1.56\times10^{-26}\ kg/m^3$

$500\ photons/cm^3 = 500\times10^6\ photons/m^3$, so the mass of each photon would be

$$= \frac{1.50\times10^{-26}\ kg/m^3}{500\times10^6\ photons/m^3} = 3.01\times10^{-35}\ kg$$

or $$m_\nu = \frac{3.01\times10^{-35}\ kg}{1.60\times10^{-19}\ J/eV}\times c^2\left(\frac{m^2}{s^2}\right) = 16.9\ eV/c^2$$

13-6.



13-7. $1c\bullet s = c\times1s = 3.00\times10^8\ m/s\times1s = 3.00\times10^8\ m = 3.00\times10^5\ km$

$1c\bullet\min = c\times1\min\times60s/\min = 3.00\times10^5\ km\times60s = 1.80\times10^7\ km$

$1c\bullet h = c\times1h\times3600s/h = 1.08\times10^9\ km$

$1c\bullet day = c\times24h\times3600s/h = 2.59\times10^{10}\ km$

13-8. (a) See Figure 13-16. $1AU = 1.496\times10^{11}\ m$. $R = 1pc$ when $\theta = 1''$, so $R = \dfrac{1AU}{1''}$

(Problem 13-8 continued)

or $R = \frac{1AU}{1"} \times \frac{3600"}{1°} \times \frac{180°}{\pi rad} = 3.086 \times 10^{16} m = 1pc$

$$1pc = \frac{3.086 \times 10^{16} m}{9.45 \times 10^{15} m / c \bullet y} = 3.26 c \bullet y$$

(b) When $\theta = 0.01"$, $R = 100 pc$ and the volume of a sphere with that radius is

$V = \frac{4}{3}\pi R^3 = 4.19 \times 10^6 pc^3$. If the density of stars is $0.08 / pc^3$, then the number of stars in the sphere is equal to $0.08 / pc^3 \times 4.19 \times 10^6 pc^3 = 3.4 \times 10^5$ stars.

13-9. $L = 4\pi r^2 f$ $\qquad m_1 - m_2 = 2.5 \log \; f_1 / f_2$

Thus, $L_p = 4\pi r_p^2 f_p$ and $L_B = 4\pi r_B^2 f_B$ and $L_p = L_B$

$\therefore \; r_p^2 f_p = r_B^2 f_B \rightarrow r_B^2 = r_p^2 \;\; f_p / f_B$

$\log \;\; f_p / f_B \;\; = \frac{1.16 - 0.41}{2.5} = 0.30 \rightarrow f_p / f_B = 2.00$

Because $r_p = 12 pc$, $\quad r_B = r_p \;\; f_p / f_B \;^{1/2} = 12\sqrt{2} = 17.0 pc$

13-10. (a) $M = 0.3 M_\odot$ $\qquad T_e = 3300K$ $\qquad L = 5 \times 10^{-2} L_\odot = 1.93 \times 10^{25} W$

(b) $M = 3.0 M_\odot$ $\qquad T_e = 13,500K$ $\qquad L = 10^2 L_\odot = 3.85 \times 10^{28} W$

(c) $R \sim M \rightarrow R = \alpha M \rightarrow \alpha = R_\odot / M_\odot$

$$R_{0.3} = \alpha \;\; 0.3 M_\odot \;\; = \frac{R_\odot}{M_\odot} \;\; 0.3 M_\odot \;\; = 0.3 R_\odot = 2.09 \times 10^8 m$$

Similarly, $R_{3.0} = 3.0 R_\odot = 2.09 \times 10^9 m$

$t_L \sim M^{-3} \rightarrow t_L = \beta M^{-3} \rightarrow t_{L_\odot} / M_\odot^{-3} = t_{L_\odot} M_\odot^3$

(d) $t_L \;\; 0.3 \;\; = \beta \;\; 0.3 M_\odot \;\; ^{-3} = t_{L_\odot} M_\odot^3 \;\; 0.3 M_\odot \;\; ^{-3} = \;\; 0.3 \;\; ^{-3} t_{L_\odot}$

or $t_L \;\; 0.3 \;\; = 37 t_{L_\odot}$. $\quad$ Similarly, $t_L \;\; 3.0 \;\; = 0.04 t_{L_\odot}$

13-11. Angular separation $\theta = \dfrac{S}{R} = \dfrac{\text{distance between binaries}}{\text{distance Earth}}$

$$\theta = \frac{100 \times 10^{6}\,km}{100c\bullet y} = \frac{10^{11}\,m}{100c\bullet y\ \ 3.15\times 10^{7}\,s/y} = 1.057\times 10^{-7}\,rad$$

$$\theta = 6.06\times 10^{-6}\,\text{degrees} = 1.68\times 10^{-9}\,\text{arcseconds}$$

13-12. Equation 13-18:

$${}^{56}_{26}Fe \rightarrow 13\,{}^{4}_{2}He + 4n. \quad m_{^{56}Fe} = 55.939395u, \quad m_{^{4}He} = 4.002603u, \quad m_n = 1.008665u.$$

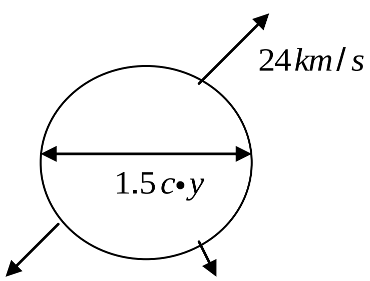
Energy required: $13m_{^{4}He} + 4m_n - m_{^{56}Fe} = 0.129104u.$

$1u = 931.49432 MeV/c^2 \quad \therefore 0.129104u \rightarrow 120 MeV$

Equation 13-19: ${}^{4}_{2}He \rightarrow 2\ {}^{1}H + 2n \qquad m_{^{1}H} = 1.007825$

Energy required: $2m_{^{1}H} + 2m_n - m_{^{4}He} = 0.020277u = 28.3 MeV$

13-13.



(a) $r = \dfrac{1.5c\bullet y}{2}$; assuming constant expansion rate,

$$\text{Age of Shell} = \frac{1.5c\bullet y/2}{2.4\times 10^{4}\,m/s} = 2.95\times 10^{11}\,s = 9400\,y$$

(b) $L_{star} = 12L_{\odot} \qquad T_{e\ star} = 1.4T_{e\odot}$

$$R \propto M \rightarrow R = \alpha M \qquad T_e \propto M^{1/2} \rightarrow T_e = \beta M^{1/2} \qquad L \propto M^4 \rightarrow L = \gamma M^4$$

$$\therefore\ \alpha = R_{\odot}/M_{\odot}, \qquad \beta = T_{e\odot}/M_{\odot}^{1/2}, \qquad \gamma = L_{\odot}/M_{\odot}^{4}$$

$$R_{star} = \frac{R_{\odot}}{M_{\odot}} m_{star}, \qquad T_{e\ star} = \frac{T_{e\odot}}{M_{\odot}^{1/2}} M_{star}^{1/2}, \qquad L_{star} = \frac{L_{\odot}}{M_{\odot}^{4}} M_{star}^{4}$$

Using either the T_e or L relations, $R_{star} = \dfrac{M_{star}}{M_{\odot}} R_{\odot} = \left(\dfrac{T_{e\ star}}{T_{e\odot}}\right)^2 = R_{\odot} = 1.4\ ^2 R_{\odot} = 1.96R_{\odot}$

or $R_{star} = \left(\dfrac{L_{star}}{L_{\odot}}\right)^{1/2} = 1.86R_{\odot}$

13-14. $R_S = 2GM / c^2$ (Equation 13-24)

(a) Sun $R_S = 2\times 6.67\times 10^{-11}\times 1.99\times 10^{30} / c^2 = 2.9\times 10^3 m \approx 3km$

(b) Jupiter $m_J = 318 m_E$ $R_S = 2.8m$

(c) Earth $R_S = 8.86\times 10^{-3} m$ $\approx 9mm!$

13-15. $M = 2M_\odot$

(a) (Equation 13-22) $R = 1.6\times 10^{14} M^{-1/3} = 1.6\times 10^{14}\ (2M_\odot)^{-1/3} = 1.01\times 10^4 m$

(b) $0.5 rev/s = \pi rad/s = \omega$

$\mathrm{K} = \frac{1}{2} I\omega^2$ where for a sphere

$$I = \frac{2}{5}MR^2 = \frac{1}{2}\left(\frac{2}{5}\times 2M_\odot \times (1.01\times 10^4)^2\right) = 8.0\times 10^{38} J$$

(c) $d\mathrm{K} = I\omega d\omega = I\omega^2\left(\frac{d\omega}{\omega}\right)$ where $\frac{d\omega}{\omega} = \frac{1}{10^8} / day$

$$= 2\mathrm{K}\left(\frac{d\omega}{\omega}\right) = \frac{2\ (8.0\times 10^{38} J)}{(10^8 d)\ (8.64\times 10^5 s/d)} = 1.85\times 10^{25} J/s \rightarrow L = 1.85\times 10^{25} W$$

13-16. Milky Way contains $\approx 10^{11}$ stars of average mass $M_\odot$, therefore the visible mass =

$1.99\times 10^{30}\times 10^{11} = 1.99\times 10^{41} kg \approx 10\%$ of total

(a) Mass of a central black hole = $9\times 1.99\times 10^{41} = 1.8\times 10^{42} kg$

(b) Its radius would be $R_S = 2GM / c^2$ (Equation 13-24).

$$R_S = 2\times 6.67\times 10^{-11}\times 1.8\times 10^{42} / c^2 = 2.6\times 10^{15} m \approx 17{,}000 AU$$

13-17. $v = 72{,}000 km/s.$

(a) $v = Hr \rightarrow r = \frac{v}{H} = \frac{72{,}000 km/s}{21.2 km/s / 10^6 c\bullet y} = 3.40\times 10^9 c\bullet y$

(b) From Equation 13-29 the maximum age of the galaxy is:

$1/H = 4.41\times 10^{17} s = 1.4\times 10^{10} y$

(Problem 13-17 continued)

$$1/H = r/v \rightarrow \Delta(1/H) = \Delta r/v \quad \therefore \quad \frac{\Delta(1/H)}{1/H} = \frac{\Delta r}{r} = 10\%$$

so the maximum age will also be in error by 10%.

13-18. The process that generated the increase could propagate across the core at a maximum rate of *c*, thus the core can be at most $1.5y \times 3.15 \times 10^7 s/y \times 3.0 \times 10^8 m/s = 1.42 \times 10^{16} m$ $= 9.45 \times 10^4 AU$ in diameter. The Milky Way diameter is $\approx 60{,}000 c \bullet y = 3.8 \times 10^9 AU$.

13-19. Combining Hubble's law (Equation 13-28) and the definition of the redshift (Equation 13-27) yields

$$z = \frac{H_o r}{c} = \frac{\lambda - \lambda_0}{\lambda_0} \quad \Rightarrow \quad \lambda = \left(\frac{H_o r}{c} + 1\right)\lambda_0$$

(a) $r = 5 \times 10^6\ \mathrm{c \cdot y}$

$$\lambda = \left(21.7 \frac{\mathrm{km}}{\mathrm{s \cdot 10^6\ c \cdot y}} \times \frac{5 \times 10^6\ \mathrm{c \cdot y}}{3 \times 10^8\ \mathrm{m/s}} + 1\right) 656.3\,\mathrm{nm}$$

$$\lambda = 656.5\,\mathrm{nm}$$

(b) $r = 50 \times 10^6\ \mathrm{c \cdot y}$

Similarly, $\lambda = 658.7\,\mathrm{nm}$

(c) $r = 500 \times 10^6\ \mathrm{c \cdot y}$

Similarly, $\lambda = 680.0\,\mathrm{nm}$

(d) $r = 5 \times 10^9\ \mathrm{c \cdot y}$

Similarly, $\lambda = 893.7\,\mathrm{nm}$

13-20. Equation 13-33: $\rho_c = \dfrac{3H^2}{8\pi G} = \dfrac{3}{8\pi\left(1/H\right)^2 G}$

$$\rho_c = \frac{3}{8\pi\left(1.5\times10^{10}\,y\times3.15\times10^{7}\,s/y\right)^2\left(6.67\times10^{-11}\,Nm^2/kg^2\right)} = 8.02\times10^{-27}\,kg/m^3$$

(This is about 5 hydrogen atoms/m^3 !)

13-21. Present size $\approx 10^{10}c\bullet y = S_p \approx \dfrac{1}{T} \rightarrow S_p = \alpha\dfrac{1}{T}$ with $T = 2.7K \quad \therefore \quad \alpha = 2.7\times10^{10}c\bullet yK$

(a) 2000 years ago, $S = S_p$

(b) 10^6 years ago, $S = S_p$

(c) 10 seconds after the Big Bang, $S \approx \left(2.7\times10^{10}c\bullet yK\right) / \left(10^9 K\right) \left(2.7\times10^{-9}S_p\right) \approx 25c\bullet y$

(d) 1 second after the Big Bang, $S \approx \left(2.7\times10^{10}c\bullet yK\right) / \left(5\times10^9 K\right) \left(5.4\times10^{-10}S_p\right) \approx 5c\bullet y$

(e) 10^{-6} seconds after the Big Bang,

$$S \approx \left(2.7\times10^{10}c\bullet yK\right) / \left(5\times10^{12} K\right) \left(5.4\times10^{-13}S_p\right) \approx 0.005c\bullet y \approx 6.4\times10^4\,AU$$

13-22. $\rho\left(\text{Planck time}\right) = \dfrac{m_{pl}}{\ell_{pl}^3} = \dfrac{5.5\times10^{-8}\,kg}{\left(10^{-35}\right)^3 m^3} = 5.5\times10^{97}\,kg/m^3$

$\rho\left(\text{proton}\right) = \dfrac{1.67\times10^{-27}\,kg}{\left(10^{-15}\right)^3 m^3} = 1.67\times10^{18}\,kg/m^3$

$\rho\left(\text{osmium}\right) = 2.45\times10^4\,kg/m^3$

13-23. Wien's law (Equation 3-11): $\lambda_{\max} = \dfrac{2.898mm\bullet K}{T} = \dfrac{2.898mm\bullet K}{2.728K} = 1.062mm$

(this is in the microwave region of the EM spectrum)

13-24. Muon rest energy $= 208m_e = 106MeV/c^2$. The universe cooled to this energy (average) at about $10^{-3}s$ (see Figure 13-34). 2.728K corresponds to average energy = 10^{-3} *eV*.

Therefore, $m = \dfrac{10^{-3}eV\times1.6\times10^{-19}\,J/eV}{c^2} = 1.8\times10^{-39}\,kg$

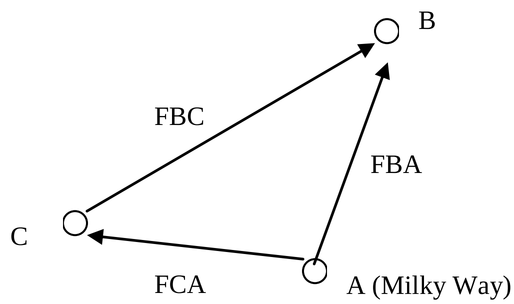
13-25. $\rho_0 = M / \; 4/3 \; \pi r^3 \; t_0 \qquad \rho \; t \; = M / \; 4/3 \; \pi r^3 \; t$

$r \; t \; = R \; t \; r \; t_0$ (Equation 13-37) \

$$\therefore \quad \rho_0 = \frac{M}{4/3 \; \pi \left[r \; t \; / R \; t \; \right]^3} = R^3 \; t \; \frac{M}{4/3 \; \pi r^2 \; t}$$

$$\rho_0 = R^3 \; t \; \rho \; t$$

13-26.



If Hubble's law applies in A, then

$v_{BA} = Hr_{BA}, \quad v_{CA} = Hr_{CA}$.

From mechanics,

$v_{BC} = v_{BA} - v_{CA} = H \; r_{BA} - r_{CA} \; = Hr_{BC}$

and Hubble's lab applies in C, as well, and by extension in all other galaxies.

13-27. At a distance *r* from the Sun the magnitude of the gravitational force acting on a dust article of radius *a* is: $F_{grav} = \dfrac{GM_\odot m}{r^2}$ where $m = (4/3)\pi a^3 \rho$. The force acting on the particle due to the Sun's radiation pressure at *r* is given by: (See Equation RP-9.)

$F_{rad} = \pi a^2 \times P_{rad} = \pi a^2 \times \dfrac{U}{3}$ where πa^2 is the cross sectional area of the particle and *U* is the energy density of solar radiation at *r*. *U* is given by: (See Equation 3-6.)

$$U = \frac{4}{c} R = \frac{4}{c} \times \frac{L_\odot}{4\pi r^2}$$

Therefore, $F_{rad} = \pi a^2 \times \dfrac{1}{3} \times \dfrac{4L_\odot}{4\pi r^2 c}$

The minimum value of *a* is obtained from the condition that $F_{grav} > F_{rad}$:

(Problem 13-27 continued)

$$\frac{GM_{\odot}m}{r^2} > \pi a^2 \times \frac{1}{3} \times \frac{4L_{\odot}}{4\pi r^2 c}$$

$$\frac{GM_{\odot}(4/3)\pi a^3 \rho}{r^2} > \pi a^2 \times \frac{1}{3} \times \frac{4L_{\odot}}{4\pi r^2 c}$$

Simplifying this expression yields: $a > \dfrac{L_{\odot}}{4\pi c G M_{\odot}\rho}$

$$a > \frac{3.84\times10^{26}\ \mathrm{W}}{4\pi(3.00\times10^{8}\ \mathrm{m/s})(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(1.99\times10^{30}\ \mathrm{kg})(5500\ \mathrm{kg/m^3})}$$

$$a > 1.40\times10^{-7}\ \mathrm{m} \quad \text{or} \quad 1.40\times10^{-5}\ \mathrm{cm}$$

Note that (i) a is very small and (ii) the magnitude of a is independent of r.

13-28. (Equation 13-31) $R_{obs} = R_{emit}(1+Z) \rightarrow r_0 = r(1+Z)$

$\rho(Z) = M/((4/3)\pi r^3) \qquad \rho_0 = M/((4/3)\pi r_0^3)$

Substituting for r_0 in the ρ_0 equation:

$$\rho_0 = M/((4/3)\pi[r(1+Z)]^3) = M/((4/3)\pi r^3(1+Z)^3)$$

$$\rho_0 = \rho(Z)/(1+Z)^3 \quad \text{or} \quad \rho(Z) = \rho_0(1+Z)^3$$

13-29. (a) H available for fusion = $M_{\odot}\times0.75\times0.13 = 2.0\times10^{30}\,kg\times0.75\times0.13 = 2.0\times10^{29}\,kg$

(b) Lifetime of H fuel = $\dfrac{2.0\times10^{29}\,kg}{6.00\times10^{1}\,kg/s} = 3.3\times10^{17}\,s$

$$= 3.3\times10^{17}\,s/3.15\times10^{7}\,s/y = 1.03\times10^{10}\,y$$

(c) Start being concerned in $1.03\times10^{10}\,y - 0.46\times10^{10}\,y = 5.7\times10^{9}\,y$

13-30. SN1987A is the Large Magellanic cloud, which is 170,000*c*•*y* away; therefore

(a) supernova occurred 170,000 years BP.

(Problem 13-30 continued)

(b) $E = K + m_o c^2 = \dfrac{m_o c^2}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

$\text{K} = 10^9 eV,\qquad m_o^2 = 9.28\times10^8 eV \rightarrow 10^9 + 9.38\times10^8 = \dfrac{9.38\times10^8}{\sqrt{1-\dfrac{v^2}{c^2}}}$ or $v = 0.875c$

Therefore, the distance protons have traveled in 170,000y

$= v \times 170{,}000y = 149{,}000c\bullet y.$ No, they are not here yet.

13-31. $M_\odot = 1.99\times10^{30} kg.$

(a) When first formed, mass of $H = 0.7M_\odot$, $m\ {}^1H = 1.007825u \times 1.66\times10^{-27} kg/u,$ thus

number of H atoms $= \dfrac{0.7\times M_\odot}{1.007825u\times1.66\times10^{-27} kg/u} = 8.33\times10^{56}$

(b) If all $H \rightarrow He$; $4\,{}^1H \rightarrow {}^4He + 26.72eV$. The number of He atoms

produced = $\dfrac{8.33\times10^6}{4}$.

Total energy produced = $\dfrac{8.33\times10^6}{4}\times 26.72MeV = 5.56\times10^{57} MeV = 8.89\times10^{44} J$

(c) 23% of max possible = $0.23\times8.89\times10^{44} J$

$t_L = \dfrac{0.23\times8.89\times10^{44}}{L_\odot} = 5.53\times10^{17} s = 1.7\times10^{10} y \qquad L_\odot = 3.85\times10^{26} W$

13-32. (a) $F = Gm_1m_2/r^2 = a_c m_2 = \left(v^2/r\right)m_2$

$v^2/r = Gm_1/r^2$ and orbital frequency $f = v/2\pi r$

Substituting for f and noting that the period $T = 1/f$, $\quad 4\pi^2 f^2 = Gm_1/r^3$

or, $T^2 = 4\pi^2 r^3/Gm_1$, which is Kepler's third law.

(b) Rearranging Kepler's third law in part (a),

(Problem 13-32 continued)

$$m_E = 4\pi^2 r_{moon}^3 / GT^2 = \frac{4\pi^2 \ 3.84\times 10^8 m \ ^3}{6.67\times 10^{-11} Nm^2/kg^2 \quad 27.3d \times 8.64\times 10^4 s/d \ ^2}$$

$$= 6.02\times 10^{24} kg$$

(c) $T = 2\pi \left[\frac{r_{sh}^3}{Gm_E}\right]^{1/2} = 2\pi\left[\frac{6.67\times 10^6 \ ^3}{6.67\times 10^{-11} \quad 6.02\times 10^{24}}\right]^{1/2} = 5.44\times 10^3 s = 1.5h$

(d) $m_{comb} = \dfrac{4\pi^2 \ 1.97\times 10^7 \ ^3}{6.67\times 10^{-11} \quad 6.46d\times 3.1\times 10^4 s/d \ ^2} = 1.48\times 10^{22} kg$

13-33. (a) $T = 12d \rightarrow \omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{12\times 24\times 3600} = 6.06\times 10^{-6}/s \ 2$

(b) For $m_1 > m_2$: reduced mass $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$, then $\mu \dfrac{v^2}{r} = G\dfrac{m_1 m_2}{r}$ and

$$\frac{m_1 m_2}{m_1 + m_2}\left(\frac{\omega^2 r^2}{r}\right) = \frac{Gm_1 m_2}{r^2} \quad \text{or} \quad m_1 + m_2 = \frac{\omega^2 r^3}{G}$$

(c) $v_1 = r_1\omega_1$, $v_2 = r_2\omega_2$, and $\omega_1 = \omega_2$ from the graph $v_1 = 200 km/s$ and $v_2 = 100 km/s$

$\therefore \quad r_1 = \dfrac{200\times 10^3 m/s}{6.06\times 10^{-6}/s} = 3.3\times 10^{10} m$ and, similarly, $r_2 = 1.6\times 10^{10} m$

$\therefore \quad r = r_1 = r_2 = 4.9\times 10^{10} m$

Assuming circular orbits, $\dfrac{m_1 v_1^2}{r_1} = \dfrac{m_2 v_2^2}{r_2}$ and $m_1 = \dfrac{r_1 v_2^2}{r_2 v_1^2} m_2$ Substituting yields,

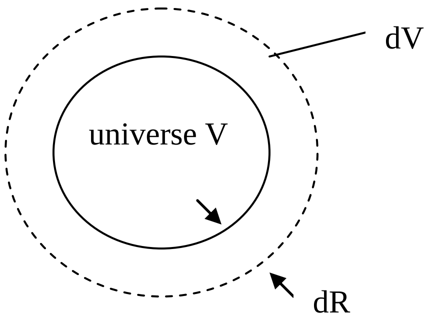
$m_1 = 6.63\times 10^{30} kg$ and $m_2 = 1.37\times 10^{31} kg$

13-34. $E = \frac{1}{2}mv^2 + \left(-GmM_{\odot}/r\right)$ $\qquad F_G = GM_{\odot}m/r^2 = mv^2/r$

or $GM_{\odot}m/r = mv^2 \rightarrow \frac{1}{2}mv^2 = \frac{1}{2}GM_{\odot}m/r$

$$\therefore \quad E = \frac{1}{2}\frac{GM_{\odot}m}{r} + \left(-\frac{GM_{\odot}m}{r}\right) = \frac{1}{2}\left(-\frac{GM_{\odot}m}{r}\right)$$

13-35.



$H = \frac{20 km/s}{10^6 c\bullet y}$

Current average density = 1H $atom/m^3$

$V = \frac{4}{3}\pi R^3 \rightarrow dV = 4\pi R^2 dR$

The current expansion rate at R is:

$$v = HR = \frac{20 km/s}{10^6 c\bullet y}\times 10^{10} c\bullet y = 20\times 10^4 km/s = 20\times 10^7 m/s$$

$$\therefore \quad dR = 20\times 10^7 m/s \times 3.16\times 10^7 s/y \times \frac{10^6 y}{10^6 y}$$

$$dV = 4\pi R^2 dR = 4\pi\times \left(10^{10}\right)^2 \left(9.45\times 10^{15} m/c\bullet y\right)^2 \times 20\times 10^7 m/s \times 3.16\times 10^7 s/y \times \frac{10^6 y}{10^6 y}$$

$$= \frac{7.07\times 10^{74} m^3}{10^6 c\bullet y} = \frac{\text{\# of H atoms}}{10^6 c\bullet y} \text{ to be added}$$

Current volume $V = \frac{4}{3}\pi \left(10^{10}\right)^3 = 8.4\times 10^{77} m^3$

$\therefore$ "new" H atoms = $\frac{7.07\times 10^{74} atoms/10^6 c\bullet y}{8.4\times 10^{77} m^3} \approx 0.001$ "new" H $atoms/m^3\bullet 10^6 c\bullet y$; no

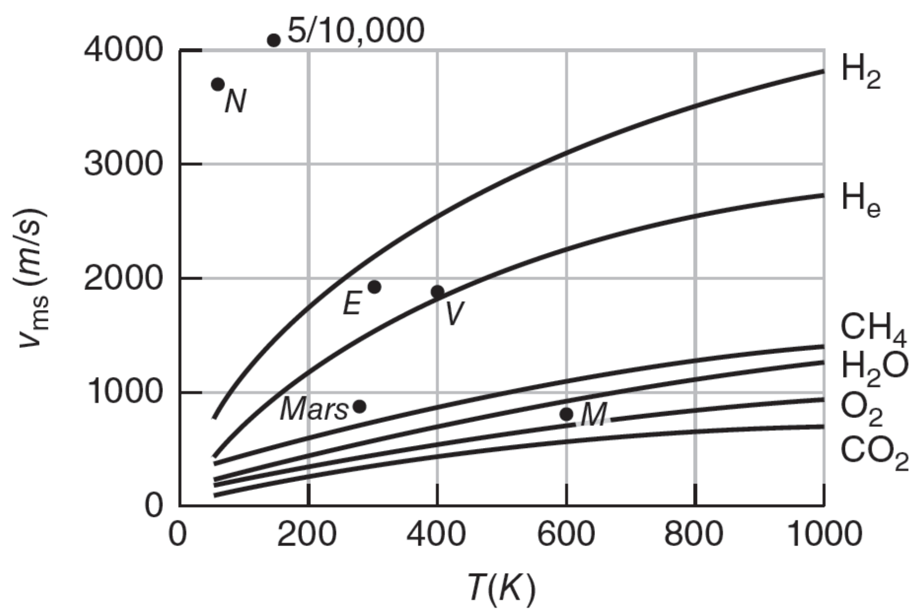
13-36. (a) Equation 8-12: $v_{rms} = \sqrt{3RT/M}$ is used to compute v_{rms} vs T for each gas ® = gas constant.

Gas	M ($\times 10^{-3} kg$)	$\sqrt{3R/M}$	v_{rms} (m/s) at T = :				
			50K	200K	500K	750K	1000K
H_2O	18	37.2	263	526	832	1020	1180
CO_2	44	23.8	168	337	532	652	753
O_2	32	27.9	197	395	624	764	883
CH_4	16	39.5	279	558	883	1080	1250
H_2	2	111.6	789	1580	2500	3060	3530
He	4	78.9	558	1770	1770	2160	2500

The escape velocities $v_{sc} = \sqrt{2gR} = \sqrt{2GM/R}$, where the planet masses M and radii R, are given in table below.

Planet	Earth	Venus	Mercury	Jupiter	Neptune	Mars
v_{esc} (*km/s*)	11.2	10.3	4.5	60.2	23.4	5.1
$v_{esc}/6$ (*m/s*)	1870	1720	750	10,000	3900	850

On the graph of v_{rms} vs T the $v_{esc}/6$ points are shown for each planet.



(Problem 13-36 continued)

(b) $v_{esc} = \sqrt{2GM/R}$ $\quad v_{Pl} = \sqrt{2GM_{Pl}/R_{Pl}}$ $\quad v_E = \sqrt{2GM_E/R_E}$

$$\therefore \quad \frac{v_{Pl}}{v_E} = \sqrt{\frac{M_{Pl}/R_{Pl}}{M_E/R_E}} = \sqrt{\frac{\alpha M_E/\beta R_E}{M_E/R_E}} = \sqrt{\frac{\alpha}{\beta}} \rightarrow v_{Pl} = \sqrt{\frac{\alpha}{\beta}}\, v_E = \sqrt{\frac{M_{Pl}/M_E}{R_{Pl}/R_E}}$$

(c) All six gases will still be in Jupiter's atmosphere and Netune's atmosphere, because v_{esc} for these is so high. H_2 will be gone from Earth; H_2 and probably He will be gone from Venus; H_2 and He are gone from Mars. Only CO_2 and probably O_2 remain in Mercury's atmosphere.

13-37. (a) α Centauri $\quad d$ in pc $= \dfrac{\text{Earth's orbit radius (in AU)}}{\sin\theta_p}$

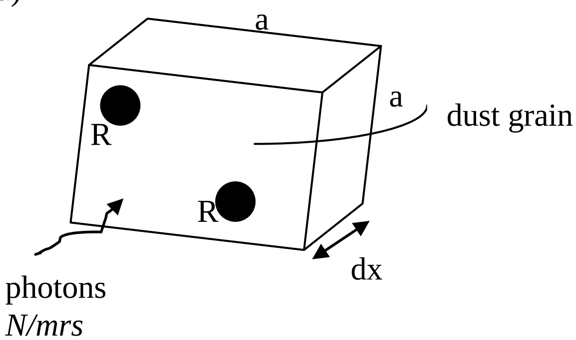
$$d = \frac{1AU}{\sin\ 0.742''} = 2.78\times10^5\, pc = 9.06\times10^5 c\bullet y$$

(b) Procyon $\quad d = \dfrac{1AU}{\sin\ 0.0286''} = 7.21\times10^5\, pc = 2.35\times10^6 c\bullet y$

13-38. Earth is currently in thermal equilibrium with surface temperature $\approx 300K$. Assuming Earth radiates as a blackbody $I = \sigma T^4$ and $I = \sigma\ 300\ ^4 = 459W/m^2$. The solar constant is $f = 1.36\times10^3 W/m^2$ currently, so Earth absorbs 459/1360 = 0.338 of incident solar energy. When $L_\odot \rightarrow 10^2 L_\odot$ then $f \rightarrow 10^2 f$. If the Earth remains in equilibrium.

$I = 0.338\times1.36\times10^3\times10^2 = \sigma T^4$ or $T = 994K = 676\ °C$ sufficient to boil the oceans away. However, the v_{rms} for H_2O molecules at $994K$ is $v_{rms} = \sqrt{3RT/M} = \sqrt{\dfrac{3\times8.31\times949}{18\times10^{-3}}} = 1146m/s = 1.15km/s$. The v_{esc} = $11.2km/s$ (see solution to problem 14-26). Because $v_{rms} \approx 0.1\ v_{esc}$, the H_2O will remain in the atmosphere.

13-39. (a)



n = *grains/cm*^{3}

total scattering area = $\pi R^2 n a^2\, dx$

which is $\dfrac{\pi R^2 a^2 n\, dx}{a^2} = \pi R^2 n\, dx$ of the

total area = fraction scattered = dN/N

$$\int_{N_0}^{N} \frac{dN}{N} = -n\pi R^2 \int_0^d dx \quad \text{or} \quad N = N_0 e^{-n\pi R^2 d}$$

From those photons that scatter at $x = 0$ (N_0), those that have not scattered again after traveling some distance $x = L$ is $N_L = N_0 e^{-n\pi R^2 L}$. The average value of L (= d_0) is given by:

$$d_0 = \frac{\int_0^\infty L \frac{dN_L}{dL}\, dL}{\int_0^\infty \frac{dN_L}{dL}\, dL} = \frac{1}{n\pi R^2} \qquad \left(\text{Note: } \frac{dN_L}{dL} = -n\pi R^2 N_0 e^{-n\pi R^2 L}\right)$$

(b) $I = I_0 e^{-d/d_0}$ near the Sun $d_0 \approx 3000 c \bullet y$ $\qquad R = 10^{-5} cm$

$$\therefore \quad 3000 c\bullet y \times 9.45 \times 10^{17}\, cm/c\bullet y = \frac{1}{n\pi \left(10^{-5}\right)^2} \qquad \therefore \quad n = 1.1 \times 10^{-12}/cm^3$$

(c) $\rho_{grains} = 2 gm/cm^3$

$$\therefore \quad \frac{m_{grains}}{cm^3 \text{ of space}} = 2 \times \frac{4}{3}\pi \left(10^{-5}\right)^3 \times 1.1 \times 10^{-12}/cm^3 = 9.41 \times 10^{-27}\, gm/cm^3$$

$$\therefore \quad \text{mass in } 300 c\bullet y = \frac{9.41 \times 10^{-27}\, gm/cm^3}{M_\odot} \times \left(9.45 \times 10^{17}\, cm/c\bullet y\right)^3 \times 300$$

$$= 0.0012 \qquad \approx 0.1\% M_\odot$$

13-40. $56\,{}_1^1H \rightarrow 14\,{}_2^{14}He \rightarrow {}_{26}^{56}Fe + 2\beta^+ \;\; +2e^-$ $\qquad 2\beta+ \;\; +2e^- \;\; = 2.04 MeV/c^2$

$$14\,m_{{}^4He} \;= 14 \times 4.002603 - m_{{}^{56}Fe} \;= 55.939395u = \Delta$$
$$= 56.036442u$$

Net energy difference (release) = $\begin{cases} 14 \times 26.72 MeV \\ \quad 2.04 MeV \\ \quad 90.40 MeV \quad = \Delta \\ \hline 466.5 MeV \end{cases}$

$2\,{}_{26}^{56}Fe \rightarrow {}_{48}^{112}Cd + 4\beta^+ \;\; +4e^-$ $\qquad 4\beta+ \;\; +4e^- \;\; = 4.08 MeV/c^2$

$2m \;\; {}^{56}Fe \;\; = 2 \times 55.939395u$ $\qquad m \;\; {}^{112}Cd \;\; = 111.902762u$

Net energy required = $2m_{{}^{56}Fe} - m_{{}^{112}Cd} \;\; = -0.023972u + 4.08 MeV = -18.25 MeV$

13-41. (a) $dt = \dfrac{1.024 \times 10^4 \pi^2 G^2 M\, dM}{hc^4}$

rearranging, the mass rate of change is $\dfrac{dM}{dt} = \dfrac{hc^4}{1.024 \times 10^{24} \;\; \pi^2 G^2 M}$

Clearly, the larger the mass *M*, the lower the rate at which the black hole loses mass.

(b) $t = \dfrac{1.024 \times 10^4 \;\; \pi^2 \;\; 6.62 \times 10^{-11} \;\;^2 \;\; 2.0 \times 10^{30} \;\;^2}{hc^4}$

$t = 3.35 \times 10^{44} s = 1.06 \times 10^{37} y$ far larger than the present age of the universe.

(c) $t = \dfrac{1.024 \times 10^4 \;\; \pi^4 \;\; 6.67 \times 10^{-11} \;\;^2 \;\; 2.0 \times 10^{30} \times 10^{12} \;\;^2}{6.63 \times 10^{-34} \;\; 3.00 \times 10^8 \;\;^4}$

$t = 3.35 \times 10^{68} s = 1.06 \times 10^{61} y$